Deep-Space Navigation With Differenced Data Types Part II: Differenced Doppler Information Content

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This article is the second in a series investigating the use of differenced range and Doppler data types for deep-space navigation. Quantitative analysis of X-band (8.4-GHz) differenced Doppler data recently acquired from the Magellan spacecraft has indicated that this data type is much more accurate than was previously thought. While differenced Doppler will be used by Magellan to support navigation during the Venus mapping mission, it might also be useful for navigation in heliocentric space flight as well. This article presents a brief investigation of the ability of differenced Doppler to determine the angular coordinates of spacecraft at interplanetary distances. A simple, analytic error covariance analysis is developed and used to compute approximate angular accuracy statistics, taking into account the effects of station frequency offset calibration errors on the data. The results indicate that it is theoretically possible to determine spacecraft right ascension to an accuracy of 30 to 40 nrad using about 4 hours of data, while declination may be determined to accuracies of 80 to 300 nrad, except for declinations within 5 deg of the Earth's equator, where declination accuracy is degraded severely due to unfavorable measurement geometry. If these accuracies could be delivered operationally, differenced Doppler, like differenced range, may be able to serve as a medium-accuracy supplementary or back-up data type to Delta-Differenced One-Way Range (ΔDOR).

I. Introduction

Delta-Differenced One-Way Range (ΔDOR) angular tracking measurements for interplanetary spacecraft navigation currently utilize differential tracking of a spacecraft and a nearby stable radio source (normally a quasar), in order to reduce or eliminate the effects of error sources such as station clock offsets and transmission media calibration errors. Differenced (two-way minus three-way) Doppler is another data type that can measure spacecraft angular coordinates and their rates of change. If the information content of differenced Doppler is sufficient to deliver angular accuracies of 50 to 100 nrad, it may be capable of serving as a supplementary or backup data type to ΔDOR, which is very accurate but operationally cumbersome.

Differenced Doppler is a relatively simple data type that can be acquired without interruption of spacecraft telemetry, returned to the Jet Propulsion Laboratory (JPL)

Space Flight Operations Facility (SFOF), and processed for navigation use in near-real time. No quasar tracking is needed to form differenced Doppler measurements, as is required for ΔDOR. The current Deep Space Network (DSN) ΔDOR system has been shown to be theoretically capable of delivering 20- to 25-nrad angular accuracies [5], but the data require a lengthy processing procedure, which can take from 12 to 24 hours, before completed ΔDOR measurements can be made available for navigation data processing.

Differenced Doppler data utilizing X-band (8.4 GHz) for both the uplink and downlink frequencies have been acquired from the Magellan spacecraft throughout its flight from Earth to Venus. Quantitative analysis of this data has conservatively indicated a measurement noise level (1σ) of 0.08–0.13 mm/sec,[1] as opposed to a preflight analysis that predicted a measurement noise level (1σ) of about 0.32 mm/sec.[2] Much of the previous work [1] investigating navigation applications of differenced Doppler focused on the use of differenced Doppler in planetary orbiter missions, especially Magellan.[3] During earlier theoretical development of radio interferometry and differenced data schemes for navigation, differenced Doppler was evaluated as a candidate data type for interplanetary spacecraft navigation, although it has never been used operationally in this capacity [2,3]. This is largely because differenced Doppler data taken at S-band (2.3-GHz) frequencies are not accurate enough to really be of much use (Magellan is the first deep-space probe to use X-band uplink and downlink frequencies in addition to S-band). Since the Magellan X-band differenced Doppler data have been found to be more than four times more accurate than S-band differenced Doppler,[4] it would be interesting once again to consider the deep-space navigation accuracy potential of this data type.

This article presents a brief investigation of the theoretical ability of X-band (8.4-GHz) differenced Doppler to determine the geocentric angular coordinates of a distant spacecraft. A simple error covariance analysis of a differenced Doppler tracking pass is developed and used to compute approximate angular navigation accuracy statistics for one or two passes of data taken from DSN intercontinental baselines. The formulation of the error analysis is identical to the weighted least-squares estimator formulation used in the first article in this series, which investigated differenced range information content [4]. A bias parameter representing the frequency offset calibration error between the two stations participating in the tracking pass is included in the analysis, since this is normally the largest systematic error source in differenced Doppler data. This parameter is estimated along with the spacecraft angular coordinates, in order to get some indication of how accurately frequency offset calibration errors can be estimated from the data signature over a single pass.

II. Error Covariance Analysis

Figure 1 illustrates the tracking configuration for both differenced Doppler and range, utilizing two-way data and three-way data.[5] The differenced Doppler observable consists simply of the difference of the downlink range rates between the two participating DSN stations. The differenced range observable, denoted here as $\Delta\rho$, has been derived previously [3] and can be written as

$$\Delta\rho = r_B \cos\delta \cos H_B + z_B \sin\delta \tag{1}$$

where

$r_B =$ baseline component normal to the spin axis of Earth

$z_B =$ baseline component parallel to the spin axis of Earth

$H_B =$ baseline hour angle, $\alpha_B - \alpha$

$\alpha =$ spacecraft right ascension

$\alpha_B =$ baseline right ascension, $\alpha_g + \lambda_B$

$\alpha_g =$ right ascension of the Greenwich meridian

$\lambda_B =$ baseline longitude

$\delta =$ spacecraft declination

The differenced Doppler (range-rate) observable is simply the derivative of Eq. (1)

[1] G. R. Kronschnabl, "Magellan Differenced Doppler Data Quality Evaluation," JPL Interoffice Memorandum 3140-GRK-90-011 (internal document), Jet Propulsion Laboratory, Pasadena, California, May 24, 1990.

[2] J. S. Border, "An Error Analysis for Magellan Differenced (F2-F3) Doppler Measurements," JPL Interoffice Memorandum 335.3-87-158 (internal document), Jet Propulsion Laboratory, Pasadena, California, October 30, 1990.

[3] Several studies of differenced Doppler navigation performance have been conducted in the past four years by E. K. Ussery, R. K. Russell, J. Ellis, J. B. McNamee, and others, which have been documented in JPL Interoffice Memoranda.

[4] G. R. Kronschnabl, op. cit.

[5] This figure also appears in the companion article, "Deep-Space Navigation With Differenced Data Types Part I: Differenced Range Information Content," in this issue.

$$\Delta\dot{\rho} = -\omega r_B \cos\delta \sin H_B \tag{2}$$

The baseline vector components used in formulating Eqs. (1) and (2) are expressed as a function of cylindrical coordinates, which can be expressed in terms of the cylindrical coordinates of the two stations comprising the baseline. Figure 2 illustrates the station and spacecraft position coordinate definitions used in this analysis.[6] Using the station coordinates shown in Fig. 2, the baseline coordinates are

$$r_B = \left\{ \left(r_{s_1} + r_{s_2}\right)^2 - 2r_{s_1}r_{s_2}\left[1 + \cos(\lambda_1 - \lambda_2)\right] \right\}^{1/2}$$

$$z_B = z_{s_1} - z_{s_2} \tag{3}$$

$$\lambda_B = \tan^{-1}\left(\frac{r_{s_1}\sin\lambda_1 - r_{s_2}\sin\lambda}{r_{s_1}\cos\lambda_1 - r_{s_2}\cos\lambda_2} \right)$$

where

$r_{s_1}, r_{s_2} =$ station distances from the spin axis of Earth

$z_{s_1}, z_{s_2} =$ station distances from the equator of Earth

$\lambda_1, \lambda_2 =$ station longitudes

The differenced Doppler observable model, Eq. (2), can be used to develop analytic expressions for the error covariance matrix associated with weighted least-squares estimates of the spacecraft coordinates δ and α. The measurement equation assumed for the estimation process is

$$z = \Delta\dot{\rho} + f + \nu \tag{4}$$

where

$z =$ observed value of differenced Doppler measurement

$\Delta\dot{\rho} =$ actual differenced Doppler value

$f =$ station frequency offset calibration error

$\nu =$ random variable representing measurement noise

[6] This figure also appears in the article, "Deep-Space Navigation With Differenced Data Types Part I: Differenced Range Information Content," in this issue.

A series of N independent measurements can be combined into a linear matrix equation relating small perturbations of the measurements from their predicted values to small perturbations of the spacecraft coordinates and the frequency offset from their a priori values

$$\Delta\vec{z} = A\Delta\vec{x} + \vec{\nu} \tag{5}$$

where

$\Delta\vec{z} =$ vector of measurement residuals (actual−predicted)

$\Delta\vec{x} = [\Delta\delta, \Delta\alpha, \Delta b]^T$

$\vec{\nu} = [\nu_1, \nu_2, \cdots, \nu_N]^T$

The A matrix, or differential correction matrix, contains the partial derivatives of the measurements with respect to the estimated parameters:

$$A = \begin{bmatrix} (\partial z_1/\partial\vec{x}) \\ (\partial z_2/\partial\vec{x}) \\ \cdot \\ \cdot \\ (\partial z_N/\partial\vec{x}) \end{bmatrix} \tag{6}$$

In this analysis, the spacecraft angular coordinates δ and α are assumed to be constant over the duration of the tracking pass, a reasonable assumption for spacecraft at interplanetary distances and the time periods (about 12 hours) of interest here. Using Eq. (6), the information array J associated with the estimates of δ, α, and f derived from a series of differenced Doppler measurements, can be written as

$$J = A^T A\cdot(1/\sigma^2_{\Delta\dot{\rho}}) = (N/\sigma^2_{\Delta\dot{\rho}})\sum_{i=1}^{N}(\partial z_i/\partial\vec{x})^T(\partial z_i/\partial\vec{x}) \tag{7}$$

where

$\sigma^2_{\Delta\dot{\rho}} =$ differenced Doppler measurement error variance

The error covariance matrix Λ for the estimated parameters can be expressed as a simple function of the information array

$$\Lambda = E\left[(\vec{x} - \hat{\vec{x}})(\vec{x} - \hat{\vec{x}})^T\right] = [\Lambda_o^{-1} + J]^{-1} \tag{8}$$

In Eq. (8), Λ_o is the a priori error covariance matrix. It has been shown previously [6] that the information array can be approximated by an integral expression if the time interval between measurements is constant and is also small, relative to the total tracking pass duration. Equation (7) then becomes

$$J \approx \frac{1}{\sigma^2_{\Delta\dot{\rho}}\Delta t}\int_{t_1}^{t_2}(\partial z/\partial\vec{x})^T(\partial z/\partial\vec{x})dt \tag{9}$$

where

$t_1, t_2 =$ tracking pass start and stop times, respectively

$\Delta t =$ time interval between measurements

The integration specified in Eq. (9) can be carried out with a change of variable, by expressing the baseline hour angle as

$$H_B = \omega(t - t_o) \tag{10}$$

where

$\omega t_o = \alpha$

$\omega =$ Earth rotation rate

The variable of integration and its associated limits, assuming a symmetric tracking pass about $H_B = 90$ deg,[7] become

$$dt = dH_B/\omega \tag{11}$$

$$H_{B_1}, H_{B_2} = \frac{\pi}{2} - \Psi, \frac{\pi}{2} + \Psi \tag{12}$$

where $\Psi \triangleq$ tracking pass half-width.

Equation (9) can be integrated using Eqs. (2), (4), (11), and (12). The partial derivatives needed to carry out the integration are obtained from the differenced Doppler measurement model, Eq. (5). They are

$$\partial z/\partial(\delta, \alpha, f) = [\omega r_B \sin H_B, \omega r_B \cos\delta\cos H_B, 1] \tag{13}$$

The completed differenced Doppler information array is

$$J = \begin{bmatrix} (\omega r_B \sin\delta)^2(\Psi + 1/2\sin 2\Psi) & 0 & 2\omega r_B \sin\delta\sin\Psi \\ 0 & (\omega r_B\cos\delta)^2(\Psi - 1/2\sin 2\Psi) & 0 \\ 2\omega r_B\sin\delta\sin\Psi & 0 & 2\Psi \end{bmatrix}\left(\frac{1}{\omega\sigma^2_{\Delta\dot{\rho}}\Delta t}\right) \tag{14}$$

To complete the development of the error covariance matrix, Eq. (8), the a priori error covariance matrix Λ_o must be specified. In this analysis, it is assumed that some a priori estimate of the station frequency offset f is available, representing the result of calibration procedures performed prior to the tracking pass. The inverse of the a priori error covariance matrix can then be written as

$$\Lambda_o^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \frac{1}{\sigma_f^2} \end{bmatrix} \tag{15}$$

where σ_f is the one-sigma uncertainty of the a priori frequency offset calibration.

Substituting Eq. (15) into the error covariance matrix expression, Eq. (8), along with the information-array elements from Eq. (14), the diagonal elements of the error covariance matrix (the variances associated with the estimates of spacecraft declination, right ascension, and frequency offset obtained from the differenced Doppler pass) are found to be

$$\sigma_\delta^2 = \left(\frac{\sigma^2_{\Delta\dot{\rho}}\Delta t}{\omega r_B^2 \sin^2\delta}\right)$$

$$\times \frac{2\Psi + \omega\Delta t(\sigma_{\Delta\dot{\rho}}/\sigma_f)^2}{(\Psi + 1/2\sin 2\Psi)[2\Psi + \omega\Delta t(\sigma_{\Delta\dot{\rho}}/\sigma_f)^2] - 4\sin^2\Psi} \tag{16}$$

[7] The companion article, "Deep-Space Navigation With Differenced Data Types Part I: Differenced Range Information Content," appearing in this issue, describes the reasoning behind this choice.

$$\sigma_\alpha^2 = \left(\frac{\sigma_{\Delta\dot{\rho}}^2 \Delta t}{\omega r_B^2 \cos^2\delta}\right) \frac{1}{\Psi - 1/2 \sin 2\Psi} \tag{17}$$

$$\sigma_f^2 = (\sigma_{\Delta\dot{\rho}}^2 \omega \Delta t) \times \frac{\Psi + 1/2\sin 2\Psi}{(\Psi + 1/2\sin 2\Psi)[2\Psi + \omega\Delta t(\sigma_{\Delta\dot{\rho}}/\sigma_f)^2] - 4\sin^2\Psi} \tag{18}$$

Inspection of Eq. (16) reveals that the uncertainty of the declination estimate is proportional to $(1/\sin\delta)$, eventually becoming infinite when $\delta = 0$. This aspect of differenced Doppler navigation accuracy behavior has been demonstrated previously [3], and it represents another reason that this data type has not seen actual use, since the declination accuracy in the near-zero declination regime will always experience significant degradation, regardless of the accuracy of the data.

III. Navigation Accuracy Results

In order to compute estimation accuracies for δ, α, and f using Eqs. (16)–(18), suitable values must be chosen for the differenced Doppler measurement accuracy, data rate, and a priori frequency offset uncertainty. Based on the quantitative analysis of the Magellan X-band (8.4-GHz) data mentioned earlier, the differenced Doppler measurement uncertainty was chosen to be 0.1 mm/sec, with a data rate (Δt) of one point every 60 sec, the rate at which Doppler data are normally generated by the DSN Doppler system. Frequency offsets between the different DSN stations are calibrated using radio metric data obtained from the Global Positioning System (GPS), usually to an accuracy of 0.02 mm/sec or better.[8] A value of 0.015 mm/sec was chosen to represent current a priori frequency offset calibration uncertainty; to investigate possible improvements in differenced Doppler navigation performance in the future, a value of 0.003 mm/sec (a factor of 5 improvement) was used to compute an additional set of navigation accuracies. The DSN baseline coordinates used in all computations were those determined by Moyer[9] for the baselines formed between the three 70-m Deep Space Stations (DSSs): DSS 43 (Canberra), DSS 14 (Goldstone), and DSS 63 (Madrid).

[8] J. B. McNamee, "The Use of Differenced Doppler Data to Support Magellan Navigation During the Mapping Mission," JPL Interoffice Memorandum MGN-NAV-90-013 (internal document), Jet Propulsion Laboratory, Pasadena, California, May 25, 1990.

[9] T. D. Moyer, "Station Location Sets Referred to the Radio Frame," JPL Interoffice Memorandum 314.5-1334 (internal document), Jet Propulsion Laboratory, Pasadena, California, February 24, 1989.

The final tracking parameter that needs to be addressed is the tracking pass width 2Ψ, attainable from DSN intercontinental baselines. It has been shown previously [4] that over the declination range of −25 deg to +25 deg, which is roughly spanned by the ecliptic plane (where most interplanetary spacecraft trajectories lie), the tracking pass width for DSN Canberra–Goldstone baselines is very nearly constant, having a value of about 60 deg (about 4 hours), assuming an elevation cutoff of 10 deg. The tracking pass width for DSN Madrid-Goldstone baselines, on the other hand, varies from about 90 deg (about 6 hours) at a declination of 25 deg to essentially zero at −25 deg.

Figure 3 portrays the right ascension accuracy results obtained for a single differenced Doppler pass from the Canberra–Goldstone and Madrid–Goldstone baselines individually, and for the combined data from both baselines. The degradation in the accuracy obtained from the Madrid–Goldstone baseline at low declination angles occurs because the tracking pass width for this baseline decreases markedly as δ decreases, since both the Madrid and Goldstone DSN complexes are located at northern latitudes. Figures 4 and 5 show the accuracy of the frequency offset estimate obtained from a differenced Doppler pass, for the Canberra–Goldstone and Madrid–Goldstone baselines, respectively. It is clear from both Figs. 4 and 5 that differenced Doppler data of 0.1 mm/sec accuracy are essentially unable to determine the frequency offset any better than it can already be determined a priori from GPS data, at least for the tracking pass durations considered here.

Figures 6 and 7 illustrate the declination accuracy obtained for a differenced Doppler pass from the Canberra–Goldstone and Madrid–Goldstone baselines, respectively. It can be seen that declination accuracy is very sensitive to the a priori frequency offset uncertainty, in complete contrast to right ascension accuracy. The declination accuracy that can be achieved by combining the Canberra–Goldstone and Madrid–Goldstone differenced Doppler passes is shown in Fig. 8. Even with up to 8 hours of data from two different baselines, differenced Doppler data do not appear to have the capability to deliver 50-nrad declination accuracy except for spacecraft declinations of less than about −12 deg and greater than 10 deg.

IV. Discussion

It must be remembered that the results presented above represent theoretical navigation accuracies for an idealistic model of spacecraft motion. Additional work will have to be performed, using much more complete models for the spacecraft trajectory and the differenced Doppler data type, to establish realistic estimates of the actual spacecraft navigation accuracies that might be achieved operationally using differenced Doppler in combination with other tracking data. In this analysis, the effects of Earth orientation errors and baseline coordinate errors on the differenced Doppler data were not considered. Through DSN Very Long Baseline Interferometry (VLBI) quasar observations, DSN baseline coordinates can presently be determined to an accuracy of 5 to 10 cm, while Earth orientation uncertainty is presently known to an accuracy of about 30 nrad. If Earth orientation error uncertainties of 30 nrad (1σ) and baseline coordinate error uncertainties of 5 cm (1σ) were included in this analysis, the individual baseline declination accuracies given in this article would be degraded by 5 to 15 percent for an a priori frequency offset uncertainty of 0.015 mm/sec, and the right ascension accuracies would be degraded by 25 to 40 percent for the same a priori frequency offset uncertainty. (These figures were arrived at through a formal sensitivity analysis of the effects of Earth orientation/baseline coordinate errors on differenced Doppler data.)

When the effects of Earth orientation/baseline coordinate error uncertainties are taken into consideration, the theoretical angular accuracies obtained from differenced Doppler are about 45 to 50 nrad in right ascension and 90 to 310 nrad in declination, except for declination angles within 5 deg of the Earth's equator. If the station frequency offset calibration uncertainty available from GPS was reduced to 0.003 mm/sec, and Earth orientation uncertainty reduced to 10 nrad (which should be accomplished in the near future with GPS and VLBI-based calibration data [7]), these accuracies would improve to 30 to 40 nrad in right ascension and a range of 30 to 150 nrad in declination for the same declination regime stated above (declination magnitude greater than 5 deg). These statistics are essentially the same as those for which no Earth orientation or baseline coordinate errors are assumed.

Another potentially significant error source that has not yet been considered is troposphere calibration error, whose effect on Doppler data is inversely proportional to the square of the elevation angle at which the spacecraft is being tracked. While its effects are generally insignificant at high elevation angles, this error source may cause significant degradation of angular coordinate estimates derived from differenced Doppler data acquired at elevation angles less than about 15 deg.[10] Further work must be done to quantify the potential impact of this error source on differenced Doppler data quality.

[10] C. D. Edwards, Tracking Systems and Application Section, Jet Propulsion Laboratory, private communication.

Overall, it appears that differenced Doppler can determine right ascension very accurately, and declination as well for declination magnitudes above about 10 deg. While this study has focused on the ability of differenced Doppler to determine spacecraft angular coordinates, it should also be pointed out that differenced Doppler can also be used to accurately estimate spacecraft angular rates, a capability especially useful for tasks such as maneuver determination and post-maneuver orbit redetermination. Due to the poor measurement geometry that exists for differenced Doppler for spacecraft at declinations near zero, a mission requiring sub-50-nrad angular accuracy would probably have to make use of other data types, such as ΔDOR, Connected-Element Interferometry (CEI), Differenced One-Way Range (DOR), or Differenced (Two-Way Minus Three-Way) Range. When used in combination with one or more of these data types, differenced Doppler data may enhance the ability of the DSN to determine spacecraft angular coordinates and make it possible to determine angular coordinates with less tracking time than would be required when using a single data type.

V. Summary and Conclusions

The information content of X-band (8.4-GHz) differenced Doppler data for determining spacecraft angular coordinates was investigated using a simple analytic model for the error covariance matrix obtained from one or two differenced Doppler tracking passes made from DSN intercontinental baselines. The analysis took into account the effects of station frequency offset and calibration errors by including a frequency bias as an estimated parameter, in addition to the declination and right ascension of the spacecraft being tracked. The values chosen for the a priori frequency bias uncertainty and data accuracy were based on the current accuracies of DSN differenced Doppler and GPS-based frequency offset calibrations in the Magellan mission. In addition, performance improvements that may be realized through better frequency offset calibration accuracy were considered.

The analysis showed that the ability of differenced Doppler to determine spacecraft right ascension appears to be virtually insensitive to station frequency offset calibration errors. The results indicate that the present DSN X-band (8.4-GHz) differenced Doppler system is theoretically capable of determining right ascension to accuracies of 30 to 40 nrad with 4 hours of data from the DSN. For an assumed a priori frequency offset uncertainty (1σ) of 0.015 mm/sec (about 5.0×10^{-14} sec/sec), which is representative of present GPS-based frequency offset calibration accuracy, spacecraft declination could theoretically be determined to an accuracy of 80–300 nrad with 4 hours of DSN differenced Doppler data, except for declinations within 5 deg of the Earth's equator. If the a priori measurement bias uncertainty was reduced to 0.003 mm/sec (1.0×10^{-14} sec/sec), the resulting declination accuracy was in the 30–150 nrad range, again with the exception of declinations within 5 deg of the equator.

Earth orientation and baseline coordinate errors were not explicitly treated in the analysis, but if they were included at their present levels of uncertainty (about 30 nrad for Earth orientation and 5 cm for baseline coordinates), the accuracy results stated here would be about 90 to 310 nrad (5 to 15 percent degradation) in declination and 45 to 50 nrad (25 to 40 percent degradation) in right ascension, depending upon the spacecraft declination. If Earth orientation errors could be kept to 10 nrad or less, which should be possible with GPS and VLBI-based calibration data, their effects on differenced Doppler would be much smaller, producing total degradations of less than 5 percent. Additional work is required to determine the effects of troposphere calibration errors on differenced Doppler data quality and to develop realistic estimates of the actual spacecraft navigation accuracies that can be achieved using differenced Doppler in combination with other data types.

Acknowledgments

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References

[1] S. R. Poole, M. P. Ananda, and C. E. Hildebrand, "Radio Interferometric Measurements for Accurate Planetary Orbiter Navigation," *AAS Advances in the Astronautical Sciences*, vol. 40, part 1, Univelt Inc., San Diego, 1980.

[2] V. J. Ondrasik and K. H. Rourke, "An Analytical Study of the Advantages Which Differenced Tracking Data May Offer for Ameliorating the Effects of Unknown Spacecraft Accelerations," *DSN Progress Report*, JPL Technical Report 32-1526, vol. IV, May–June 1971, pp. 61–70, August 15, 1971.

[3] W. G. Melbourne and D. W. Curkendall, "Radio Metric Direction Finding: A New Approach to Deep Space Navigation," presented at the AAS/AIAA Astrodynamics Specialist Conference, Jackson Hole, Wyoming, September 7–9, 1977.

[4] S. W. Thurman, "Deep-Space Navigation with Differenced Data Types, Part I: Differenced Range Information Content," *TDA Progress Report 42-103* (this issue), July–September 1990, Jet Propulsion Laboratory, Pasadena, California, pp. 47–60, November 15, 1990.

[5] S. W. Thurman, "Comparison of Earth-Based Radio Metric Data Strategies for Deep-Space Navigation," paper AIAA-90-2908, presented at the AIAA/AAS Astrodynamics Conference, Portland, Oregon, August 20–22, 1990.

[6] T. W. Hamilton and W. G. Melbourne, "Information Content of a Single Pass of Doppler Data From a Distant Spacecraft," *JPL Space Programs Summary 37-39*, vol. III, March–April 1966, pp. 18–23, May 31, 1966.

[7] A. P. Freedman, "Determination of Earth Orientation Using the Global Positioning System," *TDA Progress Report 42-99*, July–September 1989, Jet Propulsion Laboratory, Pasadena, California, pp. 1–11, November 15, 1989.

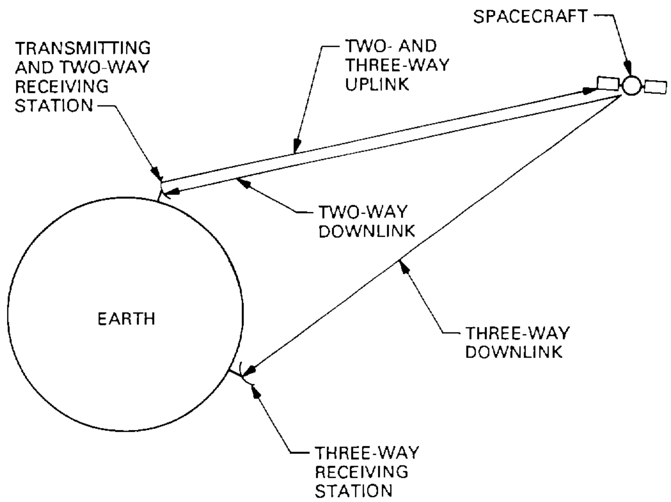

Fig. 1. Two-way and three-way data configuration.

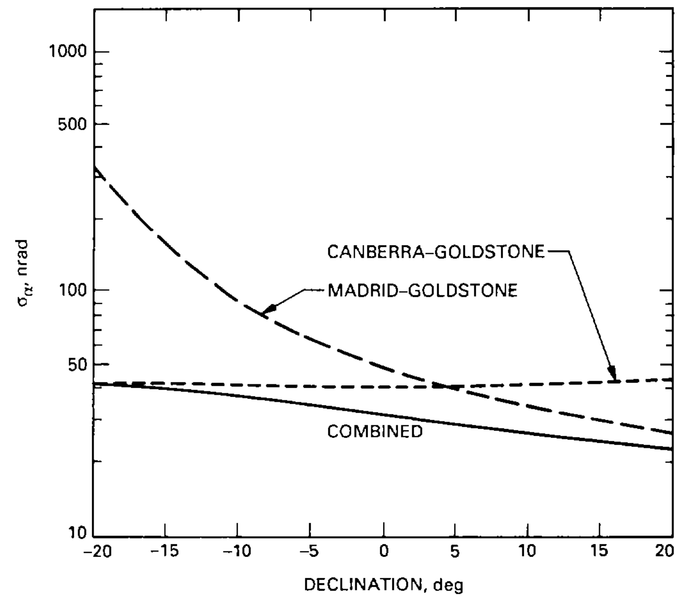

Fig. 3. Right ascension estimation accuracy for DSN baselines.

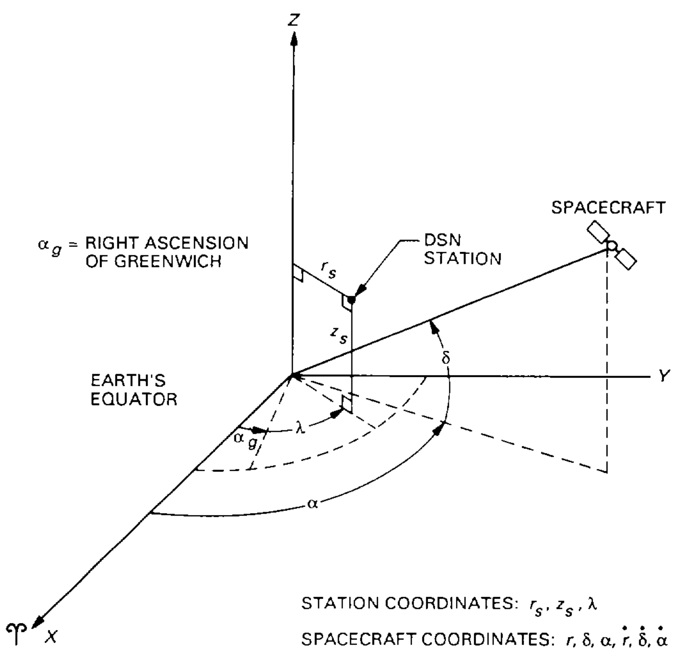

Fig. 2. Spacecraft and tracking station coordinates.

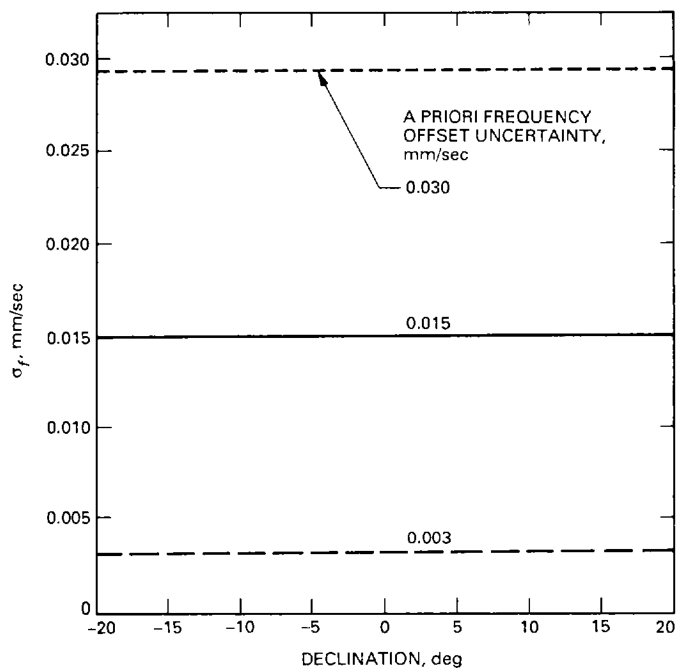

Fig. 4. Frequency offset estimation accuracy, DSN Canberra–Goldstone baseline.

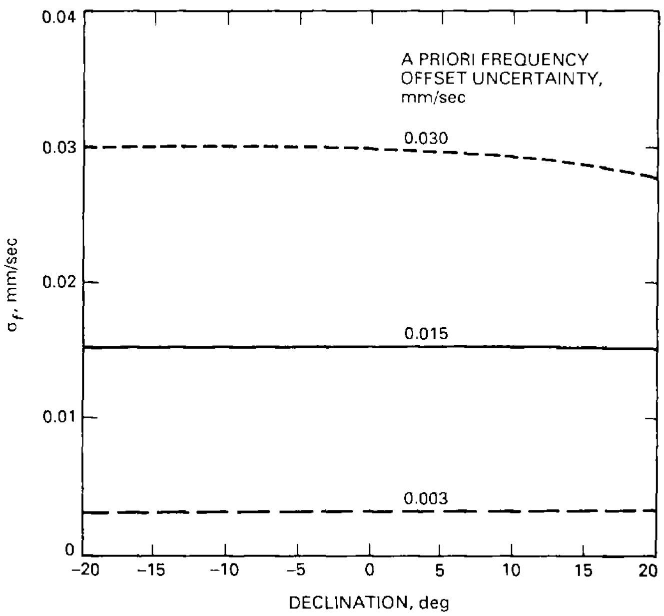

Fig. 5. Frequency offset estimation accuracy, DSN Madrid–Goldstone baseline.

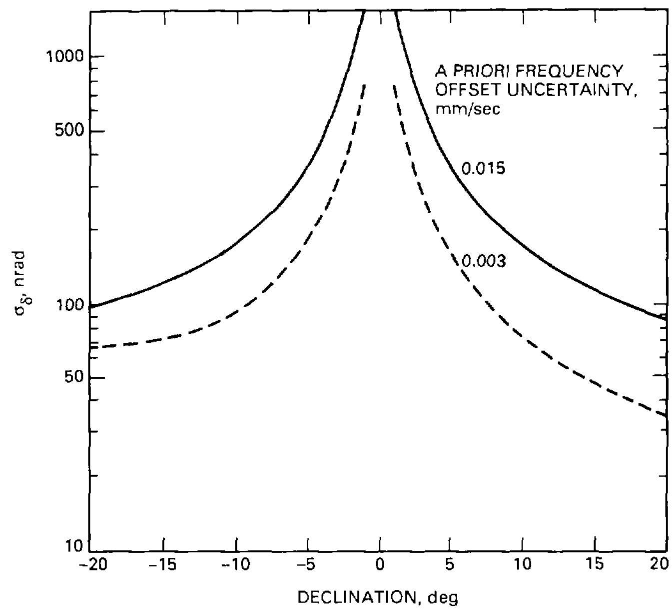

Fig. 7. Declination estimation accuracy, DSN Madrid–Goldstone baseline.

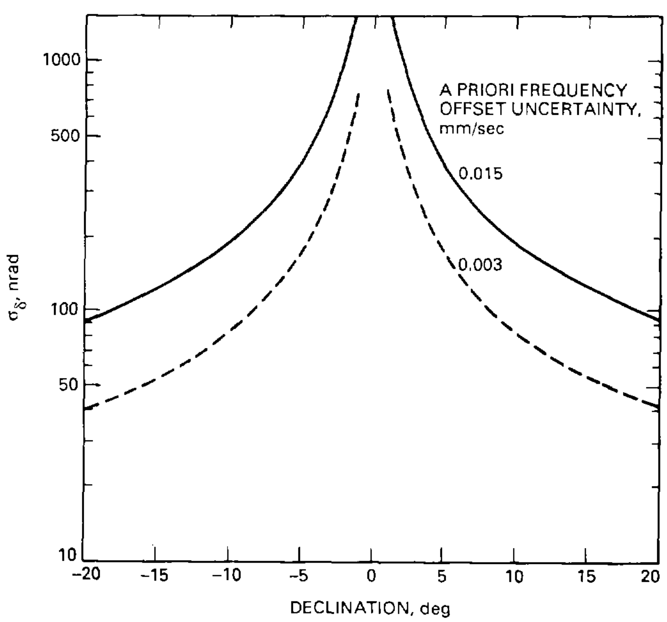

Fig. 6. Declination estimation accuracy, DSN Canberra–Goldstone baseline.

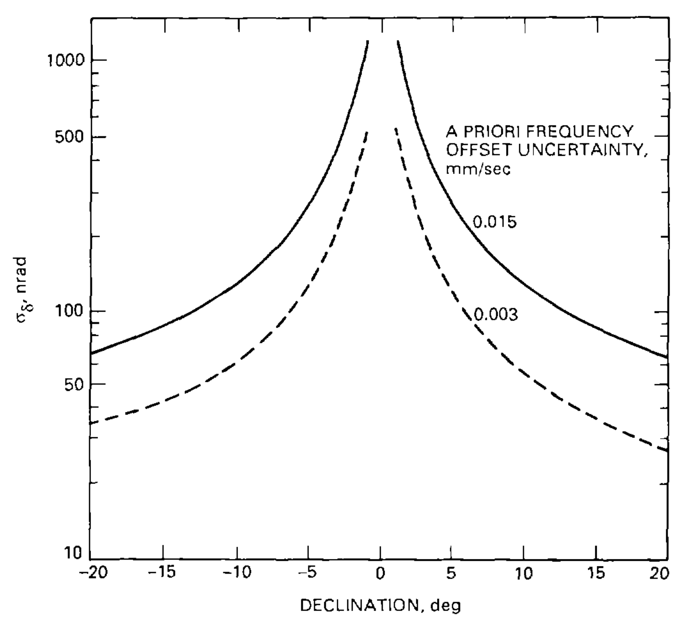

Fig. 8. Declination estimation accuracy, combined DSN baselines.