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Photon Jitter Mitigation for the Optical Channel

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We consider a pulse-position-modulated (PPM) optical communications channel where photon arrival times are offset by a random jitter. This models jitter introduced by timing synchronization or detector delay, which is a significant degradation for very short slot widths. We derive the symbol likelihoods in the presence of inter-slot interference (ITI) as well as a number of practical approximations to the likelihoods. The impact on performance of a coded system is illustrated, where we show that using the proper likelihoods yields large gains over a conventional receiver that does not model the ITI. We show that, for large PPM orders and moderate jitter, inter-symbol interference (ISI) may be ignored in the receiver while incurring only small losses.

I. Introduction

On an ideal optical communications channel, the throughput may be increased without bound by decreasing the slot width while keeping the average power constant. However, this would require a laser transmitter that could confine a pulse to an arbitrarily short slot, a detector that could precisely reproduce the photon arrival times, and a receiver that could precisely allocate photon counts to bins corresponding to signal or noise slots. Each of these components has physical limitations that don't allow transmission and reproduction of arbitrarily short slots. Peak power constraints limit the minimum pulse width a laser can produce without sacrificing average power. At the detector, each incident photon experiences a random delay from the time of arrival to the time the detector produces a pulse in response to that photon; hence, the arrival time may only be estimated—see, e.g., [1,2] or a report by Moision and Farr[2] for characterizations of this jitter for practical detectors. Finally, the receiver introduces errors in attempting to partition time into slots corresponding to the pulsed laser slots. Errors in this synchronization contribute to uncertainty in the location of a photon relative to the symbol boundaries.

These uncertainties, which we collectively refer to as photon jitter, are negligible for channels with sufficiently long slots. However, in the push to extend the communications channel throughput by narrowing the slot width, photon jitter will lead to significant degradation if ignored. In this article, we

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[1] Communications Architectures and Research Section.

[2] B. Moision and W. Farr, "Communication Limits due to Photon Detector Jitter," JPL Technical Report (internal document), Jet Propulsion Laboratory, Pasadena, California, June 2007.

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derive symbol likelihoods for a Poisson pulse-position-modulation (PPM) channel in the presence of photon jitter. We also find several approximations to these likelihoods. We illustrate the impact of photon jitter on the capacity and on the performance of a coded system.

Sequence detection for PPM in the presence of inter-symbol interference (ISI) or inter-slot interference (ITI) has been treated for the indoor wireless infrared channel; see, e.g., [3]. On the indoor wireless channel, the photon counts are large and the channel is modeled as having a deterministic pulse shape with additive white Gaussian noise. Our channel of interest differs in that we operate in a low-photon-count regime with photon-counting detectors, where the statistics are appropriately modeled as Poisson. In [5], ISI introduced by a detector pulse response that extends over several slots was considered. Webb statistics were used to model the output of the detector, but random timing jitter was presumed negligible. A number of articles have treated the transmission and reception of overlapped-PPM, originally studied in [6], wherein ITI is intentionally introduced in order to expand the number of symbols per second. In all of these prior analyses, jitter is presumed negligible when deriving optimum receivers or symbol likelihoods.

In [7,8], sequence detection is performed in the absence of slot or symbol synchronization, jointly estimating the timing offset and the data sequence. Our model differs in that an independent offset is observed each symbol, modeling the output from a synchronization algorithm utilizing a symbol-decision-based feedback loop. It also accommodates non-ideal transmitted laser pulse shapes and detector jitter. This model most closely captures the dominant constraints on current technology for a deep-space optical channel.

Recent work by Kachelmyer and Boroson [9] investigated the capacity in detector jitter for a photon-counting channel. They assumed a receiver restricted to quantizing photon arrival times to a slot, that photons are equally likely to fall outside a slot boundary independent of their arrival time, and that the magnitude of the jitter is less than the duration of a slot. We lift these assumptions in computing the capacity, putting detector and synchronization jitter in a common framework and, furthermore, focus on the mitigation of ISI/ITI. The impact of ISI is treated in [9] by computing a series of approximations to the ISI capacity. We do not address capacity in ISI, but determine bounds on the performance of a coded system using maximum a posteriori detection of PPM in ISI, illustrating the impact of ignoring ISI is small for large PPM orders and moderate ISI.

This article is organized as follows. In Section II, we introduce the channel models and derive the symbol likelihoods as well as several approximations to the likelihoods. In Section III, we discuss computation of the channel capacity. In Section IV, we illustrate numerical results, comparing the performance of detector and timing jitter, the use of various approximations to the likelihoods, and the impact of ISI.

II. Channel Model/Symbol Likelihoods

In each channel use, one of M PPM symbols is transmitted by sending a pulse in the ith slot of an M slot word. Throughout, the slot width is normalized to 1. Let $\lambda_i(t) = n_s p(t - i + 1) + n_b$, the incident photon intensity function when the ith slot, $i \in \{1, \cdots, M\}$, is pulsed, where $p(t)$ is a unit pulse on $[0, 1]$ (a more accurate pulse shape may be substituted without loss of generality in the following analysis). A collection of photons arrive at times $\{s_j\}, j = 1, 2, \cdots, N$. In the presence of timing jitter, each photon is observed at time $t_j = s_j + \delta$, where δ is independent of the photon arrivals and drawn from density f_δ.

The distribution of observed signal photon arrival times for a pulse transmitted in the first slot is given by $f(t) = (p \star f_\delta)(t)$. The *observed* intensity (as opposed to the incident) when the ith symbol is transmitted is $\lambda'_i(t) = n_s f(t - i + 1) + n_b$. We model only inter-*slot*-interference and assume there is no inter-*symbol*-interference. This simplifies the analysis and complexity of the resulting maximum-likelihood receiver. In Section IV.B, ISI is introduced, and we show that the loss in making the assumption of no ISI is small.

Throughout, we model the jitter offset with an exponential distribution

$$f_\delta(\delta) = \frac{1}{2\alpha} e^{-|\delta|/\alpha}$$

which yields

$$f(t) = \begin{cases} e^{t/\alpha}(1 - e^{-1/\alpha})/2, & t < 0 \\ \left(2 - e^{(t-1)/\alpha} - e^{-t/\alpha}\right)/2, & 0 \le t \le 1 \\ e^{-t/\alpha}(e^{1/\alpha} - 1)/2, & t > 1 \end{cases}$$

Figure 1 illustrates $f(t)$ for $\alpha \in \{0, 0.1, 0.2, 0.4\}$.

Suppose the receiver partitions time into bins of duration Δt, and counts the number of photon arrivals in each bin. Let $\Delta_j = [(j-1)\Delta t, j\Delta t]$, the jth bin, and k_j be the photon count in that bin. The symbol likelihoods given the observed photon counts, $\{p(\{k_j\}|\lambda_i)\}_{i=1}^M$, are given by

$$\begin{aligned} p(\{k_j\}|\lambda_i) &= \int p(\{k_j\}, \delta|\lambda_i) d\delta \\ &= \int p(\{k_j\}|\lambda_i, \delta) f_\delta(\delta) d\delta \\ &= \int \prod_j \frac{\exp\left(-\int_{\Delta_j} \lambda_i(u-\delta)du\right) \left(\int_{\Delta_j} \lambda_i(u-\delta)du\right)^{k_j}}{k_j!} f_\delta(\delta) d\delta \end{aligned} \tag{1}$$

In the following sections we consider two cases: when the bins are infinitesimal and when they are finite.

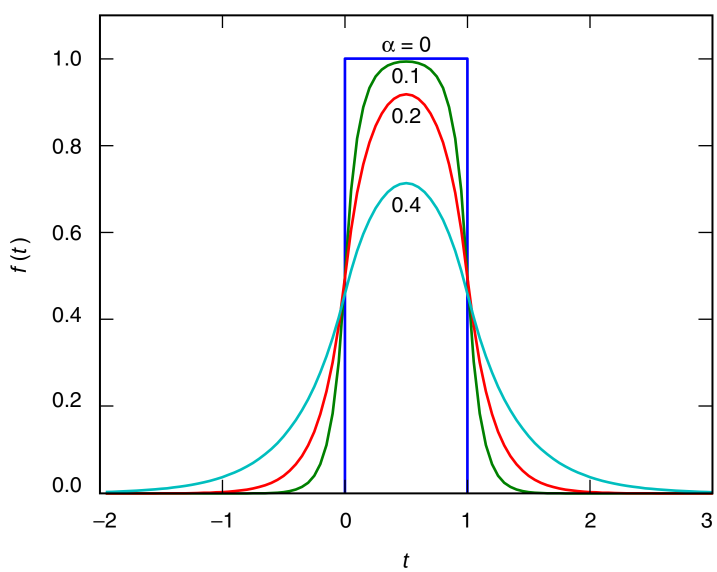

Fig. 1. $f(t)$ **for** $\alpha \in \{0.0, 0.1, 0.2, 0.4\}$.

A. Infinitesimal Bins

Let Δt approach 0 so that $\lambda_i(t)$ is constant over a bin and the probability of more than one photon in a bin is negligible. At the receiver we observe the photon arrival times $\{t_j\}, j = 1, \cdots, N$, from which we obtain the symbol likelihoods

$$
\begin{aligned}
p(\{t_j\}, N|\lambda_i) &= \int p(\{t_j\}, N|\lambda_i, \delta) f_\delta(\delta) d\delta \\
&= \exp\left(-\int \lambda(t)dt\right) \int f_\delta(\delta) \prod_{j=1}^{N} \lambda_i(t_j - \delta) d\delta \\
&= K \int f_\delta(\delta) \prod_{j=1}^{N} \left(1 + \frac{n_s}{n_b} p(t_j - \delta - i + 1)\right) d\delta \qquad (2)
\end{aligned}
$$

where, throughout, K denotes any constant that is not a function of λ_i and, hence, may be factored out of comparisons of conditional symbol likelihoods. We will refer to a receiver using Eq. (2) as an *energy-matched filter*.

1. Detector Jitter. Detector jitter may be modeled as an independent offset to each photon arrival: $t_j = s_j + \delta_j$, where the δ_j are independent of one another as well as of the arrival times $\{s_j\}$, and are identically distributed. In the presence of detector jitter, we have

$$
\begin{aligned}
p\big(\{t_j\}, N|\lambda_i\big) &= \int \cdots \int p\big(\{t_j\}, N|\{\delta_j\}, \lambda_i\big) p\big(\{\delta_j\}\big) d\{\delta_j\} \\
&= \exp\left(-\int \lambda(t)dt\right) \int \cdots \int \prod_{j=1}^{N} \lambda_i(t_j - \delta_j) p(\delta_j) d\{\delta_j\} \\
&= K \prod_j \left(1 + \frac{n_s}{n_b} f(t_j - i + 1)\right) \qquad (3)
\end{aligned}
$$

Analogous to the case of timing jitter, we refer to a receiver using Eq. (3) in the presence of detector jitter as an *energy-matched filter*. In the presence of timing jitter, Eq. (3) may be used as an approximation to Eq. (2). We refer to a receiver using Eq. (3) in timing jitter as an *approximate matched filter*.

B. Finite Bins

Practical considerations, e.g., the bandwidth required to transmit the arrival times of all photon arrivals, and limitations on locating distinct photon arrival times, will force the bins to have finite durations. In this section, we derive the symbol likelihoods for the case of finite bins and find some useful approximations. Let $m = j\Delta t - i, l = m + 1 - \delta$. Conditioned on δ and λ_i, k_j is Poisson, with mean

$$\begin{aligned}
\lambda_{i,j}(\delta) &= \int_{\Delta_j} \lambda_i(u-\delta)du \\
&= \int_{(j-1)\Delta t}^{j\Delta t} \big(n_b + n_s p(u - i - \delta + 1)\big)du \\
&= n_b\Delta t + n_s \int_{l-\Delta t}^{l} p(t)dt \\
&= n_b\Delta t + n_s \begin{cases} 0, & l \le 0 \\ l, & 0 < l \le \min(\Delta t, 1) \\ 1, & \min(\Delta t, 1) < l < \Delta t \\ \Delta t, & \Delta t < l < \max(\Delta t, 1) \\ 1 - l + \Delta t, & \max(\Delta t, 1) < l \le 1 + \Delta t \\ 0, & 1 + \Delta t < l \end{cases}
\end{aligned}$$

Solving for the marginals yields

$$\begin{aligned}
p(k_j|\lambda_i) &= \frac{1}{k_j!}\int_{-\infty}^{\infty} \exp\big(-\lambda_{i,j}(\delta)\big)\big(\lambda_{i,j}(\delta)\big)^{k_j} f_\delta(\delta)d\delta \\
&= \frac{1}{2\alpha k_j!}\Bigg(e^{(m+1)/\alpha}\int_{m+1}^{\infty}\big(\lambda_{i,j}(m+1-l)\big)^{k_j}\exp\big(-\lambda_{i,j}(m+1-l) - l/\alpha\big)dl \\
&\quad + e^{-(m+1)/\alpha}\int_{-\infty}^{m+1}\big(\lambda_{i,j}(m+1-l)\big)^{k_j}\exp\big(-\lambda_{i,j}(m+1-l) + l/\alpha\big)dl\Bigg)
\end{aligned}$$

In a conventional receiver, $\Delta t = 1$ and the distributions reduce to

$$p(k_j = k|\lambda_i) = p_0(k)p(k; m), j = i + m, i - m$$

where

$$p(k;m) = \begin{cases} e^{-1/\alpha} + \frac{e^{-1/\alpha} e^{n_b} e^{-n_b/\alpha n_s}}{\alpha n_s} h(k;b), & m = 0 \\ 1 + \frac{e^{-|m|/\alpha}(e^{-1/\alpha} - e^{1/\alpha})}{2} + \frac{e^{-(|m|+1)/\alpha} e^{n_b}}{2\alpha n_s} \left(e^{-n_b/\alpha n_s} h(k;b) + e^{2/\alpha} e^{n_b/\alpha n_s} h(k;a)\right), & |m| \geq 1 \end{cases}$$

$$p_0(k) = e^{-n_b} n_b^k / k!$$

$$h(k;a) = \frac{1}{n_b^k} \int_{n_b}^{n_s+n_b} \lambda^k e^{a\lambda} d\lambda$$

$$a = (\alpha n_s + 1)/(\alpha n_s)$$

$$b = (\alpha n_s - 1)/(\alpha n_s)$$

The contributions of signal photons to the likelihoods decay exponentially as $-|m|/\alpha$. For α sufficiently small, the probability of a signal photon arrival two slots from the signal slot is negligible, i.e., $p(k_j|\lambda_i) \approx p_0(k)$ for $|m| > 1$. Making the approximation that the symbol likelihoods factor, i.e., that $p(\{k_j\}|\lambda_i) \approx \prod_j p(k_j|\lambda_i)$, we have

$$p\big(\{k_j\}|\lambda_i\big) \approx K p(k_{i-1};1) p(k_i;0) p(k_{i+1};1) \tag{4}$$

We refer to a receiver using Approximation (4) as a *finite bin* receiver.

1. Poisson Approximation. The mean photons in the jth bin, conditioned on λ_i (but not on δ) are given by

$$\lambda'_{i,j} = \int_{(j-1)\Delta t}^{j\Delta t} \lambda'_i(t) dt$$

$$= n_b \Delta t + \frac{\alpha n_s}{2} \begin{cases} e^{m/\alpha}(e^{1/\alpha} - 1)(1 - e^{-\Delta t/\alpha}), & m \leq -1 \\ \frac{2(m+1)}{\alpha} + e^{-(m+1)/\alpha} + e^{m/\alpha}(e^{-\Delta t/\alpha} - e^{(1-\Delta t)/\alpha} - 1), & -1 < m \leq \min(0, \Delta t - 1) \\ \frac{2\Delta t}{\alpha} + e^{-(m+1)/\alpha}(1 - e^{\Delta t/\alpha}) + e^{m/\alpha}(e^{-\Delta t/\alpha} - 1), & \min(0, \Delta t - 1) < m < 0 \\ \frac{2}{\alpha} + e^{(m-\Delta t)/\alpha}(1 - e^{1/\alpha}) + e^{-m/\alpha}(e^{-1/\alpha} - 1), & 0 < m < \max(0, \Delta t - 1) \\ \frac{2(\Delta t - m)}{\alpha} + e^{(m-\Delta t)/\alpha} + e^{-(m+1)/\alpha}(1 - e^{\Delta t/\alpha} - e^{1/\alpha}), & \max(0, \Delta t - 1) \leq m < \Delta t \\ e^{-m/\alpha}(1 - e^{-1/\alpha})(e^{\Delta t/\alpha} - 1), & \Delta t \leq m \end{cases}$$

In a conventional receiver, $\Delta t = 1$, in which case the means reduce to

$$\lambda'_{i,j} = n_b + \frac{\alpha n_s}{2} \begin{cases} \frac{2}{\alpha} + 2e^{-1/\alpha} - 2, & m = 0 \\ e^{-|m|/\alpha}\left(e^{1/\alpha} - 1\right)(1 - e^{-1/\alpha}), & |m| \geq 1 \end{cases} \tag{5}$$

We may approximate the photon count in the jth bin as Poisson, with mean $\lambda'_{i,j}$. In practice, this would allow one to estimate just the mean $\lambda'_{i,j}$ of the bin, rather than the distributions $p(k_j|\lambda_i)$. For sufficiently small α, we may also take $\lambda'_{i,j} \approx n_b$ for $|m| > 1$. Using Eq. (5) for the means, and assuming the symbol likelihoods factor and that $\lambda'_{i,j} \approx n_b$ for $|m| > 1$, yields the approximation

$$p\big(\{k_j\}|\lambda_i\big) \approx K \left(1 + \frac{n_s}{n_b}\left(1 + \alpha(e^{-1/\alpha} - 1)\right)\right)^{k_i} \left(1 + \frac{\alpha n_s}{2n_b}\left(1 - e^{-1/\alpha}\right)^2\right)^{k_{i+1}+k_{i-1}} \tag{6}$$

where K is a constant independent of i. By assuming the photon intensity is deterministic and the counts Poisson, we obtain the *energy matching* receiver of [10]. We refer to a receiver using Approximation (6) as a *Poisson approximation* receiver.

III. Capacity

Let $X \in \{1, \cdots, M\}$ be the random transmitted PPM symbol, N the number of observed photons, and T the random vector of photon arrival times. The mutual information of the Poisson-PPM ITI channel with equally likely transmitted PPM symbols is given by

$$I(X; T, N) = E_{X,T,N} \log_2 \frac{Mp(T, N|X)}{\sum_{i=1}^{M} p(T, N|X = i)} \quad \text{bits/symbol} \tag{7}$$

The equiprobable-input mutual information corresponds to the capacity of the channel with input symbols restricted to being transmitted equiprobably. In the presence of ITI (and no ISI), symbols should not be sent equiprobably if one wants to maximize the mutual information, since pulses in the first and last slots are more likely to be received with good reliability. However, our interest is in applying a receiver derived for the ITI channel to a practical channel with ISI. Moreover, in practice, system constraints may enforce an equiprobable distribution. With a slight abuse of notation, we refer to Eq. (7) as the capacity of the channel: $C_{\text{ITI}}(\alpha, n_s, n_b, M) = I(X; T, N)$ for the ITI channel or $C_{\text{ISI}}(\alpha, n_s, n_b) = I(X; T, N)$ for the ISI channel, where the statistics of T reflect the channel model. In numerical results, Eq. (7) is estimated by means of a sample mean. Since the ISI channel can be generated by summing the outputs of the ITI channel, we have $C_{\text{ITI}}(\alpha, n_s, n_b, M) \geq C_{\text{ISI}}(\alpha, n_s, n_b)$.

IV. Error Rates, Losses

All simulation results in this section are reported for an $M = 16$ Poisson PPM channel with $n_b = 0.2$ and exponentially distributed jitter with $\alpha = 0.2$. Figure 2 illustrates symbol-error rates (SERs) obtained using the energy-matched filter given by Eq. (2), as well as four approximations: no modeling or compensation for the ISI, using the approximate matched filter of Eq. (3), the finite-bin receiver of Approximation (4), and the Poisson approximation of Approximation (6). In the case of no compensation, the receiver computes symbol likelihoods assuming no ITI and Poisson statistics:

$$p\big(\{k_j\}|\lambda_i\big) \approx K \left(1 + \frac{\hat{n}_s}{\hat{n}_b}\right)^{k_i} \tag{8}$$

where the means are estimated as

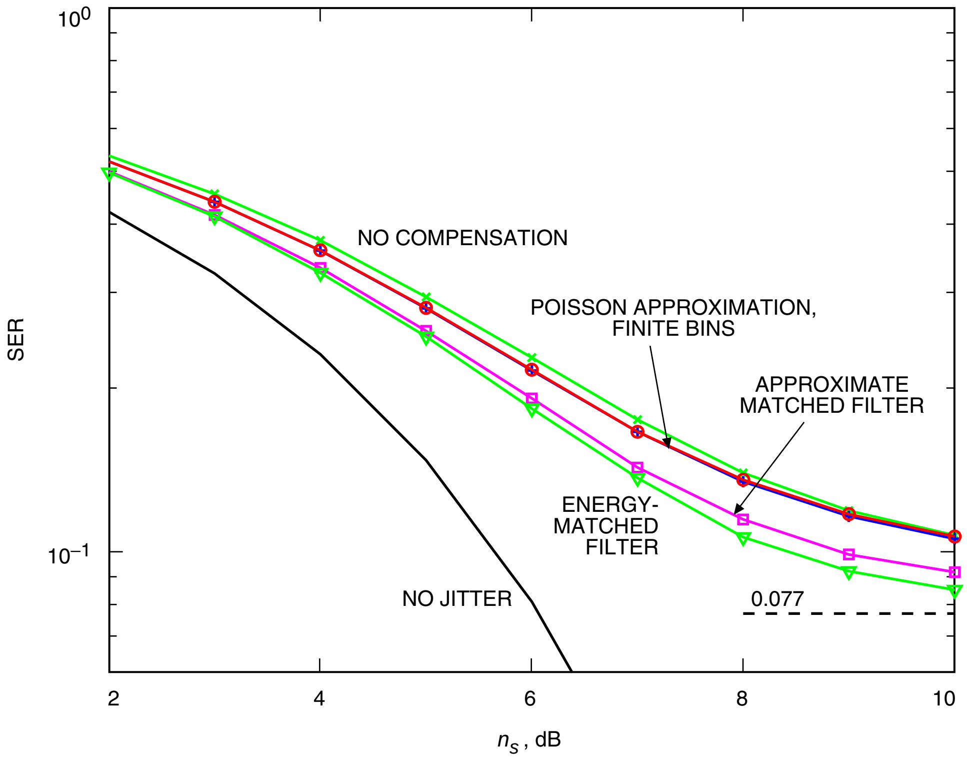

Fig. 2. Symbol-error rates, ITI, α = 0.2, M = 16, n_b = 0.2.

$$\hat{n}_b = \frac{1}{M-1}\sum_{j\neq i}\lambda'_{i,j}$$

$$\approx n_b + \frac{\alpha n_s}{(M-1)}\left(1 - e^{-1/\alpha}\right)^2$$

$$\hat{n}_s = \lambda'_{i,i} - \hat{n}_b$$

$$\approx n_b + n_s\left(1 + \alpha(e^{-1/\alpha} - 1)\right) - \hat{n_b}$$

corresponding to the perfect knowledge of the observed means in a signal and (average) non-signal slot. The performance in the absence of ITI is also illustrated for comparison.

An asymptote occurs in the SER at ≈0.077, which can be seen as follows. Suppose a pulse is transmitted in position i. In the limit of large signal power, if $|\delta| > 0.5$ for $i \notin \{1, M\}$, $\delta > 0.5$ for $i = 1$, or $\delta < -0.5$ for $i = M$, a symbol error will be made. Since the SER is non-increasing in the signal power, this yields a bound on the symbol-error rate,

$$\text{SER} \geq \left(\frac{M-1}{M}\right)e^{-1/(2\alpha)}$$

which is approached as $n_s \to \infty$. For $M = 16, \alpha = 0.2$, the symbol-error rate approaches the asymptote 0.077, as illustrated in Fig. 2. This presumes α is not a function of n_s, whereas in a practical system jitter would decrease with increasing signal power. Nonetheless, there would remain some mean residual jitter even at high signal powers.

Symbol-error rates alone can be misleading, as we are typically interested in the performance of a coded system. Figure 3 illustrates codeword-error rates for a coded system consisting of the serial concatenation of a convolutional code and accumulator, mapped through a bit interleaver (see [11] for a detailed description of the code). One codeword corresponds to 3780 $M = 16$ PPM symbols. Measured relative to the no-ITI case at a word-error rate (WER) of 10^{-4}, there is a loss of 2.2 dB with no compensation. The energy-matched filter recovers 1.2 dB of this loss. Using Eq. (3), Approximation (4), or Approximation (6) costs, respectively, 0.2, 0.35, and 0.45 dB relative to a matched filter. Note that the relative gains are not predicted by the SER curves. Relative gains in SER and WER may differ since the SER depends only on the sign of the difference of the likelihoods, whereas the WER for a soft-input decoder will depend on the sign and magnitude of the difference. Figure 3 also illustrates the capacity of the ITI-free and the ITI channels. The loss in capacity when ITI is introduced is 0.91 dB, mirroring the loss in system performance of 0.97 dB. This suggests using the capacity loss to quantify the degradation due to ITI.

A. Detector Jitter

The word-error rate in the presence of detector jitter using either no compensation, Approximation (8), or the matched filter, Eq. (3), is illustrated in Fig. 4, along with the corresponding data for timing jitter. Since the photon offsets within a symbol are independent, the loss for the same jitter statistics are less than for timing jitter. There is no asymptote in the log domain for the SER since the statistics approach a deterministic waveform in the presence of Gaussian noise with large n_s, for which the SER decreases.

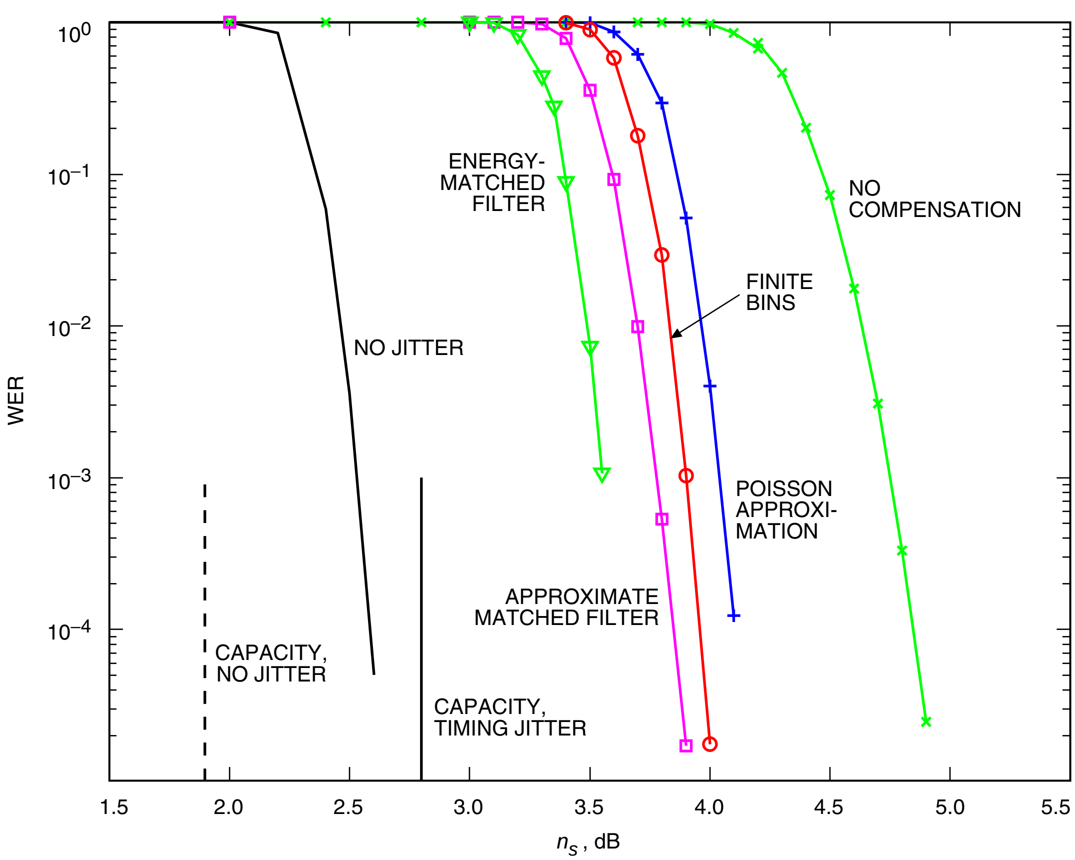

Fig. 3. Word-error rates, ITI, $\alpha = 0.2$, $M = 16$, $n_b = 0.2$.

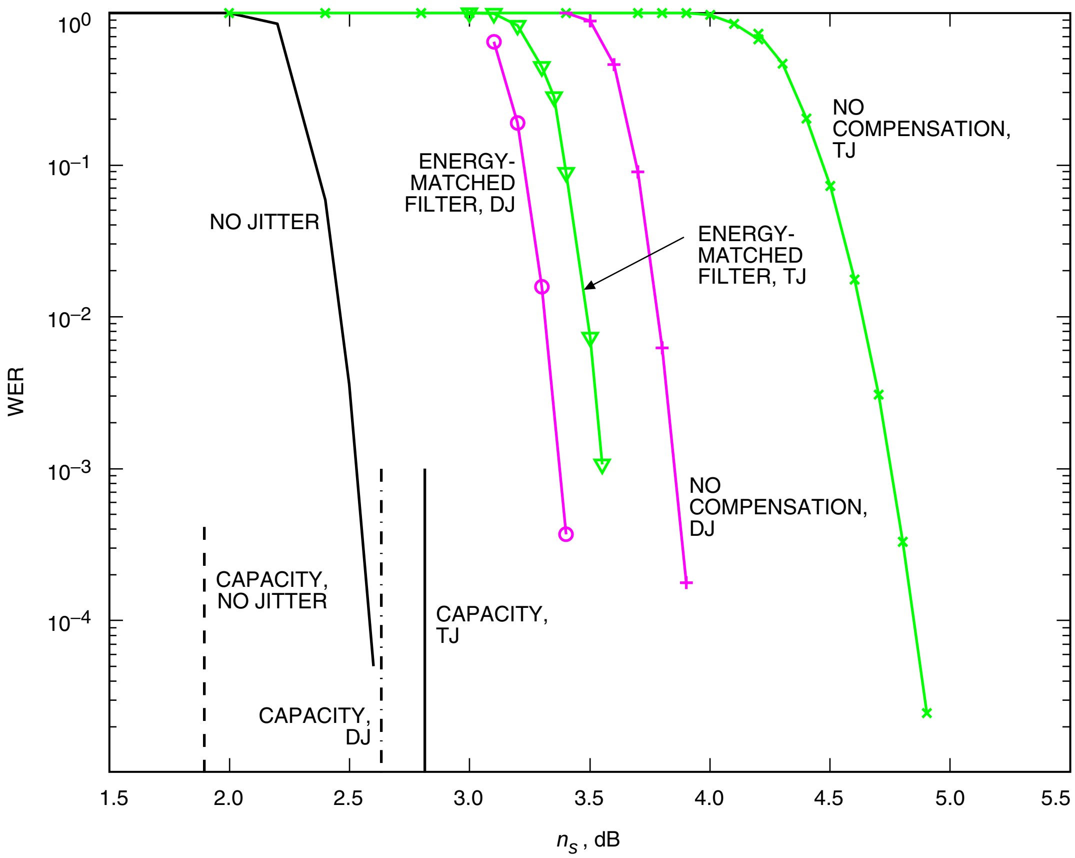

Fig. 4. Word-error rates, detector jitter (DJ), and timing jitter (TJ), ITI, α = 0.2, M = 16, n_b = 0.2.

B. The Effects of Inter-*Symbol* Interference

We have to now neglect ISI, modeling only ITI. This essentially assumes that each symbol is observed over an independent, parallel channel. In this section, we examine the loss when using the receiver derived for the ITI channel on a channel with ISI (ITI is always presumed present). Let $\{x_i\}$ denote the sequence of transmitted pulse locations, where each $x_i \in \{1, \cdots, M\}$, and let the received intensity of the Poisson process be given by

$$n_b + n_s \sum_i p(t - x_i + 1 - iM)$$

Maximum a posteriori (MAP) detection of PPM in the presence of ISI follows as an extension to our prior results by incorporating memory into the decoding trellis; see, e.g., [4,12]. However, the number of states goes as M^ν, where ν is the memory, in symbols, of the ISI, which would be prohibitively complex for a high-order PPM channel (this assumes a trellis defined over PPM symbols—one could alternately use a trellis over slots, but we do not address that here).

We bound the performance of a code concatenated with MAP detection of PPM in ISI as follows. Let MAP-ISI and MAP-ITI refer to the MAP receivers for PPM in ISI and ITI, respectively (the MAP-ITI receiver is the energy-matched filter described earlier). We assume that, since $C_{\rm ISI} \leq C_{\rm ITI}$, the performance of a coded system concatenated with MAP-ISI on the ISI channel will be worse than coded performance with MAP-ITI on the ITI channel. Furthermore, it is clear that MAP-ISI on the ISI channel will perform better than MAP-ITI on the ISI channel. This yields upper and lower bounds on the performance of MAP-ISI on the ISI channel. Figure 5 overlays several of the error rates illustrated in Fig. 3 with the performance of the same receivers in ISI. The losses incurred for this case are bounded by ≈0.1 dB for the approximate matched filter; 0.2 dB for the energy-matched filter, finite bins, or the Poisson approximation; and 0.05 dB for no compensation. Hence, ignoring intersymbol interference in this case costs no more than 0.2 dB. This loss will decrease with increasing M and increase with increasing α. Modulations that insert an inter-symbol guard time between symbols, e.g., for slot synchronization, would further reduce the gap between ITI and ISI performance.

C. Loss as a Function of RMS Jitter

Simulation results illustrated in Figs. 3 and 4 demonstrate that the degradation in performance tracks well with the loss in capacity. Figure 6 illustrates the loss in capacity as a function of the root-mean-square (RMS) of the jitter, $\alpha\sqrt{2}$, for timing and detector jitter. The loss is defined as

$$\text{loss}_{\text{dB}} = 10 \log_{10} \frac{n_s}{n_s'} \tag{9}$$

where n_s, n_s' are the signal powers satisfying $C_{\text{ITI}}(0, n_s, n_b, M) = C_{\text{ITI}}(\alpha, n_s', n_b, M) = M/(2\log_2 M)$, i.e., the capacity corresponding to a rate 1/2 error-correction-code. Note that Eq. (9) gives a *lower bound* on the loss. There is also a small additional loss when ISI is introduced, and there would be larger losses if approximations to the likelihoods were used.

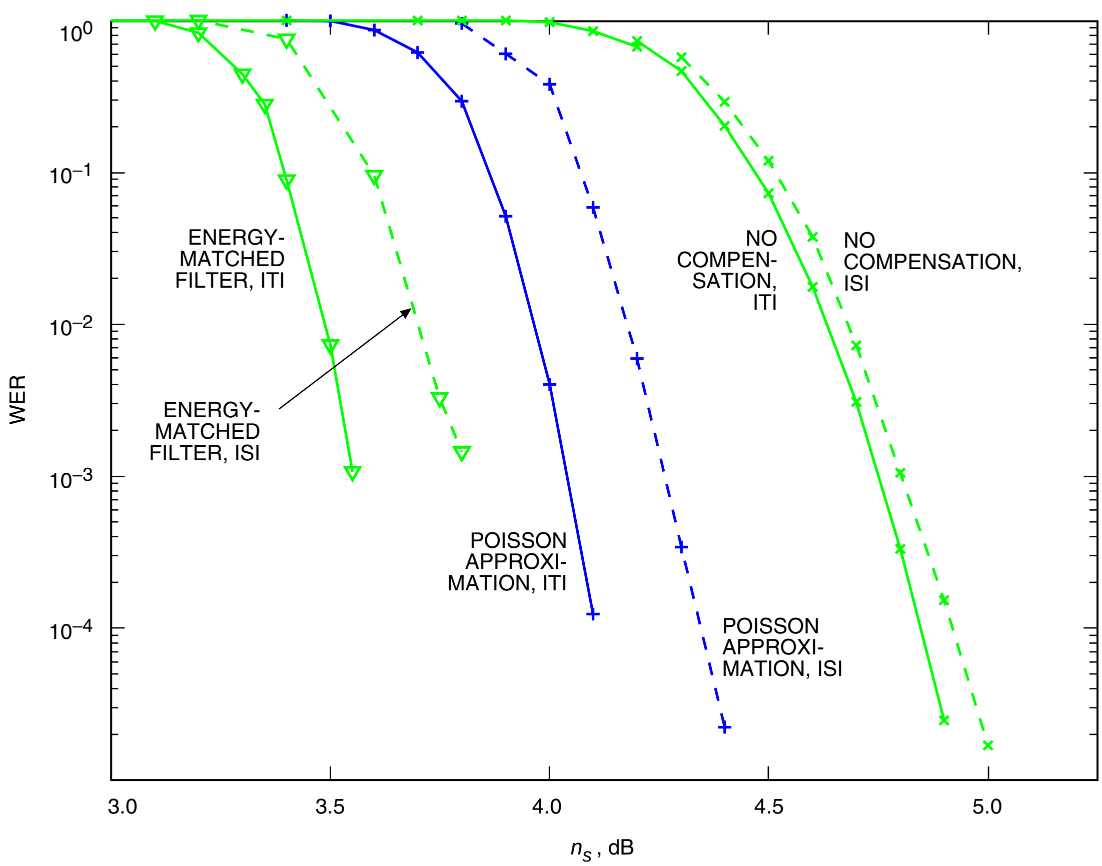

Fig. 5. Word-error rates, ITI (solid) and ISI (dashed), α = 0.2, M = 16, n_b = 0.2. MAP performance in ISI is bounded by the solid and dashed lines for the energy-matched filter.

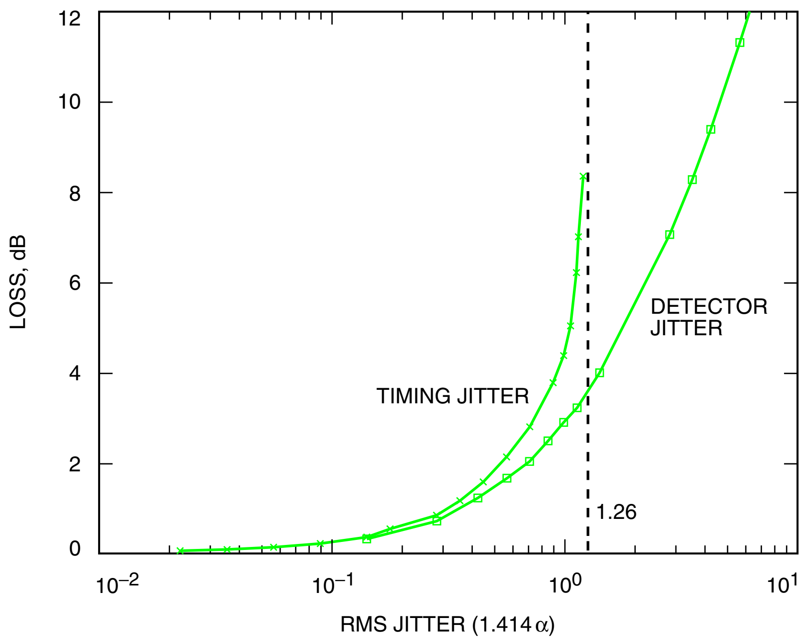

Fig. 6. Loss in capacity as a function of RMS jitter as a fraction of a slot width, $M = 16$, $n_b = 0.2$.

We observe a threshold in the tolerable RMS timing jitter for a given code rate beyond which no increase in the signal power will allow us to close the link. This appears for a similar reason as that for the SER asymptote. Suppose that the channel mutual information is non-decreasing in n_s. At sufficiently large n_s, the only uncertainty in the pulse position comes from random jitter. In the limit of large n_s, the channel is equivalent to the transmission of an integer $X \in \{1, 2, \cdots, M\}$ in the presence of jitter, where the received sample is $Y = X + \delta$. The mutual information of this channel, $I(X; Y|\alpha)$, yields the largest supportable data rate as a function of α (for any signal power n_s). We find that $I(X; Y|\alpha = 0.89) = 0.5$. Hence, for $\alpha \geq 0.89$, no additional signal power will allow us to close the link with a rate 1/2 code and $\text{loss}_{\text{dB}} \to \infty$, an asymptote illustrated in Fig. 6. The location of the asymptote is a function of α and M but has no dependence on n_b. We observe no such asymptote for detector jitter.

V. Conclusions

In the presence of ITI introduced by random jitter, utilizing the exact symbol likelihoods can lead to significant gains over a conventional receiver that assumes no ITI is present. The exact symbol likelihoods are, however, complex. A sequence of approximations to the likelihoods was introduced that trade off complexity and performance. The maximum-likelihood receiver for the ITI channel may be used on the ISI channel with small losses, while saving significantly on complexity.

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