Electronic Beam Steering of Semiconductor Injection Lasers

J. Katz
Communications Systems Research Section

A theoretical analysis of the problem of beam steering of semiconductor injection lasers is presented. The required modifications of the dielectric constant profile of the laser structure are derived, and a practical method for implementing the needed modifications is outlined.

I. Introduction

Many applications of lasers in general, and semiconductor injection lasers in particular, call for the controlled deflection, or steering, of the radiation pattern emitted by them. That is the case, for example, in pointing and tracking subsystems of optical communication links or optical radars and in systems for optical data recording and retrieval.

In many cases the beam steering is performed by optical-mechanical systems of mirrors, lenses and scanners. Several physical phenomena such as the electrooptic, acoustooptic or piezoelectric effects can also be utilized for building beam deflectors (Ref. 1). In this paper we present a theoretical analysis of yet another important way of achieving this goal by incorporating the steering mechanism within the semiconductor laser itself via the modification of its index of refraction. Such a monolithic configuration has the potential advantages of higher reliability and considerable savings in size and weight of the system. Another method, which is a subject of a separate publication, is by controlling the individual phases of lasers in a phase-locked array configuration, in a similar fashion to microwave phased arrays (Ref. 2).

In a paper published recently (Ref. 3) electronic steering of a semiconductor laser beam via the modification of the dielectric constant has been demonstrated. The radiation pattern was deflected ±14° with respect to the normal of the laser facet. Since the far field radiation pattern was about 6°, this represents deflection of about ±2 beamwidths.

The purpose of this paper is twofold: first, to provide a theoretical analysis of the problem, and second, to outline the approach for implementing the beam steering method.

Section II reviews the relation between the far-field radiation pattern of the laser (i.e., the beam to be deflected) and the near-field pattern (i.e., the field distribution at the laser facet), which is basically a Fourier transform relation. Then the wave equations for the modified laser field are derived, establishing the general relation between the unperturbed dielectric constant profile of the laser cavity, the desired amount of beam deflection and the specific modifications of the dielectric constant profile that must be implemented. Section III gives several specific examples of dielectric constant profiles which represent several types of semiconductor lasers. In all cases it is found that an antisymmetric modification of the imaginary part of the dielectric constant can cause beam deflection. Finally, Section IV outlines a method for achieving the desired modification of the dielectric constant profile, thus serving as a general guideline for this application.

II. Modification of the Laser Field and Dielectric Constant Needed for Beam Deflection

In this section we establish the relations between the desired beam deflection and the modifications of the laser field that need to be employed for this purpose. The schematic configuration (Fig. 1) shows a top view of a semiconductor injection laser and its emitted radiation pattern. Deflection of the beam by angle θ is equivalent to shifting the far-field pattern by this angle.

The relation between the far-field radiation pattern and the near-field radiation pattern (i.e., the field distribution at the laser facet – the (xy) plane) has been thoroughly investigated (Refs. 4-9). Basically it is found that the far-field pattern is the Fourier transform of the near-field pattern times an obliquity function $g(\phi)$:

$$U(\phi) \propto g(\phi) \int_{-\infty}^{\infty} E(y)\, e^{-ik(\phi)y} dy \tag{1}$$

where ϕ is a general angle in the (yz) plane (see Fig. 1), E and U are the near-and far-field patterns, respectively, and $k(\phi)$ is given by

$$k(\phi) = \frac{2\pi}{\lambda} \sin \phi \tag{2}$$

The different works cited above give different forms for $g(\phi)$. A good approximation, commonly used in the literature (Ref. 10) is $g(\phi) \sim \cos \phi$. In the following analysis we will not take this factor into account since the additional attenuation caused by it when the beam is deflected can be compensated, in principle, by increasing the total current through the device.

The equation governing the near-field distribution of the laser near field E is the Helmholtz equation:

$$\frac{d^2E}{dy^2} + (\epsilon\, k^2 - \beta^2) E = 0 \tag{3}$$

where ϵ is the dielectric constant profile along the y direction, β is the propagation constant, $k = 2\pi/\lambda$ and λ is the wavelength of the radiation in vacuum. The notations needed in this paper and the relations between the dielectric constant, index of refraction and gain and loss coefficients of a particular material are summarized in Appendix A. The reduction of the general three-dimensional formulation of the wave equation in the laser structure to the one-dimensional problem considered here (as implied by Eq. 3) is outlined in Appendix B. In all the following analysis, the superscript 0 indicates the value of the unperturbed parameter, i.e., its value when the laser beam is not deflected, and all unsuperscripted parameters are valued in the deflected case. Also the subscripts r and i refer to the real and imaginary parts, respectively, of the parameter.

Using the well-known rules of the Fourier transform we know that in order to obtain a beam deflection by an angle θ, i.e.,

$$U[k(\phi)] = U^0\, [k(\phi) - k(\theta)] \tag{4}$$

the near-field pattern has to be multiplied by a phase factor, namely,

$$E(y) = E^0(y)\, e^{ik(\theta)y} \tag{5}$$

It is clear that in order for such an operation to take place, the dielectric constant profile of the laser has to be modified, so that the modified waveguide will support the modified near-field pattern. The magnitude and shape of the modification are derived below:

Using Eq. (5) in Eq. (3), noting that E^0 by definition satisfies the following equation:

$$\frac{d^2E^0}{dy^2} + [\epsilon^0\, k^2 - (\beta^0)^2]\, E^0 = 0 \tag{6}$$

we obtain

$$\epsilon\, k^2 - \beta^2 = \frac{[\epsilon^0\, k^2 - (\beta^0)^2 + k^2(\theta)]\, E^0(y) - 2ik(\theta)\dfrac{dE^0}{dy}}{E^0(y)} \tag{7}$$

Equating the real and imaginary parts of Eq. (7) and using the results of Appendix C which establishes the relations between the propagation constants of the optical mode in the laser waveguide and the field distribution, we obtain the following basic equations, relating the needed modification of the real and imaginary parts of the dielectric constant to the desired amount of beam deflection:

$$\Delta\epsilon_r + \frac{\int_{-\infty}^{\infty} \Delta\epsilon_r |E^0|^2 dy}{\int_{-\infty}^{\infty} |E^0|^2 dy} = -\frac{2k(\theta)}{k^2} \frac{\frac{dE_r^0}{dy} E_i^0 - E_r^0 \frac{dE_i^0}{dy}}{(E_r^0)^2 + (E_i^0)^2} \tag{8}$$

and

$$\Delta\epsilon_i = \frac{2}{k^2}[\beta_r^0 \Delta\beta_i + \beta_i^0 \Delta\beta_r] - \frac{2k(\theta)}{k^2} \frac{\frac{dE_r^0}{dy} E_r^0 + \frac{dE_i^0}{dy} E_i^0}{(E_r^0)^2 + (E_i^0)^2} \tag{9}$$

where

$$\Delta\epsilon_r = \epsilon_r - \epsilon_r^0 \tag{10a}$$

$$\Delta\epsilon_i = \epsilon_i - \epsilon_i^0 \tag{10b}$$

$$\Delta\beta_r = \beta_r - \beta_r^0 \tag{10c}$$

$$\Delta\beta_i = \beta_i - \beta_i^0 \tag{10d}$$

are the changes of the corresponding parameters from the unmodified case.

As noted in Appendix C, significant simplification of Eqs. (8) and (9) results if the modifications of the dielectric function is an antisymmetric function; i.e.,

$$\Delta\epsilon_r(y) = -\Delta\epsilon_r(-y) \tag{11a}$$

$$\Delta\epsilon_i(y) = -\Delta\epsilon_i(-y) \tag{11b}$$

In this case Eq. (8) is reduced to

$$\Delta\epsilon_r = -\frac{2k(\theta)}{k^2} \frac{\frac{dE_r^0}{dy} E_i^0 - E_r^0 \frac{dE_i^0}{dy}}{(E_r^0)^2 + (E_i^0)^2} \tag{12}$$

while Eq. (9) is reduced to

$$\Delta\epsilon_i = -\frac{2k(\theta)}{k^2} \frac{\frac{dE_r^0}{dy} E_r^0 + \frac{dE_i^0}{dy} E_i^0}{(E_r^0)^2 + (E_i^0)^2} \tag{13}$$

Equations (12) and (13) give explicit expressions for $\Delta\epsilon_r$ and $\Delta\epsilon_i$, respectively, while Eqs. (8) and (9) are integral equations for these quantities.

In the following section we will apply Eqs. (12) and (13) to several specific examples of semiconductor laser waveguides.

III. Specific Examples of Dielectric Constant Modification

In this section we present three examples of beam-steering in one-dimensional waveguide structures. In the first example the unperturbed waveguide has a pure real index guiding; in the second case, a pure imaginary index guiding, and in the last example, a general complex index guiding. It is found that in all cases the beam deflection can be achieved by establishing an antisymmetric modification of the imaginary part of the dielectric constant profile along the laser junction plane (i.e., y direction in Fig. 1).

A. Pure Real Index Guiding — the "Sech²" Profile

We consider the following dielectric constant profile, shown in Fig 2a:

$$\epsilon^0(y) = \tilde{\epsilon}\left[1 + \delta \cdot \left(\mathrm{sech}^2 \sqrt{\frac{\delta \cdot \tilde{\epsilon}}{2}} ky\right)\right] \tag{14}$$

where $\tilde{\epsilon}$ is the dielectric constant at $y \to \pm\infty$ and δ is a constant. Although this profile represents only a first-order approximation to real life devices, it is analyzed here since the simple analytic solution that one obtains in this case serves to illuminate the basic underlying requirements of beam steering.

Defining the dimensionless coordinate

$$\zeta \equiv \sqrt{\frac{\delta \cdot \tilde{\epsilon}}{2}} ky \tag{15}$$

the wave equation (6) is reduced to

$$\frac{d^2 E^0}{d\zeta^2} + 2\,\mathrm{sech}^2(\zeta)\, E^0(\zeta) = \frac{2}{\delta \cdot \tilde{\epsilon}}\left[\frac{(\beta^0)^2}{k^2} - \tilde{\epsilon}\right] E^0(\zeta) \tag{16}$$

which is known to have a solution of the form

$$E^0(y) = \operatorname{sech}(\zeta) = \operatorname{sech}\left(\sqrt{\frac{\delta\tilde{\epsilon}}{2}}\,ky\right) \tag{17}$$

with the propagation constant β^0 given by

$$(\beta^0)^2 = k^2\,\tilde{\epsilon}\left(1 + \frac{\delta}{2}\right) \tag{18}$$

Using Eqs. (17) and (18) in Eqs. (12) and (13) we obtain

$$\Delta\epsilon_r = 0 \tag{19}$$

$$\Delta\epsilon_i(y) = \sqrt{2\delta\tilde{\epsilon}}\,\frac{k(\theta)}{k}\tanh\left(\sqrt{\frac{\delta\tilde{\epsilon}}{2}}\,ky\right) \tag{20}$$

or, using the definition of Eq. (2):

$$\Delta\epsilon_i(y) = \sqrt{2\delta\tilde{\epsilon}}\,(\sin\theta)\tanh\left(\sqrt{\frac{\delta\tilde{\epsilon}}{2}}\,ky\right) \tag{21}$$

The profile of $\Delta\epsilon_i$ (Eq. 21) is shown in Fig. 2b. It is an antisymmetric function, as expected. In this case there is no change in the real part of the dielectric constant profile.

B. Pure Imaginary Index Guiding — the Quadratic Profile

We consider the following dielectric constant profile:

$$\epsilon^0(y) = \begin{cases} \epsilon(0) - a^2y^2 & |y| < \dfrac{S}{2} \\ \epsilon(0) - a^2\left(\dfrac{S}{2}\right)^2 & |y| > \dfrac{S}{2} \end{cases} \tag{22}$$

This example describes with a reasonable accuracy the guiding mechanism of a contact-stripe laser with "medium" stripe widths S (Ref. 10, Ch. 7.10). In the following we will neglect the effects of the field distribution tails at $|y| > S/2$, which is a good approximation for not too narrow stripe widths.

The complex parameter a is given by Ref. 10, Ch. 7.10

$$a = a_r + ia_i = (1+i)\sqrt{\frac{\lambda\sqrt{\epsilon_r^0}\,g_m}{\pi S^2}} \tag{23}$$

where g_m is the gain in the center of the laser stripe. The parameters in Eq. (23) are depicted in Figs. 3a and 3b.

The fundamental mode of the electric field in this waveguide is given by

$$E^0(y) = e^{-K^2y^2(1+i)} \quad |y| > \frac{S}{2} \tag{24}$$

where K is defined by

$$K^4 \equiv \frac{k\sqrt{\epsilon_r^0}\,g_m}{2S^2} \tag{25}$$

Using the results of Appendix C, we find the real and imaginary parts of the wave propagation constant:

$$(\beta_r^0)^2 = k^2\epsilon_r^0 - \frac{K^2}{2} \tag{26}$$

$$\beta_i^0 = \frac{k\sqrt{\epsilon_r^0}\,g_m - K^2}{2\beta_r^0} \tag{27}$$

It is interesting to note from Eq. (26) that because of the gain guiding, a wave can propagate although its velocity is higher than the velocity of light in the material, a phenomenon that is impossible in real index waveguides.

Using Eqs. (2), (24)-(27) in Eqs. (12), (13) we obtain the needed modification in the dielectric profile:

$$\Delta\epsilon_r = -4\left(\frac{K^2}{k}\right)\cdot(\sin\theta)\cdot y \quad |y| < \frac{S}{2} \tag{28}$$

and

$$\Delta\epsilon_i = -\Delta\epsilon_r = 4\left(\frac{K^2}{k}\right)\cdot(\sin\theta)\cdot y \quad |y| < \frac{S}{2} \tag{29}$$

We note again that antisymmetrical modifications of the index of refraction are needed. Another interesting feature is that in this case the modification in the real part of the dielectric constant has the same magnitude and the opposite sign as the modification of the imaginary part of the dielectric constant.

C. Complex Index Guiding — Slab Waveguide Profile

This example can be used to describe the behavior of many generic types of lasers such as the Buried-Heterostructure laser (Ref. 11), the Channelled-Substrate-Planar laser (Ref. 12), and the Deep Diffusion Stripe laser (Ref. 13). The dielectric constant profile in such structures, as shown in Fig. 4, is given by

$$\epsilon_r = \begin{cases} \epsilon_{r2} & |y| < \frac{S}{2} \quad (30a) \\ \epsilon_{r1} & |y| > \frac{S}{2} \quad (30b) \end{cases}$$

$$\epsilon_i = \begin{cases} \epsilon_{i2} & |y| < \frac{S}{2} \quad (30c) \\ \epsilon_{i1} & |y| > \frac{S}{2} \quad (30d) \end{cases}$$

where ϵ_{r1}, ϵ_{r2}, ϵ_{i1} and ϵ_{i2} are constants. Typical stripe widths in lasers of these types are usually – although not always – narrower than in gain-guided lasers described in the previous example (2-5 μm vs 5-15μm).

In this case the field solutions are given by

$$E^0(y) = \begin{cases} \cos h^0 y & (31a) \\ \cos\left(h^0 \frac{S}{2}\right) e^{q^0 S/2}\, e^{-q^0 |y|} & (31b) \end{cases}$$

where h and q are, in general, complex numbers, and their values are determined by the eigenvalue equation (Ref. 1, Ch. 19):

$$\left(h^0 \frac{S}{2}\right) \tan\left(h^0 \frac{S}{2}\right) = \left(q^0 \frac{S}{2}\right) \tag{32}$$

Following the calculations outlined in Appendix C, the propagation constant is given by

$$(\beta_r^0)^2 = \epsilon_{r2}\, k^2 - (h_r^0)^2 = \epsilon_{r1}\, k^2 + (q_r^0)^2 \tag{33}$$

$$\beta_i^0 = \frac{k^2}{2\beta_r^0} \left[\epsilon_{i1} + (\epsilon_{i2} - \epsilon_{i1})\, \Gamma_y\right] \tag{34}$$

where Γ_y is the fraction of the mode energy contained under the stripe width S.

Using Eqs. (33), (34) in Eqs. (12), (13) we obtain

$$\Delta\epsilon_r = \begin{cases} -\dfrac{2k(\theta) h_i^0}{k^2 \cos^2 (h_r^0 y)} \left[(h_r^0 y) + \dfrac{1}{2} \sin (2\, h_r^0 y)\right] & |y| < \dfrac{S}{2} \\ \pm \dfrac{2 q_i^0 k(\theta)}{k^2} & |y| > \dfrac{S}{2} \end{cases}$$

(35a, 35b)

and

$$\Delta\epsilon_i = \begin{cases} \dfrac{2k(\theta) h_r^0}{k^2} \tan (h_r^0 y) & |y| < \dfrac{S}{2} \quad (36a) \\ \mp \dfrac{2 q_r^0 k(\theta)}{k^2} & |y| > \dfrac{S}{2} \quad (36b) \end{cases}$$

Note that in this case the modification of the real part of the dielectric constant is much smaller than the modification of the imaginary part of the dielectric constant, since $|h_i^0| << h_r^0$ and $|q_i^0| << q_r^0$.

IV. Implementation of Beam Steering via Asymmetric Current Injection Across the Laser Stripe

In the last sections it has been shown that antisymmetric modifications of the dielectric constant have to be established across the laser structure if its beam is to be deflected. The major contribution to the beam deflection comes from modifying the imaginary part of the dielectric constant, since it is a well-known fact that Fourier transform of real functions are always symmetric, regardless of the nature of the (real) function. As shown in Appendix A (Eq. A-6), the imaginary part of the dielectric constant ϵ_i is related to the gain in the laser medium g:

$$g = \frac{2\pi}{\lambda} \frac{\epsilon_i}{\sqrt{\epsilon_r}} = k \frac{\epsilon_i}{\sqrt{\epsilon_r}} \tag{37}$$

In addition, it is known that the gain in the laser medium is related to the carrier density N in its active layer. A commonly used formula is (Ref. 14)

$$g(N) = \frac{A}{\beta_r^0} (N - N_{om}) \tag{38}$$

where A is a proportionality constant and N_{om} is the carrier density needed for transparency (i.e., $g = 0$). For GaAs at room temperature $A \cong 1.6 \cdot 10^{-6}$ cm^3-sec^{-1} and $N_{om} \cong 7.5 \cdot 10^{17}$ cm^{-3}.

Equations (37), (38) suggest a possible method for modifying the dielectric constant via modifying the distribution of the carrier density across the laser structure. This can be done by splitting the stripe contact of the laser to several parallel stripes, and passing different amounts of current through each stripe, as shown schematically in Fig. 5. Although the exact formulas are quite lengthy (Ref. 10, Ch. 7.7), the carrier density profile in the active region due to current injection in a stripe contact is basically a bell-shaped function whose width and height are roughly proportional to the stripe width and to the current density through the stripe, respectively.

An example of carrier distribution is shown in Fig. 6, where the total current of 250 mA is divided between two stripe contacts whose width is 2 μm and whose center-to-center separation is 8 μm (the diffusion length is taken to be 3.6 μm). The currents ratio in the two stripes is $\gamma:(1-\gamma)$. For $\gamma = 0.5$, the distribution is symmetric and no beam deflection is expected. However, for $\gamma < 1/2$, an antisymmetric component whose shape is shown in Fig. 7 is established across the structure, resulting in a beam deflection. If we approximate the laser structure by the quadratic gain medium (example 2 of previous section), and assume that we want to deflect the beam by 0.1 rad (5.7°), and that $g_m = 100$ cm^{-1}, $\sqrt{\epsilon_r} = 3.6$ and $\lambda = 0.9$ μm, then we need a gain difference between the two edges of the stripe of (see Eqs. 25, 29 and 37)

$$\Delta g = g\left(y = \frac{S}{2}\right) - g\left(y = -\frac{S}{2}\right)$$

$$= \sqrt{\frac{k \cdot g_m}{\sqrt{\epsilon_r}}} \sin\theta \cong 140 \text{ cm}^{-1} \qquad (39)$$

From Eq. (38) we see that the requirement of Eq. (39) corresponds to establishing a carrier density difference of about $\Delta N = 8 \cdot 10^{17}$ cm^{-3}. Using the results of Figs. 6 and 7 we see that a current splitting ratio of approximately 3:7 is needed (see Appendix D for a detailed derivation). From Eq. (28) we see that a change in the real part of the dielectric constant has also to be established. This change may be automatically effected through the plasma effect, where, for GaAs we have (Ref. 10, Ch 2):

$$\delta\epsilon_r \cong -1.1 \cdot 10^{-20} \Delta N \qquad (40)$$

It is seen that $\Delta N = 8 \cdot 10^{17}$ translates to $\delta\epsilon_r \approx 9 \cdot 10^{-3}$, which reasonably compares with the needed value of $\delta\epsilon_r \cong 2 \cdot 10^{-2}$.

In other structures, such as the slab waveguide (example 3 in the previous section), the requirements for beam deflection are even less stringent in terms of the gain and carrier gradients that need to be established across the laser structure, and thus larger beam deflection angles are feasible.

As a final note it should be emphasized that it is virtually impossible to achieve the exact modification, as required by equations such as (28), (29), (35), (36), since we do not have a direct local access and control to the active region. Furthermore, the above equations were derived using simplified laser models and thus are also not exact. However, the experimental results reported in Ref. 3, where beam deflection was obtained using only a double-stripe contact, indicate that practical structures which adequately approximate the exact theoretical requirements are feasible. Since the results of Ref. 3 also show that the amount of beam deflection depends on both the current ratio and magnitude, a more refined model of the laser operation above threshold (see, for example, Ref. 15) must also be incorporated into the analysis.

V. Conclusions

Electronic beam steering of semiconductor lasers is very useful in many applications, and thus it is important to understand the underlying relationships between the physical parameters of the device and the amount of deflection of its radiation pattern.

In this report the problem of beam deflection of semiconductor injection lasers has been theoretically investigated. It was found that beam deflection can be achieved by tailoring the profile of the current injected into the laser active region via the modifications in the dielectric constant that accompany such current distribution changes. The magnitude of the modifications possible are sufficient for beam deflections of several degrees and in certain laser structures even more. Among the systems that could greatly benefit – in terms of size and weight reduction – from the application of electronic beam steering are pointing and tracking subsystems of optical communication links, optical radars, and optical data recording/retrieval systems.

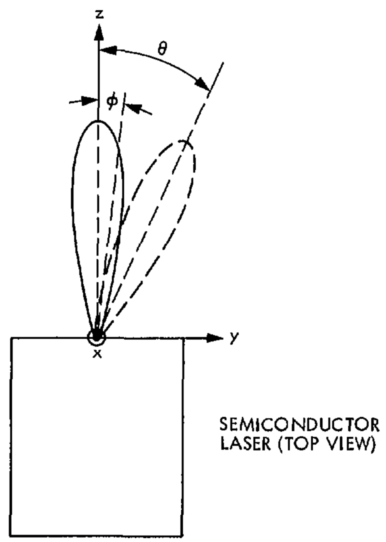

Fig. 1. Schematic configuration of semiconductor laser beam steering

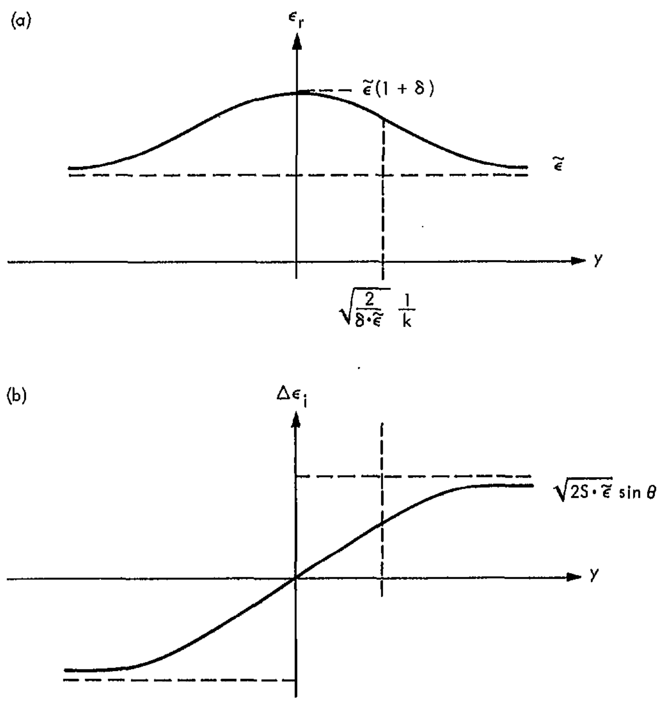

Fig. 2. (a) A sech^2-law dielectric constant distribution; (b) modification of the dielectric constant needed for a beam deflection of an angle θ

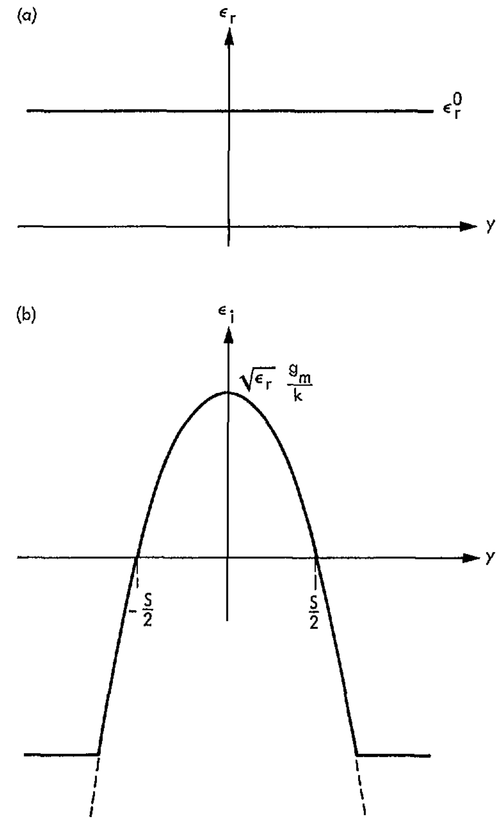

Fig. 3. Dielectric constant distribution in a quadratic index pure gain guiding medium: (a) real part, (b) imaginary part

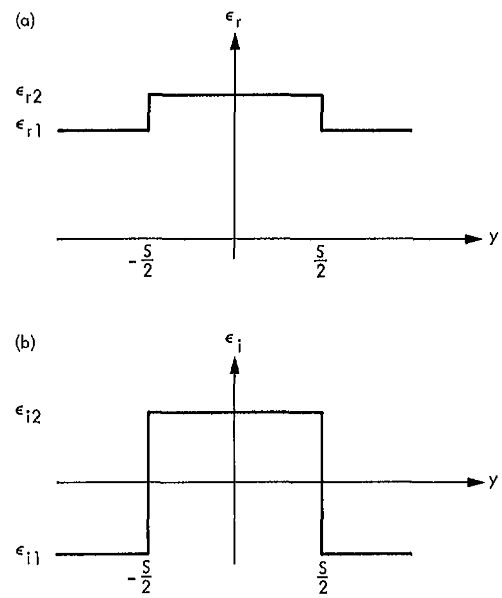

Fig. 4. Dielectric constant distribution in a one-dimensional three-layer slab waveguide: (a) real part, (b) imaginary part

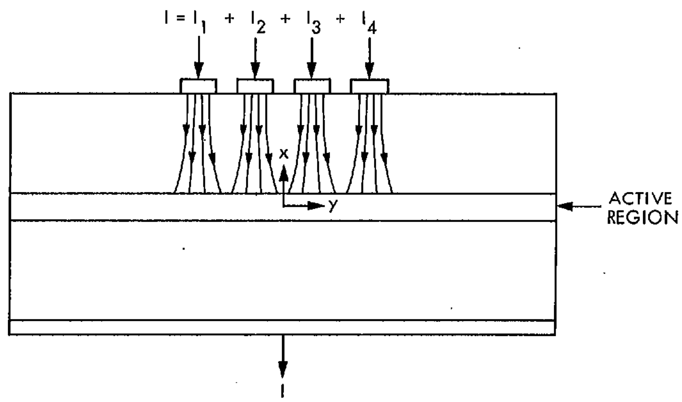

Fig. 5. Schematic configuration of multiple stripe laser structure which makes carrier density profiling possible

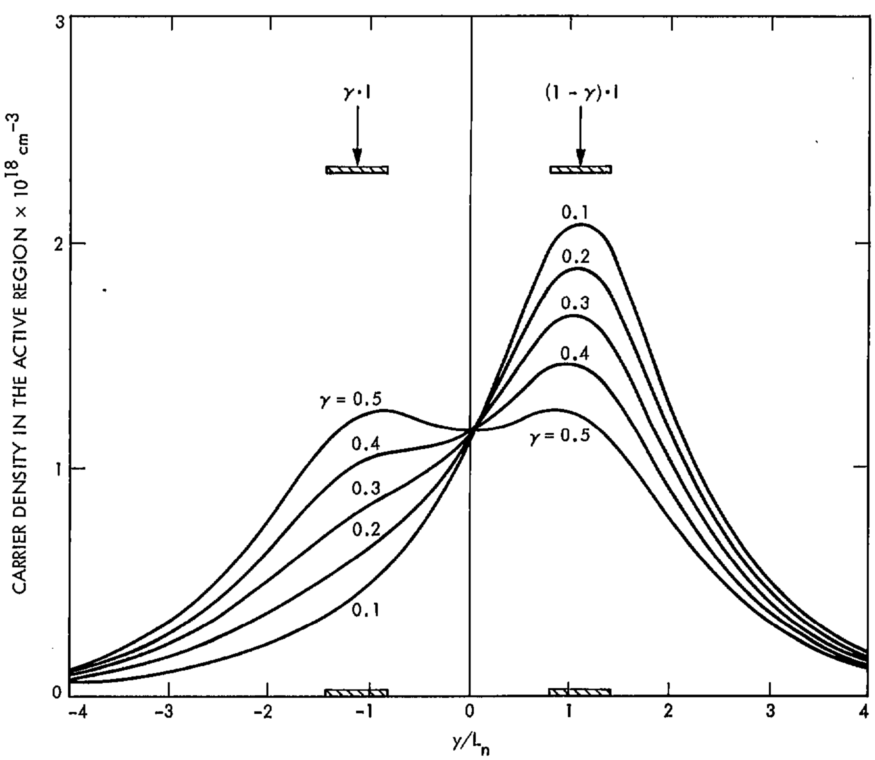

Fig. 6. Distribution of the carrier density in the GaAs active region due to current injection in a double-stripe structure; the currents ratio is γ:(1$-\gamma$)

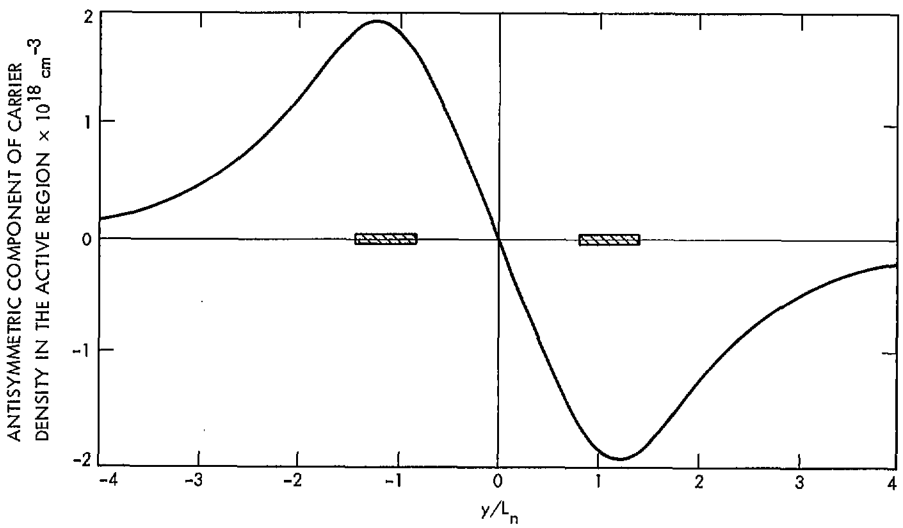

Fig. 7. Distribution of the antisymmetric component of the carrier density in the case of maximum asymmetry (i.e., $\gamma \rightarrow 0$)

Appendix A

In this appendix we establish the notations used in the text for describing the dielectric properties of the material. A material is characterized by a dielectric constant ϵ, whose relation to the index of refraction n of the material is given by

$$\epsilon = n^2 \tag{A-1}$$

Since both ϵ and n are generally complex quantities, Eq. (A-1) can be rewritten as

$$\epsilon_r + i\epsilon_i = (n_r + in_i)^2 \tag{A-2}$$

where the subscripts r and i refer to the real and imaginary parts, respectively, of each quantity. Since in virtually all the relevant applications involving dielectric waveguides the condition $|\epsilon_i| << \epsilon_r$ (and hence also $|n_i| << n_r$) is met, we can use the following approximate relations between the real and imaginary part of ϵ and n:

$$n_r \cong \sqrt{\epsilon_r} \tag{A-3a}$$

$$n_i \cong \frac{\epsilon_i}{2\sqrt{\epsilon_r}} \tag{A-3b}$$

and

$$\epsilon_r \cong n_r^2 \tag{A-4a}$$

$$\epsilon_i \cong 2n_r n_i \tag{A-4b}$$

The meaning of the imaginary part of the index of refraction is readily understood when the propagation of a plane wave through a medium with an index of refraction n along the Z direction is considered)

$$E = e^{i(\omega t - knZ)} = e^{kn_iZ} e^{i(\omega t - kn_rZ)} \tag{A-5}$$

where $k \equiv \omega/c$. We see that n_r determines the phase velocity of the wave, and n_i determines if the medium is lossy ($n_i < 0$) or with gain ($n_i > 0$).

The magnitude of the power gain (or loss) coefficients, expressed in units of (length^{-1}), is given by

$$g = 2 \cdot kn_i = \frac{4\pi}{\lambda} n_i = \frac{2\pi}{\lambda} \frac{\epsilon_i}{\sqrt{\epsilon_r}} \tag{A-6}$$

Appendix B

In this appendix we briefly outline the derivation of the wave equation (3) used in this paper. This outline also serves as a good demonstration on how real life three-dimensional problems, which usually do not have analytic solutions, can be reduced to tractable one-dimensional problems. Here we follow the treatment of Ref. 10, Ch. 7.10.

The laser structure cross-section is depicted in Fig. B-1. Its active region has a width d and a dielectric constant $\epsilon(y)$. It is sandwiched between two low index of refraction cladding layers whose dielectric constant is assumed to be fixed at ϵ_{clad}. The laser cavity is along the z axis. Since we are interested in variations along the y direction, we want to factor out the x and z dependence in the problem. In the following we explain how this is done.

We start with the general three-dimensional wave equation. Since semiconductor lasers are known to emit light predominantly in the TE modes, only the equation for $\mathscr{E}_y$ is considered:

$$\nabla^2 \mathscr{E}_y = \frac{\epsilon}{c^2} \frac{\partial^2 \mathscr{E}_y}{\partial t^2} \tag{B-1}$$

where ϵ is the dielectric constant of the medium and c is the light velocity in vacuum.

Next we assume solutions with harmonic time dependence that are propagating along the laser cavity z with a propagation constant β:

$$\mathscr{E}_y(x,y,z,t) = \tilde{\mathscr{E}}(x,y)\, e^{i(\omega t - \beta_z z)} \tag{B-2}$$

Using Eq. (B-2) in Eq. (B-1), with the definition

$$k \equiv \frac{\omega}{c}$$

we obtain the following equation, which does not contain any z dependence:

$$\nabla_t^2 \tilde{\mathscr{E}}_y(x,y) + (\epsilon k^2 - \beta_z^2)\, \tilde{\mathscr{E}}_y(x,y) = 0 \tag{B-3}$$

where

$$\nabla_t^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} .$$

In order to factor out the x dependence, we can, to a first approximation, assume field solutions of the form:

$$\tilde{\mathscr{E}}_y(x,y) = E_y^x(x)\, E_y^y(y) \tag{B-4}$$

This approximation is good for modes that are not too close to cutoff (i.e., modes that are "reasonably" well guided). Furthermore, since the variation of ϵ along the x direction occurs over distances much shorter than along the y direction (typically fractions of a micron vs several microns), we can neglect the small y dependence of E_y^x [as implied in Eq. (B-4)], and use separation of variables and write the following equation for $E_y^x(x)$:

$$\frac{\partial^2 E_y^x(x)}{\partial x^2} + \beta_x^2\, E_y^x(x) = 0 \tag{B-5}$$

where β_x^2 is a separation constant. Using Eqs. (B-4), (B-5) in (B-3), multiplying the resulting equation by $[E_y^x(x)]^*$ (where * denotes complex conjugation) and integrating over x, results in the following equation:

$$\frac{d^2 E_y^y(y)}{dy^2} + [k^2\, \Gamma_x \epsilon(y) + (1 - \Gamma_x)\epsilon_{clad} - \beta_z^2 - \beta_x^2]\, E_y^y(y) = 0 \tag{B-6}$$

where Γ is the fraction of the field intensity confined to the active region, i.e.,

$$\Gamma_x \equiv \frac{\int_{-d/2}^{d/2} |E_y^x(x)|^2\, dx}{\int_{-\infty}^{\infty} |E_y^x(x)|^2\, dx} \tag{B-7}$$

Equation (B-6) is the desired one-dimensional wave equation. The following few notational changes are employed in order to bring it to the form used in the text (Eq. 3). First, we drop the subscript and the superscript from the field notation since it is clear to which component we are referring. Second, we scale the actual dielectric constant of the active region and use the effective quantity: $\epsilon(y)$ in the text refer to ϵ_{eff} of this appendix, where

$$\epsilon_{eff} \equiv \Gamma_x \epsilon(y) + (1 - \Gamma_x)\, \epsilon_{clad} \tag{B-8}$$

and last, the propagation constant β in the text is given by

$$\beta^2 = \beta_z^2 + \beta_x^2 \tag{B-9}$$

It is worthwhile noting that as the mode becomes more confined to the active region (either by increasing d or by increasing the difference between $\epsilon(y)$ and ϵ_{clad}), Γ approaches unity, and then $\epsilon_{eff} \to \epsilon(y)$ and $\beta \to \beta_z$.

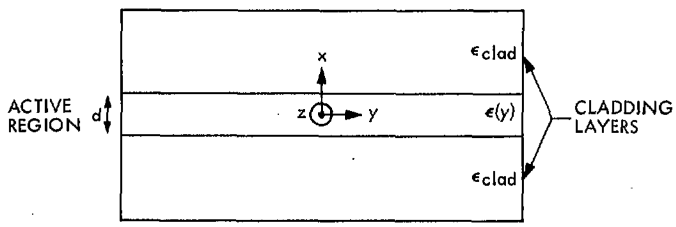

Fig. B-1. Schematic cross section of a semiconductor injection laser

Appendix C

In this appendix we derive the relations between the propagation constants and the field distributions. These relations are required for evaluating some of the effects of the modification of the dielectric constant ϵ_x that are needed for beam steering.

We start with the wave equation for the unperturbed field E^0 (Eq. 6):

$$\frac{d^2E^0}{dy^2} + [\epsilon^0 k^2 - (\beta^0)^2]\, E^0 = 0 \tag{C-1}$$

where the superscript 0 denotes the unperturbed quantities. Multiplying Eq. (C-1) by $(E^0)^*$, integrating over y and assuming along the reasoning of Appendix A that

$$(\beta^0)^2 \equiv (\beta_r^0 + i\beta_i^0)^2 \cong (\beta_r^0)^2 + 2i\beta_r^0\beta_i^0 \tag{C-2}$$

we obtain

$$(\beta_r^0)^2 = \frac{k^2 \int_{-\infty}^{\infty} \epsilon_r^0\, |E^0|^2\, dy - \int_{-\infty}^{\infty} \left|\frac{dE^0}{dy}\right|^2 dy}{\int_{-\infty}^{\infty} |E^0|^2\, dy} \tag{C-3}$$

and

$$\beta_i^0 = \frac{k^2 \int_{-\infty}^{\infty} \epsilon_i^0\, |E^0|^2\, dy}{2\beta_r^0 \int_{-\infty}^{\infty} |E^0|^2\, dy} \tag{C-4}$$

where $\epsilon^0 = \epsilon_r^0 + i\epsilon_i^0$ is the unperturbed dielectric constant of the medium.

The modified (perturbed) field is given by [Eq. (5)]

$$E = E^0\, e^{ik(\theta)y} \tag{C-5}$$

From Eq. (C-5) we see that

$$|E|^2 = |E^0|^2 \tag{C-6}$$

(i.e., the magnitude of the field is not changed), and that

$$\left|\frac{dE}{dy}\right|^2 = k^2(\theta)\,|E^0|^2 + \left|\frac{dE^0}{dy}\right|^2 \tag{C-7}$$

From Eqs. (C-3), (C-4) we can write equivalent equations for β_r and β_i,

$$\beta_r^2 = \frac{k^2 \int_{-\infty}^{\infty} \epsilon_r |E|^2\, dy - \int_{-\infty}^{\infty} \left|\frac{dE}{dy}\right|^2 dy}{\int_{-\infty}^{\infty} |E|^2\, dy} \tag{C-8}$$

$$\beta_i = \frac{k^2 \int_{-\infty}^{\infty} \epsilon_i |E|^2\, dy}{2\beta_r \int_{-\infty}^{\infty} |E|^2\, dy} \tag{C-9}$$

Using Eqs. (C-3), (C-4), (C-6) and (C-7) in Eqs. (C-8), (C-9), we obtain:

$$\beta_r^2 = (\beta_r^0)^2 - k^2(\theta) + k^2 \frac{\int_{-\infty}^{\infty} \Delta\epsilon_r |E^0|^2\, dy}{\int_{-\infty}^{\infty} |E^0|^2\, dy} \tag{C-10}$$

and

$$\beta_i = \left(1 - \frac{\Delta\beta_r}{\beta_r^0}\right)\left[\beta_i^0 + \frac{\int_{-\infty}^{\infty} \Delta\epsilon_i |E^0|^2\, dy}{\int_{-\infty}^{\infty} |E^0|^2\, dy}\right] \tag{C-11}$$

where

$$\Delta\epsilon = \epsilon - \epsilon^0 = (\epsilon_r - \epsilon_r^0) + i\,(\epsilon_i - \epsilon_i^0) \equiv \Delta\epsilon_r + i\Delta\epsilon_i \tag{C-12}$$

is the perturbation on the dielectric constant, and $\Delta\beta_r$ can be calculated from Eq. (C-10) using the approximation $\beta_r^2 - (\beta_r^0)^2 \cong 2\,\beta_r^0\,\Delta\beta_r$:

$$\Delta\beta_r \equiv \beta_r - \beta_r^0 \cong \frac{1}{2\beta_r^0}\left[k^2 \frac{\int_{-\infty}^{\infty} \Delta\epsilon_r |E^0|^2\, dy}{\int_{-\infty}^{\infty} |E^0|^2\, dy} - k^2(\theta)\right] \tag{C-13}$$

A special case is when the real and/or the imaginary parts of the perturbation on the dielectric constant are antisymmetric functions.

First, if

$$\Delta\epsilon_r(y) = -\,\Delta\epsilon_r(-y) \tag{C-14}$$

then, from Eq. (C-13) we see that

$$\Delta\beta_r \cong -\frac{k^2(\theta)}{2\beta_r^0} \tag{C-15}$$

If furthermore we also have

$$\Delta\epsilon_i(y) = -\,\Delta\epsilon_i(-y)\,, \tag{C-16}$$

then from Eq. (C-11) we see that

$$\Delta\beta_i = \beta_i^0\,\frac{k^2\theta}{2(\beta_r^0)^2} \tag{C-17}$$

Appendix D

In this appendix we derive the current ratio needed to establish a given antisymmetric carrier density. Let us denote by $F(\zeta)$ the carrier profile resulting from current injection through a single stripe centered at $\zeta = 0$. From Ref. 10 Ch. 7.7 we can see that

$$F(\zeta) = F(-\zeta) \qquad \text{(D-1)}$$

As a first approximation we assume that the problem is linear (for a detailed analysis, refer to Ref. 10, Ch. 7.7) and thus we have two stripes whose centers are separated a distance of $2a$ apart and the currents ratio through the stripes is $\gamma:(1 - \gamma)$, then the resulting carrier profile is

$$f(y) = \gamma F(y - a) + (1 - \gamma) F(y + a) \qquad \text{(D-2)}$$

Decomposing $f(y)$ into its symmetric and antisymmetric components, making use of Eq. (D-1), we obtain

$$f(y) = f_{sym}(y) + f_{\substack{anti\\sym.}}(y) \qquad \text{(D-3)}$$

where

$$f_{sym} = \frac{1}{2}\left[F(y + a) + F(y - a)\right] \qquad \text{(D-4)}$$

$$f_{\substack{anti\\sym.}} = \left(\frac{1}{2} - \gamma\right)\left[F(y + a) - F(y - a)\right] \qquad \text{(D-5)}$$

From Fig. 7 we see that

$$max\,|F(y + a) - F(y - a)| \cong 4 \cdot 10^{18}\,\mathrm{cm}^{-3} \qquad \text{(D-6)}$$

and from the discussion following Eq. (39) we see that we need $\Delta N \cong 8 \cdot 10^{17}\ \mathrm{cm}^{-3}$. Solving

$$8 \cdot 10^{17} \cong \left(\frac{1}{2} - \gamma\right) 4 \cdot 10^{18}$$

we obtain $\gamma \cong 0.3$.

References

1. See, for example, A. Yariv, *Quantum Electronics*, 2nd Edition, Wiley, N.Y., 1975.
2. Katz, J., "Phase Control and Beam Steering of Semiconductor Laser Arrays", *TDA Progress Report 42-68*, Jet Propulsion Laboratory, Pasadena, Calif. (this issue).
3. Scifres, D. R., Streifer, W., and Burnham, R. D., "Beam Scanning with Twin-Stripe Injection Lasers," *Appl. Phys. Lett., 33*, pp. 702-704, 1978.
4. Hockham, G. A., "Radiation from a Solid-State Laser," *Electron. Lett., 9*, pp. 389-391, 1973.
5. Lewin, L., "Obliquity Factor for Radiation from Solid State Laser," *Electron. Lett., 10*, pp. 134-135, 1974.
6. Butler, J. K., and Zoroofchi, J., "Radiation Fields of GaAs-(AlGa)As Injection Lasers," *IEEE J. Quant. Electron., QE-10*, pp. 809-815, 1974.
7. Lewin, L., "Obliquity-Factor Correction to Solid-State Radiation Patterns," *J. Appl. Phys., 46*, pp. 2323-2324, 1975.
8. Lewin, L., "A Method for Calculation of the Radiation Pattern and Mode Conversion Properties of a Solid State Heterojunction Laser," *IEEE Trans. Microwave Theory Tech., MTT-23*, pp. 516-535, 1975.
9. Dias, P., Portnoi, E. L., Raikh, M. E., and Ryvkin, B. S., "Far-Field Radiation of a Heteorepitaxial Waveguide with Smooth Refractive Index Profile," *Sov. Tech. Phy. Lett., 2*, pp. 424-425, 1976.
10. Casey, H. C., Jr., and Panish, M. B., *Heterostructure Lasers*, Academic Press, N.Y., 1978, Ch. 2.7.
11. Tsukada, T., "GaAs – $Ga_{1-x}Al_xA_s$ Buried Heterostructure Injection Lasers," *J. Appl. Phys. 45*, pp. 4899-4906, 1974.
12. Aiki, K., Nakamura, M., Kuroda, T., Umeda, J., Ito, R., Chinone, N., and Maeda, M., "Transverse Mode Stabilized $Al_xGa_{1-x}A_s$ Injection Lasers with Channeled-Substrate-Planar Structure," *IEEE J. Quant. Electron., QE-14*, pp. 89-94, 1978.
13. Ueno, M., and Yonezu, H., "Stable Transverse Mode Oscillation in Planar Stripe Laser with Deep Zn Diffusion," *IEEE J. Quant, Electron., QE-15,* pp. 1189-1196, 1979.
14. Kressel, H., and Butler J. K., *Semiconductor Lasers and Heterojunction LEDs*, Academic Press, N.Y., 1977, ch. 17.
15. Wilt, D., and Yariv, A., "A Self-Consistent Static Model of the Double-Heterostructure Laser," *IEEE J. Quant Electron., QE-17*, pp. 1941-1949, 1981.