Use of the VLBI Delay Observable for Orbit Determination of Earth-Orbiting VLBI Satellites

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Very long-baseline interferometry (VLBI) observations using a radio telescope in Earth orbit were performed first in the 1980s. Two spacecraft dedicated to VLBI are scheduled for launch in 1995; the primary scientific goals of these missions will be astrophysical in nature. This article addresses the use of space VLBI delay data for the additional purpose of improving the orbit determination of the Earth-orbiting spacecraft. In an idealized case of quasi-simultaneous observations of three radio sources in orthogonal directions, analytical expressions are found for the instantaneous spacecraft position and its error. The typical position error is at least as large as the distance corresponding to the delay measurement accuracy but can be much greater for some geometries.

A number of practical considerations, such as system noise and imperfect calibrations, set bounds on the orbit-determination accuracy realistically achievable using space VLBI delay data. These effects limit the spacecraft position accuracy to at least 35 cm (and probably 3 m or more) for the first generation of dedicated space VLBI experiments. Even a 35-cm orbital accuracy would fail to provide global VLBI astrometry as accurate as ground-only VLBI. Recommended changes in future space VLBI missions are unlikely to make space VLBI competitive with ground-only VLBI in global astrometric measurements.

I. Introduction

Very long-baseline interferometry (VLBI) is a radio astronomy technique that achieves high angular resolution by means of the simultaneous recording of signals from artificial or natural radio sources at widely separated radio telescopes, and then cross-correlating those signals at a central processing facility [1]. This technique has been used with ground radio telescopes for about 25 years. Currently its uses include high-resolution imaging of radio sources, radio-source position measurements, and monitoring of Earth-rotation parameters and continental drift (e.g., [2]). The first VLBI experiments involving a space radio telescope along with the ground radio telescopes were performed between 1986 and 1988 [3–6]. In those experiments, a Tracking and Data Relay Satellite (TDRS) was used as the space radio telescope, while large telescopes in Australia and Japan were used on the ground.

Two dedicated space VLBI satellites, with radio telescopes 7–10 m in diameter, are scheduled for launch in

1995. The VSOP (VLBI Space Observatory Programme) satellite is being developed in Japan [7], while Radioastron [8] will be a product of the Russian space agency. Radioastron will operate in a highly elliptical orbit with a perigee height of about 3000 km and an apogee height of 80,000 km, while the VSOP orbit will have a perigee height of 1000 km and an apogee height of 20,000 km. Other missions that would be launched after the year 2000 have been studied, particularly the International VLBI Satellite [9]. The primary goals of all these missions are astrophysical, with models and images of radio-source morphologies being the most important scientific output. In this article, the additional use of space VLBI data for improved orbit determination of space VLBI satellites and for radio-source position measurements ("astrometry"), first suggested in [10], are explored. However, the performance in these areas is not the justification for the space missions, and the limitations that are discussed in this article in no way imply limitations in the ability of the spacecraft to achieve their primary goals.

In theory, the longer baselines available to space radio telescopes could provide enhanced astrometry of radio sources, but only if the knowledge of the baselines has accuracy comparable to that available for ground–ground baselines. This requires position accuracies of a few centimeters for the space telescope, 2–3 orders of magnitude better than the accuracy of tens of meters achievable for high Earth orbiters using conventional Doppler tracking [11]. One possible way to obtain such high accuracy is by the use of Global Positioning System (GPS) receivers on board the spacecraft [12], although their use would be restricted somewhat for orbits outside the GPS constellation (altitudes of 20,000 km). It also has been suggested that the VLBI data acquired on space–ground baselines could be used to achieve the desired accuracy [13,14]. However, studies of this subject often have ignored potentially important error sources or have not obtained the desired positional accuracy. Even if centimeter-level baseline accuracy could not be obtained using space VLBI data, it still is of interest to determine the improvement in orbit determination that might be achieved through the use of such data.

This article represents one aspect of an effort to analyze the potential of space VLBI data for improved orbit determination. It has two main goals. The first is an analysis of the information content and the smallest possible orbit-determination errors for delay measurements in a highly idealized scenario of space VLBI observations. This framework is intended to provide a limiting case that shows the best performance that might be achieved for a hypothetical VLBI satellite. The second goal is to investigate a number of practical limitations to the accuracy achievable in a realistic observing scenario. The effect of these limitations on the ability to do astrometry using space VLBI is addressed, with emphasis on the application to the first generation of dedicated space VLBI satellites that will operate in the 1990s.

II. Assumptions and Definitions

It is assumed that VLBI observations are performed using one space-based telescope and one or more telescopes on the surface of the Earth. The space- and ground-based telescopes simultaneously observe the same set of radio sources. The VLBI data at the ground telescopes are recorded in the standard way on high-capacity videotapes. The radio-source data received by the space antenna are digitized and transmitted to a ground tracking station for recording. The clock at the tracking station that records the time of data reception at the spacecraft is initialized by means of a tone sent up from the tracking station and transponded by the spacecraft. Thereafter, the ground clock is driven by the VLBI bitstream, with the evolution of the clock error monitored and corrected by means of a two-way phase link between the tracking station and the spacecraft. The videotapes from the ground and space telescopes then are correlated at a central processing facility, where amplitude, relative phase, delay, and delay-rate observables are extracted for each observation. A spacecraft orbit reconstructed from two-way Doppler data (e.g., [11,15]) is assumed to be the a priori model used at the correlation facility. In this article, only the VLBI delay observable is considered. It is assumed that a single ground tracking station is used during the set of VLBI observations considered.

In order to visualize the delay observable and its relationship to spacecraft position and clock uncertainties, it is appropriate to define a Cartesian coordinate system whose origin is at the location of the ground tracking station in contact with the spacecraft. Instantaneous unit vectors of that coordinate system are $\hat{\imath}$ and $\hat{\jmath}$ at right angles to each other in the Earth's equatorial plane, with $\hat{k}$ directed toward the North Pole. This coordinate system is a mixture of the classical topocentric and equatorial coordinate systems and is selected because it simplifies the analysis done below in Section III. A ground radio telescope has position $\vec{r}_t = (x_t, y_t, z_t)$ relative to the tracking station, while the spacecraft position is $\vec{r} = (x, y, z)$, with the direction from the tracking station to the spacecraft indicated by the unit vector $\hat{r}$. The range between the tracking station and the spacecraft is r. Figure 1 shows the geometry for a space VLBI observation of a single radio source. The measured VLBI delay τ is given approximately by

$$\tau = \tau_g + \tau_c + \tau_p + \tau_i + \tau_s \quad (1)$$

In Eq. (1), τ_g is the geometric delay for the space–ground baseline, τ_c is the delay contribution caused by the experiment clocks, τ_p is the delay caused by propagation through the Earth's troposphere and ionosphere, τ_i is the instrumental delay, and τ_s is the delay caused by radio-source structure. Note that the propagation delays must be included because they affect the data received by the ground radio telescope, hence contributing to the delay measured on the space–ground baseline.

III. Error Analysis and Information Content in an Idealized Case

Consider a VLBI observation of a hypothetical radio source located on the x-axis. In the ideal case, assume that the radio source is a point source ($\tau_s = 0$), the propagation delays caused by the ionosphere and troposphere are known perfectly (τ_p equals the modeled propagation delay, $\tau_{p,m}$), and there is no instrumental delay ($\tau_i = 0$). Further, assume that all clocks on the ground are perfect and that there are no errors in the determinations of universal time, Earth rotation, or polar motion. In that case, the only contribution to τ_c comes from the spacecraft clock. Suppose that the best model of the spacecraft position is $\vec{r}_m$, the best model of the ground telescope location is $\vec{r}_{t,m}$, the model geometric delay is $\tau_{g,m}$, and the best model of the spacecraft clock delay is $\tau_{c,m}$. There are a number of possible ways to find the best model of the clock delay; details are not considered here, and the reader is referred to [16] for more discussion. If the delay is "tracked" in the correlator by subtracting the model delay τ_m, the measured residual delay $\tau_{r,1}$ will be given by

$$\begin{aligned} c\tau_{r,1} &\equiv (\tau - \tau_m) \\ &= c(\tau_g - \tau_{g,m}) + c(\tau_c - \tau_{c,m}) \\ &= (\vec{r} - \vec{r}_m)\cdot\hat{\imath} - (\vec{r}_t - \vec{r}_{t,m})\cdot\hat{\imath} + c(\tau_c - \tau_{c,m}) \quad (2) \end{aligned}$$

where the speed of light is given by c. If it is assumed further that the locations of the ground telescope and tracking station are known perfectly (i.e., $\vec{r}_t = \vec{r}_{t,m}$), Eq. (2) reduces to

$$c\tau_{r,1} = (\vec{r} - \vec{r}_m)\cdot\hat{\imath} + c(\tau_c - \tau_{c,m}) \quad (3)$$

The magnitude of the delay contributed by the spacecraft clock is the time it takes to transmit a tone between the spacecraft and the tracking station. The clock epoch is initialized and monitored as summarized in Section II. Because the clock's evolution is monitored precisely by means of the two-way phase link, only the spacecraft clock delay τ_c at the initialization epoch need be considered here. The transponded tone received at the tracking station at the initialization time arrives after a delay given by the light travel time, so the initial spacecraft data are tagged with a time later than the time at which the VLBI data actually were received by the space radio telescope. This delay must be modeled in the correlation of the VLBI data. Since the clock delay at the initialization time t_0 is just r_0/c, where $r_0 \equiv r(t_0)$, Eq. (3) reduces to

$$c\tau_{r,1} = (x - x_m) + (r_0 - r_{0,m}) \quad (4)$$

The simple analysis above shows that the measured delay residual depends on the errors in the spacecraft location in two different (usually not orthogonal) directions. One is the line-of-sight direction to the infinitely distant radio source ($\hat{\imath}$), while the other is the line of sight from the tracking station to the spacecraft ($\hat{r}_0$). A single measurement of $\tau_{r,1}$ cannot distinguish between the two. Simultaneous observations from other ground radio telescopes cannot contribute new information. The location of a ground radio telescope contributes only to the term $(\vec{r}_t - \vec{r}_{t,m})$ in Eq. (2), but (by assumption) this term is negligible. One can use as many ground telescopes as are available on Earth, and the result still reduces to Eq. (4), one equation in two unknowns, x and r_0.[1]

In an ideal world, one could imagine making a VLBI observation instantaneously, then slewing to another source at infinite speed and making another instantaneous observation. If observations are made of radio sources in the $\hat{\jmath}$ and $\hat{k}$ directions in this hypothetical world, the following equations are added to the system:

$$c\tau_{r,2} = (y - y_m) + (r_0 - r_{0,m}) \quad (5)$$

and

$$c\tau_{r,3} = (z - z_m) + (r_0 - r_{0,m}) \quad (6)$$

[1] Although the additional ground telescopes give no direct information about the observable, they may provide additional constraints that enable improved calibration of a variety of systematic errors in an observation.

Now, there are three equations in the four quantities x, y, z, and r_0. However, a fourth useful equation expresses r in terms of its components

$$r = \left[x^2 + y^2 + z^2\right]^{1/2} \tag{7}$$

If one assumes that the clock initialization is done at the same time as the VLBI observations, $r_0 = r$ and $r_{0,m} = r_m$ in Eqs. (4–6). Then, the delay observables from the observations of the three radio sources carry sufficient information to determine the spacecraft position.

After combining Eqs. (4–7) and doing some algebra, one finds the following result for the spacecraft position:

$$\begin{aligned}
x &= \frac{1}{2}\left[x_m - y_m - z_m - r_m + c(\tau_{r,1} - \tau_{r,2} - \tau_{r,3})\right] + \sqrt{\frac{K}{2}} \\
y &= \frac{1}{2}\left[y_m - x_m - z_m - r_m + c(\tau_{r,2} - \tau_{r,1} - \tau_{r,3})\right] + \sqrt{\frac{K}{2}} \\
z &= \frac{1}{2}\left[z_m - x_m - y_m - r_m + c(\tau_{r,3} - \tau_{r,1} - \tau_{r,2})\right] + \sqrt{\frac{K}{2}}
\end{aligned} \tag{8}$$

Here, K is given by

$$\begin{aligned}
K =& \frac{1}{2}\left[(x_m + y_m + z_m + r_m)^2 - c^2({\tau_{r,1}}^2 + {\tau_{r,2}}^2 + {\tau_{r,3}}^2)\right] \\
&+ c\tau_{r,1}(c\tau_{r,2} + y_m + z_m - x_m + r_m) \\
&+ c\tau_{r,2}(c\tau_{r,3} + x_m + z_m - y_m + r_m) \\
&+ c\tau_{r,3}(c\tau_{r,1} + x_m + y_m - z_m + r_m)
\end{aligned} \tag{9}$$

Equations (8) and (9) provide the means of finding the spacecraft position $\vec{r}$ based on the model position $\vec{r}_m$ and the three measured quantities $\tau_{r,1}$, $\tau_{r,2}$, and $\tau_{r,3}$.

The uncertainty in a component of the spacecraft position can be computed using Eqs. (8) and (9). Taking the x-component as an example, the uncertainty σ_x in x can be found from

$${\sigma_x}^2 = \left(\frac{\partial x}{\partial \tau_{r,1}}\right)^2 {\sigma_{\tau_{r,1}}}^2 + \left(\frac{\partial x}{\partial \tau_{r,2}}\right)^2 {\sigma_{\tau_{r,2}}}^2 + \left(\frac{\partial x}{\partial \tau_{r,3}}\right)^2 {\sigma_{\tau_{r,3}}}^2 \tag{10}$$

Here, the uncertainty in the measurement of the delay residual for the measurement of radio source i is $\sigma_{\tau_{r,i}}$, and the uncertainties in the three radio-source measurements have been taken to be uncorrelated. After doing some algebra, the uncertainties in the components of the spacecraft position are found to be

$$\begin{aligned}
{\sigma_x}^2 =& c^2\left\{\left[1 + \frac{x^2}{2K} - x\sqrt{\frac{2}{K}}\right]{\sigma_{\tau_{r,1}}}^2 + \frac{y^2}{2K}{\sigma_{\tau_{r,2}}}^2 + \frac{z^2}{2K}{\sigma_{\tau_{r,3}}}^2\right\} \\
{\sigma_y}^2 =& c^2\left\{\left[1 + \frac{y^2}{2K} - y\sqrt{\frac{2}{K}}\right]{\sigma_{\tau_{r,2}}}^2 + \frac{x^2}{2K}{\sigma_{\tau_{r,1}}}^2 + \frac{z^2}{2K}{\sigma_{\tau_{r,3}}}^2\right\} \\
{\sigma_z}^2 =& c^2\left\{\left[1 + \frac{z^2}{2K} - z\sqrt{\frac{2}{K}}\right]{\sigma_{\tau_{r,3}}}^2 + \frac{x^2}{2K}{\sigma_{\tau_{r,1}}}^2 + \frac{y^2}{2K}{\sigma_{\tau_{r,2}}}^2\right\}
\end{aligned} \tag{11}$$

If the simplifying assumption is made that $\sigma_{\tau_{r,1}} = \sigma_{\tau_{r,2}} = \sigma_{\tau_{r,3}} \equiv \sigma_\tau$, Eq. (11) reduces to

$$\left.\begin{aligned}
{\sigma_x}^2 &= \left[1 + \frac{r^2}{2K} - x\sqrt{\frac{2}{K}}\right]c^2{\sigma_\tau}^2 \\
{\sigma_y}^2 &= \left[1 + \frac{r^2}{2K} - y\sqrt{\frac{2}{K}}\right]c^2{\sigma_\tau}^2 \\
{\sigma_z}^2 &= \left[1 + \frac{r^2}{2K} - z\sqrt{\frac{2}{K}}\right]c^2{\sigma_\tau}^2
\end{aligned}\right\} \tag{12}$$

The expression for K [Eq. (9)] can be given in terms of the actual spacecraft position components:

$$K = \frac{1}{2}(x + y + z + r)^2 \tag{13}$$

Combining Eqs. (12) and (13) yields the following result:

$$\sigma_x{}^2 = c^2\sigma_\tau{}^2\left[1+\frac{r^2}{(x+y+z+r)^2}-\frac{2x}{(x+y+z+r)}\right] \tag{14}$$

For example, take the case where the spacecraft position is along the line of sight to one of the radio sources, say $x = r$ and $y = z = 0$. Then, evaluation of Eq. (14) shows that $\sigma_x = 0.5c\sigma_\tau$. Since the position error along the line of sight to the radio source and the position error along the line of sight to the tracking station are one and the same quantity in this case, the signature in the delay observable is doubled for a given position offset, or the position error is halved for a given delay measurement error. In order to show the range of values for σ_x, Fig. 2 is a plot of σ_x (in units of $c\sigma_\tau$) as a function of x/r, assuming that $z = 0$. Fig. 2(a) shows the results for $y > 0$, while Fig. 2(b) displays results for $y < 0$. Figures 3(a) and (b) show similar plots for σ_x under the assumption that $z = 0.5r$. In general, the results for the uncertainties along each of the three coordinate axes are similar for a given set of assumptions.

As Fig. 2 shows, σ_x diverges if $x = -r$ or $y = -r$ (or, as not shown in the figure, if $z = -r$), i.e., if the line of sight from the tracking station to the spacecraft is opposite to the direction to one of the natural radio sources. Consider the case where $x \approx -r$. In fact, $x = -r$ is not possible, since this would require the spacecraft radio telescope to look through the Earth to see the radio source hypothesized to be in the direction of the positive x-axis. However, it is possible that $x \approx -r$ in the case where the spacecraft is at a low elevation angle as seen from the tracking station and is several Earth radii distant. For $x = -r$, any error in the spacecraft position along the x-direction is exactly compensated for by an error in the time associated with the VLBI data, and an infinite position error would give no signature in the delay measurement.

The denominators of two terms in Eq. (14) vanish when $x + y + z = -r$, which includes (but is not limited to) the situation where the spacecraft position is opposite to the direction to one of the three radio sources. This is the equation of a plane. The spacecraft position must lie on the spherical surface at a range r from the tracking station; the intersection of the plane and that spherical surface is a circle on which the spacecraft position error becomes infinite. Therefore, if the hypothetical VLBI delay measurements were to be used to improve the spacecraft orbit determination, they should be made at a time when the spacecraft position lies far from that circle of singularity. If there are two or more tracking stations available, it should be possible to select the one for which the position errors dictated by the geometry of the VLBI delay measurements are minimized and to use that station for the time transfer to the spacecraft.

IV. Estimate of VLBI Delay Measurement Errors

A number of practical limitations may prevent space VLBI observations from being as useful for navigation as indicated for the idealistic case treated above. This section provides a discussion of some of those limitations. For a number of effects, contributions to the delay error σ_τ (expressed in distance units, i.e., multiplied by c) are estimated independently, then combined in quadrature. It is important to recognize that the actual method of orbit determination would involve a multiparameter fit to spacecraft and radio-source positions, as well as other quantities such as troposphere, ionosphere, and clock parameters. Therefore, discussion of individual uncertainties as though they were completely separate from one another is an oversimplification but does serve to indicate the general limitations imposed by a variety of effects.

A. Limited Precision of Delay Measurements

The precision of the single-band delay measurements can be derived from the signal-to-noise ratio (SNR) of the VLBI observations using

$$\sigma_\tau = \frac{1}{2\pi\Delta\nu(\mathrm{SNR})} \tag{15}$$

where $\Delta\nu$ is the observing bandwidth. Table 1 summarizes assumptions and predicted measurement precision for three different observing frequencies (1.6, 5, and 22 GHz) that will be used in the first generation of VLBI satellites. Assumptions are those for the current best guesses at the performance of the radio telescope on board Radioastron; the VSOP performance may be somewhat poorer. In distance units, the estimated delay precisions are 1.8 cm, 1.9 cm, and 4.5 cm at 1.6, 5, and 22 GHz, respectively. Radio sources with correlated flux densities as high as the assumed value of 0.5 Jy on baselines in the 40,000–80,000 km range probably will be rare or nonexistent at all three frequencies, so the above precisions will not be achievable on the longest baselines for most sources.

B. Ionospheric Propagation Errors

In the DSN, typical errors of 2–4 cm currently are achieved in the calibration of the ionospheric propagation delay of 8.4-GHz radio signals at the zenith [17], although there is some hope for improvement using signals from GPS satellites [18]. Even if it is assumed that the radio-source signal propagating to the spacecraft suffers no charged-particle delay because of the interplanetary medium, the delay measurement will still be corrupted by propagation through the Earth's ionosphere to the ground radio telescope. Scaling from the 8.4-GHz estimates (ionospheric delays are proportional to the inverse square of the frequency), the current ionospheric calibration errors will give respective zenith delay errors of 55–110 cm, 5–10 cm, and 0.3–0.6 cm at the three frequencies. (Radioastron also will operate at 300 MHz, where the ionospheric effects will be even larger.) At elevation angles of 30 deg, which will be much more common when several radio sources are observed in very different directions, the above delay errors should be doubled. The delay errors could be reduced substantially if there were a capability for simultaneous dual-frequency observations, as there is for ground-based astrometric and geodetic experiments. Such a capability does not exist on VSOP but might be available (at 1.6 and 5 GHz) on Radioastron. However, ground radio telescopes currently do not have dual-frequency capabilities at the space VLBI frequencies, implying that ionospheric propagation errors cannot be reduced by means of dual-frequency observations.

C. Tropospheric Propagation Errors

Troposphere fluctuations and errors in the static troposphere also will have a significant effect on delay measurements. Typical errors in the calibration of the zenith troposphere delay are about 4 cm, corresponding to 8 cm at a 30-deg elevation. By the mid-1990s, GPS calibrations have the potential for reducing these errors by a factor of 2–4 at ground radio telescopes, provided that GPS receivers are present at the telescopes. The error caused by troposphere fluctuations is on the order of 3 cm at low elevations, and not readily reducible using GPS data. In the future, this error might be reduced by using advanced water-vapor radiometers.

D. Earth-Orientation and Timing Errors

Errors in prediction of Earth orientation and Universal Time typically give errors of tens of centimeters or more in effective locations of tracking stations and radio telescopes on the Earth. However, a delay of several weeks will occur between the observations and the data correlation. With that delay, the combination of VLBI and GPS calibration measurements allows reconstruction of the Earth orientation and timing parameters to better than 2 cm per component [19], implying similar errors in the delay measurements.

E. Radio-Source Position Errors

In the idealized case, it was assumed implicitly that the positions of the radio sources observed for orbit determination were known perfectly. In fact, they are not. For a priori position errors of 1 nrad that will be characteristic of the strongest compact radio sources in the mid-1990s, the delay measurement error will be 4 cm on a 40,000-km baseline and 8 cm on an 80,000-km baseline.

F. Summary of Delay Errors

Table 2 summarizes the minimum expected error contributions (in length units) to space–ground VLBI delay measurements for Radioastron, assuming a 40,000-km baseline. (The value of 40,000 km is used because for observations of three radio sources in orthogonal directions, it is not possible for the projected baselines in all three directions to be near 80,000 km.) This table assumes the sensitivity and the improved troposphere and Earth-orientation calibrations given in the above subsections. On longer baselines, the delay error contributed by the source position uncertainties will be larger, while the correlated flux densities and consequent sensitivities probably will be lower than assumed above. Even assuming no instrumental errors on the ground, the minimum delay errors derived from the rss of the individual error contributions are 110 cm at 1.6 GHz, 12 cm at 5 GHz, and 8 cm at 22 GHz. Inspection of Figs. 2 and 3 shows that the one-dimensional spacecraft position error derived from the VLBI data for the idealized radio-source observation strategy can range from 0.5 to more than 10 times the delay measurement error, depending on the exact geometry. These results are for the hypothetical case of simultaneous observations of three radio sources, and do not include the additional considerations discussed below in Section V.

V. Other Practical Considerations in Using VLBI Delay for Orbit Determination

A. Geometry of Radio-Source Observations

The ideal case considered above assumes observations of radio sources in three orthogonal directions. This simplification makes the analytical development tractable but probably is not necessary for improved orbit-determination results. In order to solve for the spacecraft position, it is likely that the three radio-source directions need only be linearly independent (i.e., not coplanar). However, it will be necessary to determine whether, in this more general

case, there are geometries in which the position error diverges as it does in the idealized case. If the assumption of orthogonal radio-source directions could be relaxed, this would help in several ways. First, it would give a much larger set of candidate sources for observation, which is important in view of the correlated-flux limitations mentioned previously. Second, it would make it more likely that a set of radio sources providing a reasonable geometry actually can be observed; spacecraft constraints will make it very difficult to observe sources in three orthogonal directions, none of which is near the line of sight to the tracking station. Third, the total amount of slewing necessary for the space radio telescope to observe the three radio sources would be reduced, a consideration whose importance is described further in the next subsection.

B. Nonsimultaneous Observations and Propagation to a Common Reference Time

The idealized case considered in this article includes the assumption that observations of three radio sources in very different directions could be made simultaneously. Of course, this assumption is completely unrealistic. For VSOP, the minimum time necessary for the space telescope to make a 90-deg slew to a new source, settle, and begin observations, is likely to be at least 60 minutes. Radioastron should slew much more rapidly and may be able to change sources in 15 minutes, so it is used here to derive the more optimistic result. Assuming 5-min integration times, three observations would take a total of 45 minutes, with reference times (scan midpoints) spanning 40 minutes. If the reference time were chosen to be the time of the second observation, the delay residuals at the times of the first and third observations must be propagated to the time of the second observation in order to solve for the spacecraft position at that time. Typical velocity uncertainties in the reconstructed spacecraft orbit will be about 1 cm/sec [11]. In the unlikely event that the velocity errors from one second to the next were completely uncorrelated, there would be an additional position error of at least 35 cm due to this error propagation over 20 minutes. If the correlation time for velocity errors in the reconstructed orbit is much longer than 1 sec, the position error at the reference time will be considerably larger than 35 cm. For example, if the correlation time for the velocity errors were 100 seconds or more, as seems likely, the position error due solely to the propagation to a reference time 20 minutes away would be at least 3 meters.

If the orbit at the reference time is propagated to the time of another VLBI observation that might be used for astrometric purposes, the spacecraft position error at that time will be still larger than the error at the reference time. Uncertain spacecraft accelerations caused by mismodeling of the Earth's gravitational field, by errors in the model of solar pressure effects on the spacecraft, and by spacecraft maneuvers will serve to increase the spacecraft position error at times later (or earlier) than the epoch of the observations actually used for orbit determination. In addition, the above discussion is oversimplified because it assumes a fixed velocity accuracy that is used to propagate the position results found from the VLBI data. In reality, the VLBI data would be acquired simultaneously with two-way Doppler data, and both would be used to determine the spacecraft orbit. Detailed investigation of the orbit accuracy achievable in that case is beyond the scope of this article.

C. Tracking Continuity

In order for the three hypothetical VLBI observations to provide useful data for spacecraft trajectory determination, they must be referenced to the same clock. This implies, first, that the same tracking station must be used during all the VLBI observations. Second, it suggests that it may be necessary to maintain a continuous link to the tracking station between the VLBI observations. Without that continuity, there also will be clock breaks that will degrade the accuracy of the orbit determination. For a spacecraft in a fairly low orbit, such as VSOP, there will be a limited view period from a particular tracking station. In many instances, there will not be time to make three observations while the spacecraft is in view of the same station, particularly since long slews with VSOP may take up to 4 hours.

D. Possible Importance of Ranging

It was shown in Section III that in the absence of other data, three VLBI delay measurements are needed to improve on the knowledge of the spacecraft position. However, inspection of Eqs. (4–6) shows that a single VLBI delay observation can determine one component of the spacecraft position if the range r is determined accurately. Thus, a capability for ranging from the tracking stations to the VLBI spacecraft could be quite useful. In fact, any combination of three simultaneous ranging and VLBI delay measurements sampling three linearly independent directions should suffice to provide an accurate instantaneous position for the spacecraft. There is no fundamental reason that ranging observations from two different tracking stations could not be made at the same time that the spacecraft is making a VLBI observation of a radio source in a third direction. The two ranging measurements would provide accurate position components in two directions as well as supply an absolute clock time to the spacecraft. The accuracy of the VLBI delay observable still would be

limited by the effects summarized in Section IV above and by the clock error imposed by the limited accuracy of the ranging system. Two simultaneous ranging measurements with errors of 15 cm, when combined with the minimum error of 8 cm possible for a 22-GHz VLBI delay measurement, would give a total delay accuracy of about 23 cm, implying a spacecraft position uncertainty at least as large.

There are practical limitations and implications for the current generation of space VLBI satellites. VSOP and Radioastron will have limited ranging capability only from Japanese and Russian tracking stations, respectively. Radioastron will not be able to maintain a phase link during ranging observations, so simultaneous ranging/VLBI observations are not possible. Furthermore, neither spacecraft has multiple, independent downlink antennas. Therefore, it will not be possible to do simultaneous ranging from two different tracking stations. Reorientation of the downlink antenna would be necessary to do ranging from two different tracking stations; that reorientation would cause a break in the clock continuity.

E. Possible Benefits of Accurate Clocks On Board Space VLBI Satellites

Another possibility that would eliminate some of the difficulties in solving for the position of a space VLBI satellite would be the provision of a highly accurate clock on board the spacecraft. However, at centimeter wavelengths, monitoring of a two-way phase link from the ground, as was done in the first space VLBI experiments [3,4] and is planned for the first generation of dedicated missions, can provide a clock-rate accuracy nearly equivalent to that of the original clock on the ground. If a clock is flown on board the spacecraft, calibration of the absolute delay still will face the same difficulties as the calibration of the absolute delay for a clock transmitted from the ground. In either case, highly accurate ranging measurements are needed to fix the absolute clock time.

VI. Prospects (or Lack Thereof) for Improved Astrometry

The above analysis has shown that, with many caveats, it may be possible to achieve improvements in orbit determination through the use of space VLBI delay data. The degree of improvement that is achievable must be determined through numerical simulations, since the simple analytical model does not include many of the practical limitations discussed above. However, it is possible to address some aspects of the utility of the orbit determination improvement. In particular, a desirable consequence of highly accurate orbit determination would be the ability to use the spacecraft as a platform for making astrometric VLBI observations more accurately than could be done using ground baselines alone. This section addresses the feasibility of that task.

It is critical to recognize that doing VLBI with a space radio telescope is a much more expensive and complicated task than ground-only VLBI experiments. Therefore, it makes no sense to do astrometry in space–ground VLBI experiments unless it can be shown that the accuracy will be significantly better than is possible for ground-only astrometry experiments. Expected improvement by a factor of 2 or more should be the minimum requirement for attempting an astrometric experiment using space VLBI.

The current delay precision achieved in the best ground-based astrometric VLBI experiments is approximately 15 psec [20], corresponding to a distance of 4.5 mm, a factor of about 20 better than the best that could be hoped for in space–ground VLBI using the first generation of dedicated VLBI satellites. The accuracy of the determination of VLBI baselines on the ground is approximately 1 cm in the best experiments [21], also a factor of about 35 better than the best that possibly could be expected for space VLBI in the 1990s. The delay precision achieved on the ground can be used to give much better baselines, in large part because of the ability to make many observations in different directions in a short period of time. This enables multiparameter fits that give excellent solutions for such quantities as clocks and atmospheric delays. Space VLBI observations for astrometric purposes will not be competitive with ground-only observations until they are capable of such flexibility.

The astrometric angular precision possible for wide-angle VLBI astrometry, σ_ϕ, can be estimated from the baseline error σ_B and the baseline length B as

$$\sigma_\phi \sim \frac{\sigma_B}{B} \tag{16}$$

In ground-only experiments, a baseline error of 1 cm over a 10,000-km baseline corresponds to an angular error of 1 nrad. For Radioastron, the minimum possible baseline error of 35 cm over a baseline length of 40,000 km gives an angular accuracy of 9 nrad in the best possible case, not competitive with current ground-only VLBI.

It also is important to consider the possibility that accurate differential astrometric VLBI from space might be done over narrow angles without knowledge of the spacecraft orbit to within a few centimeters. One could imagine

successive, quasi-simultaneous observations of two radio sources separated in the sky by the small angle ϕ. For ϕ much smaller than a radian, the differential astrometric error in the source positions would be given approximately by

$$\sigma_\phi \approx \phi \frac{\sigma_B}{B} \tag{17}$$

The extra factor of ϕ compared with Eq. (16) is due to the fact that the effect of the baseline error becomes smaller and smaller for decreasing angular separations of the radio sources, as a larger fraction of the delay error cancels when the delay observations of the two radio sources are differenced. For angular separations on the order of 0.5 deg, differential astrometry at the 0.1-nrad level has been achieved using ground radio telescopes [22]. The space–ground VLBI differential astrometry on a 40,000-km baseline could yield a factor-of-two improvement at this angular separation only if the spacecraft position were known with an accuracy of $\sim$ 20 cm. This accuracy does not seem achievable using space VLBI delay data.

VLBI observations of two sources in the same beams of the radio telescopes might provide further error reduction and the ability to do highly accurate differential VLBI. Given the sensitivity limitations of the first generation of space VLBI telescopes, there will be few (or no) pairs of continuum sources with separations well under a degree that can be observed. Thus the narrow-angle differential VLBI might be limited to spectral-line studies of the separation of spots in water masers. Separations of different water-maser complexes in nearby external galaxies might be studied; for same-beam VLBI involving a 70-m ground telescope, these complexes would need to be separated by less than 0.01 deg. Individual water-maser sources typically span regions of 100 nrad (6×10^{-6} deg) in this galaxy to 1 nrad or less in external galaxies, so measurements of spot separations within these individual complexes also might be possible.

Ten years ago, ground-based observations of the quasar pair 1038+528A and B, separated by about 0.01 deg, achieved differential astrometric accuracy of 0.02 nrad [23]. For a 40,000-km baseline and a 0.01-deg separation, Eq. (17) predicts that the space–ground VLBI astrometry could achieve this accuracy if the spacecraft position error were less than about 5 meters. Thus, it is conceivable that the space observations might compete at 0.01-deg source separations, although the current potential for ground-only results may be significantly better than the accuracy obtained in the early 1980s. However, lack of frequency tunability will prevent Radioastron from observing most extragalactic water masers, and the shorter baselines sampled using VSOP will require position errors less than about two meters for such astrometric measurements to be more useful than the ground VLBI observations.

For source separations of 500 nrad (3×10^{-5} deg) within individual water maser complexes, Eq. (17) predicts that orbit determination accuracies of 40 m on a 20,000-km baseline would provide the potential for differential astrometric accuracies better than 10^{-3} nrad. Such orbital accuracies should be achievable using standard two-way Doppler tracking [11]. Furthermore, the sensitivities of VSOP and Radioastron probably would limit the possible astrometric accuracy to $\sim 5 \times 10^{-3}$ nrad for water masers [24]. Thus, improved orbit determination is not needed for accurate differential astrometry within individual maser complexes.

VII. Summary

An analysis has been performed to determine the information content of VLBI delay measurements on a space–ground baseline for improved orbit determination of an Earth-orbiting VLBI satellite. In the idealized case of simultaneous observations of three different radio sources in mutually orthogonal directions, expressions have been given for the expected error in the orbit determination from the VLBI data alone. Given a number of random and systematic error sources that will be very difficult to reduce in space VLBI, the delay measurement precision even in the idealized case will be at least 8 cm for the VLBI spacecraft scheduled for launch in the mid-1990s; this often corresponds to a position error much larger than 8 cm. Other practical aspects of space VLBI, particularly the long time required between observations, will make the orbit determination less accurate than the prediction for the ideal case. The long time between observations would give rise to a minimum position error of at least 35 cm, with an error of at least a few meters far more likely. Combination of VLBI data with simultaneous highly accurate ranging data from two different tracking stations shows promise for an improved determination of the instantaneous spacecraft position for a second generation of dedicated space VLBI observatories.

It is unrealistic to expect that an instantaneous position accuracy as good as 35 cm can be derived from the space VLBI delay data acquired by the first generation of space VLBI satellites; the achievable accuracy probably will be no better than 3 meters. Even under the optimistic assumption of 35-cm position accuracy, space–ground VLBI

cannot compete with ground-based VLBI for astrometry over angles of 0.1 deg or larger. It is conceivable that differential astrometric observations over smaller separations might be useful, although the lack of sensitivity of the first generation of space VLBI telescopes probably limits this utility to relative measurements of water-maser spots. Within individual maser complexes, the accuracy of the spacecraft position usually is not the limiting factor for astrometric accuracy, and orbit determination using two-way Doppler data may suffice.

Several changes could be implemented for the second generation of space VLBI observations that would provide enhanced capabilities for improved orbit determination using the space–ground VLBI delay data. First, the capability for simultaneous dual- frequency operations is needed for both the spacecraft radio telescope and the large ground telescopes, in order to solve for and eliminate the charged particle effects on signal propagation to the ground telescopes. (A number of ground telescopes currently can make simultaneous observations at 2.3 and 8.4 GHz, but this capability is of no use for space VLBI satellites that do not observe at those frequencies.) Second, the most accurate possible calibrations must be used to minimize the errors in knowledge of Earth orientation and tropospheric delay. Third, the space radio telescope must be more sensitive, implying some combination of larger bandwidth, larger diameter, and lower system temperature. Fourth, the space radio telescope must have a capability for very rapid slewing and for observing a large number of sources in a relatively short period of time. Fifth, a highly accurate ranging system, including the capabilities for simultaneous ranging from more than one direction and for simultaneous ranging and VLBI data acquisition, is necessary to realize the improvements discussed in this article. This capability could be supplied either by ranging from the ground or by a satellite system, such as GPS, preferably with higher orbits. Sixth, solar-pressure and gravitational-field models would have to be made more accurate to improve the propagation of trajectory measurements to a specific reference time. Seventh, simultaneous observations with a large number of ground telescopes could be helpful in solving for systematic calibration errors.

With all the improvements listed above, it might be possible to use space VLBI delay data to determine an instantaneous spacecraft position with an accuracy of tens of centimeters. However, this still is not accurate enough for space–ground VLBI to compete with ground-only VLBI in making global astrometric measurements. Instead, space VLBI can be used far more productively for imaging and other astrophysical measurements that depend on the longer baselines available, but do not require highly accurate measurements of the absolute delay.

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Table 1. Space VLBI delay-measurement precision.

Observing frequency, GHz	1.6	5.0	22
Correlated flux density, Jy	0.5	0.5	0.5
Observing bandwidth, MHz	32	32	32
Integration time, sec	300	300	300
Ground telescope			
Diameter, m	70	70	70
Efficiency	0.60	0.60	0.50
System temperature, K	35	35	50
Space telescope			
Diameter, m	10	10	10
Efficiency	0.50	0.50	0.30
System temperature, K	60	70	135
Sensitivity			
σ_τ, nsec	0.06	0.06	0.15
$c\sigma_\tau$, cm	1.8	1.9	4.5

Table 2. Components of space VLBI delay-measurement errors (cm) on a 40,000-km baseline.

Observing frequency, GHz	1.6	5.0	22
System noise	1.8	1.9	4.5
Ionosphere	110–220	10–20	0.6–1.2
Static troposphere	2–4	2–4	2–4
Fluctuating troposphere	3	3	3
Earth orientation	3	3	3
Radio-source position	4	4	4
RSS	110–220	12–21	8

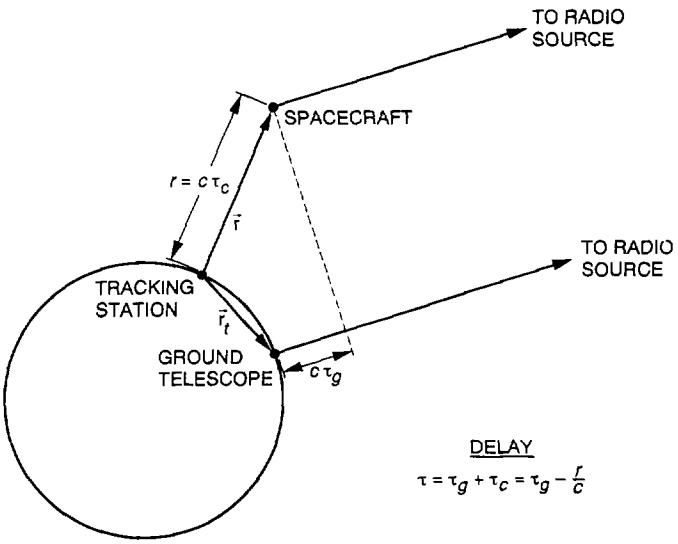

Fig. 1. Geometry of the delay measurement on a space–ground VLBI baseline.

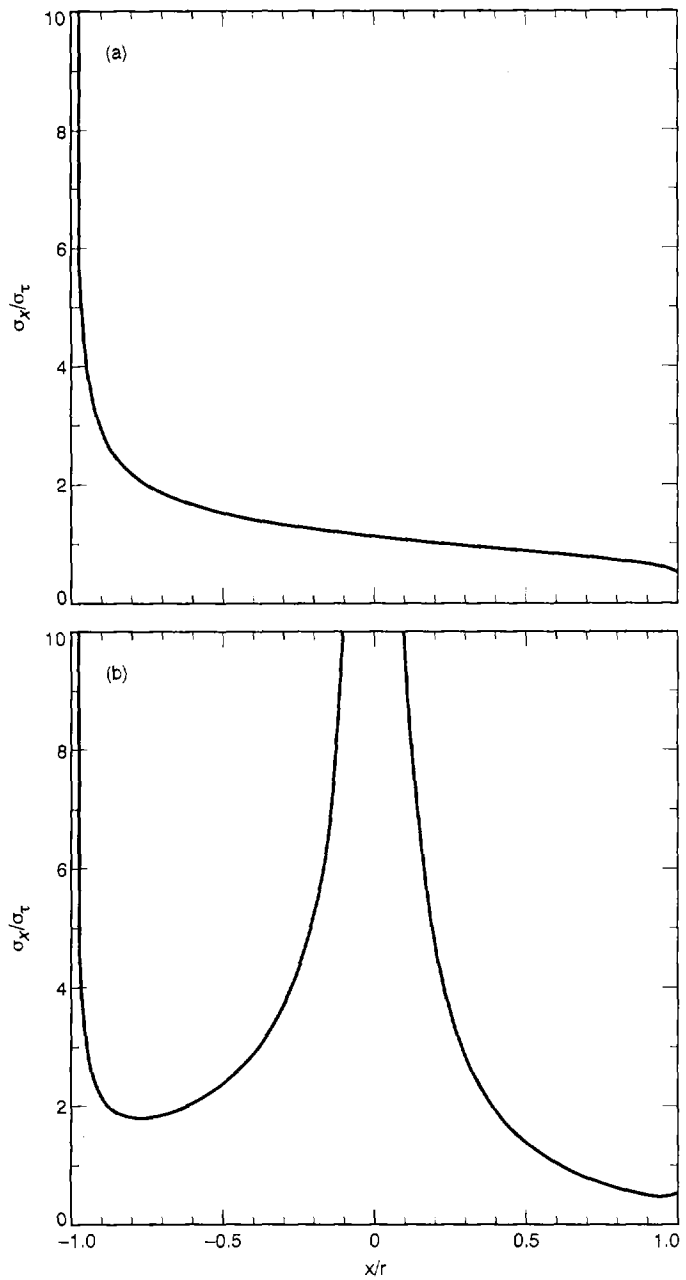

Fig. 2. Predicted minimum (one-dimensional) spacecraft position error, in units of the delay measurement error, for a highly idealized set of space–ground VLBI delay measurements: (a) $y > 0$ and $z = 0$ and (b) $y < 0$ and $z = 0$.

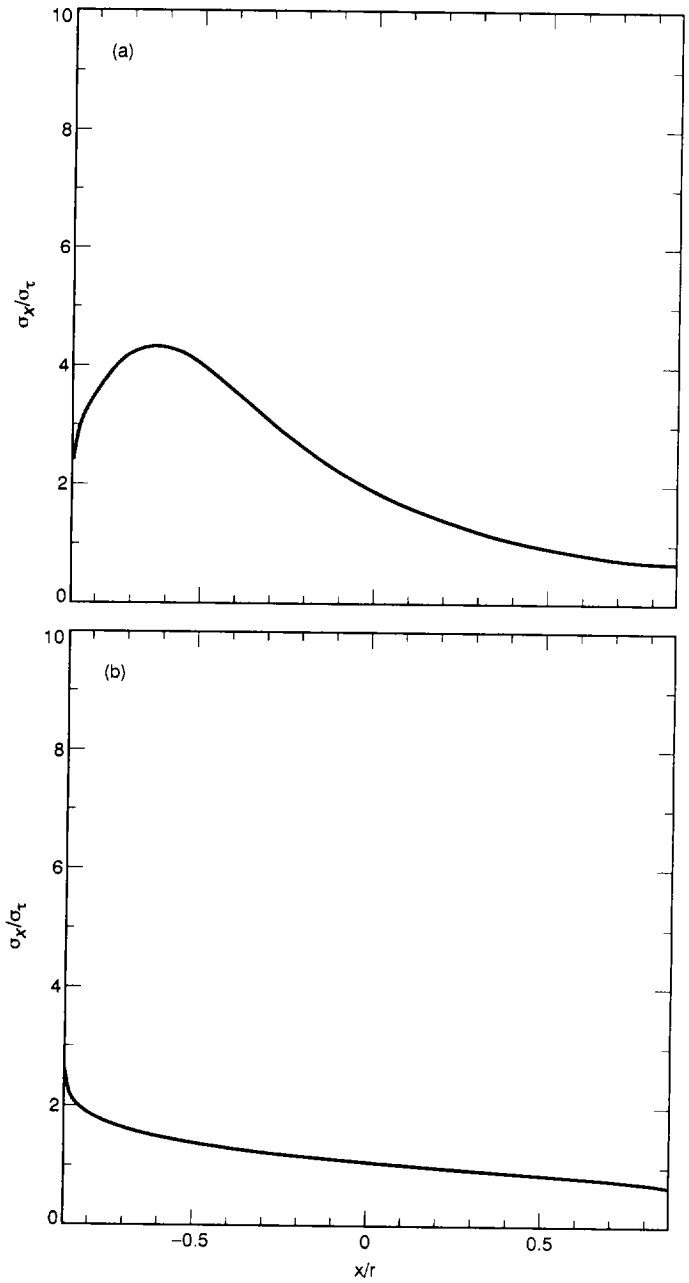

Fig. 3. Predicted minimum (one-dimensional) spacecraft position error, in units of the delay measurement error, for a highly idealized set of space–ground VLBI delay measurements: (a) $y > 0$ and $z = 0.5$ and (b) $y < 0$ and $z = 0.5$.