# A Covariance Analysis for the Determination of Baselines Observing GPS Satellites

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*Results of covariance analyses are presented for the determination of baseline vectors of length from 150 m to 4000 km. The data are doubly differenced ranges derived from the signals transmitted by the satellites of the Global Positioning System. The modeling of various error sources is described. The results indicate the low-cost baseline determination system is capable of sub-decimeter accuracy for baselines up to 1000 km in length.*

## I. Introduction

The application of Very Long Baseline Interferometry (VLBI) to the determination of baseline vectors has been investigated for over a decade (Refs. 1-5). The results of ARIES (Astronomical Radio Interferometric Earth Surveying) experiments have demonstrated an accuracy better than 10 cm for baselines up to a few hundred kilometers in length (Refs. 3, 4). The *lengths* of intercontinental baselines (~10,000 km) between the Deep Space Network stations in California, Spain, and Australia have also been determined to 10 cm accuracy using a more sophisticated VLBI system (Ref. 5).

The high precision in baseline determination is a result of observing extragalactic radio sources (quasars) with time-invariant angular positions which are accurately known. Utilizing the faint signals of quasars requires low-noise receivers coupled with high-precision clock, high-efficiency antennas of moderately large aperture, and a high-density data acquisition facility. The data from each end of the baselines, typically of the order of $10^{10}$ bits, are brought together for processing. Therefore, the operating cost of a VLBI system is quite high. For frequent monitoring of crustal motion in a seismically active area, a low-cost alternative is highly desirable.

The Global Positioning System (GPS), to be fully in operation after 1988, will consist of 18 navigation satellites evenly distributed in 6 orbit planes at an altitude of ~20,000 km. The use of signals transmitted by these GPS satellites in place of quasar signals for baseline determination has been proposed and studied by several researchers (Refs. 6, 7). The advantage of the GPS signals over quasar signals is that they are strong and encoded. This enables the use of a low-cost receiver with a small antenna and allows on-site preprocessing to reduce the data volume before being brought together for further processing. Such a system will provide low-cost operation and may become a strong candidate to supplement the quasar-based VLBI system to provide frequent baseline motion monitoring.

Pseudorange and range rate information can be easily extracted from the GPS signals without the knowledge of the signal codes by a system called SERIES (Satellite Emission

Range Inferred Earth Surveying) proposed by MacDoran (Ref. 7). This SERIES system is currently under development at the Jet Propulsion Laboratory and a series of tests on baseline determination capability is underway. This report provides a covariance analysis to estimate the accuracies with which baselines of different lengths can be determined by the SERIES approach.

Recently, the use of SERIES in the orbit determination of a low-altitude Earth satellite (e.g., TOPEX) has been investigated (Refs. 8, 9). Preliminary studies indicate a sub-decimeter altitude determination capability. While an in-flight proof-of-concept demonstration is impossible without actually implementing the whole system, including a dedicated satellite carrying a SERIES receiver, consistency between the results of the current analysis and of the baseline determination tests will provide increased confidence in predicted performance of the proposed SERIES satellite tracking system.

## II. Data Types

The GPS satellites emit signals at two L-band frequencies, 1575.42 MHz ($L_1$) and 1227.60 MHz ($L_2$), that can be used to make one-way (pseudo-) range and doppler measurements between the GPS satellites and a user receiver. A detailed description of GPS signal structure can be found in Ref. 10. When the range or doppler measurements from a GPS satellite to the two ends of a baseline are brought together and differenced, the data are analogous to VLBI data. Differenced GPS range (or doppler) reduces the sensitivity to the GPS ephemeris errors and eliminates the GPS instrument (including clock) errors. However, the clock discrepancy between the two ends of the baseline remains as an error source to the baseline determination. The clock discrepancy will be the same no matter which satellite originates the signals. Then, when simultaneous differenced range (or doppler) measurements from two GPS satellites are further differenced, the clock error will cancel. These new data types are called doubly differenced GPS range (DDGR) and doubly differenced GPS doppler (DDGD), respectively (Ref. 9). Since the angular separation between the GPS satellites can be very large, the geometrical strength is not expected to be significantly reduced from that of singly differenced data.

In the following analysis the DDGR data taken at 5-minute intervals are used. The DDGD data will be only briefly studied.

## III. Geometry

Although 18 GPS satellites will ultimately be in operation, only 6 satellites are currently in orbit, of which five are under operating conditions. Periodically, all five satellites can be in view nearly simultaneously from a station on the continental U.S. The view periods of these satellites on May 9, 1982, from a station in the state of California are shown in Fig. 1. A cutoff elevation angle of 20-deg was assumed. These view periods will be used in the following analysis. Other satellite parameters are summarized in Table 1.

Baselines to be studied are of lengths from 150 m to 4000 km. These are also listed in Table 1. A short baseline experiment is useful in providing a measure of how the instrument will perform, because the effects of atmospheric delays, of station location error, and of GPS ephemeris error will cancel nearly perfectly between the two ends of such a baseline.

## IV. Error Models

The error models are summarized in Table 2. Since dual-frequency ($L_1$ and $L_2$) observations are available, ionospheric delay is assumed to be totally removed and is not considered here. Clock errors are also totally removed by the double difference inherent in the DDGR data.

The ephemeris error of each GPS satellite is represented by a full covariance matrix generated by a 9-hour orbit fit such that the RSS position error in three orthogonal directions remains nearly at 10 meters. A diagonal covariance matrix would cause the in-track position error to increase rapidly with time, as depicted by Fig. 2, and thus would result in an unrealistically large baseline determination error.

For the tropospheric error, we assume a 5-cm zenith calibration using surface measurements (Ref. 11) or a seasonal model (Ref. 12). Azimuth homogeneity is assumed so that the error is scaled only by an elevation factor. Over short baselines the effects of the troposphere between the two ends are highly correlated. The difference in zenith tropospheric delay between the two ends of a baseline of length B can be represented by a function proportional to $B^{0.7}$ (Ref. 13). Even though this relationship was derived from observations with $B$ ranging from 1 to 20 km, the trend seems to suggest a linear extrapolation to $B \sim 80$ km. Within this range, a smaller error will result when a single zenith calibration is applied to both ends of the baseline. The proportionality factor 0.2 is twice as large as given in the reference to account for lower altitudes than the 2 km at the VLA site in New Mexico, where the data of the reference were acquired. For $B > 100$ km, the correlation breaks down to a level that independent calibration at each end of the baseline becomes a better choice.

With VLBI observation of quasars at an "infinite distance," the location error of one end of the baseline will translate one-to-one into the location of the other end, resulting in no net *baseline* error. With DDGR measurements over long baselines, the finite distances of the GPS satellites will translate the location error between the two ends of the baseline less perfectly and a net baseline error will result. The assumed 1-m error in each component of the reference station (first of the pair) is relative to the "center of the Earth" defined by the GPS satellite orbits or by the station network determining such orbits.

The effects of polar motion and UT1 errors will rotate only the baseline orientation. These effects are independent of the observation scheme and can be easily determined (Ref. 14). These errors are not included in the current analysis.

## V. Results of Covariance Analysis

Covariance analysis was performed using simulated DDGR data. The baseline vector determination errors as a function of baseline length are shown in Figs. 3-6, respectively, for the four error sources. The east, north and vertical components of these errors are relative to the second station of each baseline. All four error sources have their largest effects in the vertical component. The component along the north-south direction is least affected by data noise and GPS ephemeris error. The component along the transverse horizontal direction is least affected by tropospheric error and location error of the reference stations.

The effects of GPS emphemeris error and of the reference station location error increase with baseline length. For shorter baselines, the correlation of troposphere between the two stations decreases as baseline length, thus increasing its effects. For longer baselines, these effects level off at about 20 cm, still mostly in the vertical component. The effects of data noise remain nearly constant for most baseline lengths, with a slight increase for the longest baseline (Haystack-OVRO) due mainly to shorter common view periods of all GPS satellites from the two ends of the baseline.

The three components of the total (root-sum-square) baseline vector determination error are shown in Fig. 7, again as a function of baseline length. The shorter baselines are dominated by data noise. Tropospheric error dominates the baselines of medium length. For baselines longer than ~600 km, GPS ephemeris error becomes the dominating error source. While decimeter accuracy for the vertical component can be retained only for baselines shorter than 60 km, such accuracy can be retained for the horizontal components for baselines up to about 500 km in length.

An analysis was also performed for the DSS 14-DSS 13 baseline using DDGD data. Such data have the advantage over DDGR data in that they are more accurate[1], less affected by multipath effects and can be acquired with simpler receivers. However, it is well known that the information content of doppler data is less than the corresponding range data. This is confirmed in Fig. 8 by the larger data noise effects. Note that the 0.05-mm/s DDGD data noise at a 5-minute count time corresponds to 1 cm DDGR error, which is a factor of 10 smaller than the assumed data noise for the previous analysis using DDGR data. Such low DDGD data noise may be achievable from the GPS RF carriers.

The effects of tropospheric error are also smaller when using DDGD data. Even though the total baseline vector errors are larger than the corresponding DDGR solutions, these are not significant for longer baselines because the dominating effects of GPS ephemeris error and of reference station location error will be similar for the two data types.

The above analysis was performed for the whole view periods of the five GPS satellites in one pass. The ~6 hour data span[2] provides sufficient dynamics needed by the DDGD data. The DDGR data possess intrinsic geometrical information that yields a geometric solution from instantaneous measurements. An analysis was performed for such instantaneous determination of the DSS 14-DSS 13 baseline. The results are shown in Fig. 9. Here the dominating error source is the 10 cm DDGR data noise. This is a result of reducing the number of data points from 93 into 4. A factor of $(4/93)^{1/2}$ reduction in data noise would reduce its effects to a level comparable to the long-arc solutions, shown side-by-side in Fig. 9, indicating the geometrical strength of the DDGR measurements. The effects of other error sources are also comparable with the long-arc solutions. Preliminary test results (Ref. 15) for the same baseline are also shown in Fig. 9 for comparison. The baseline component errors of these short-arc (~1.5 hours) tests lie between the long-arc solutions and the instantaneous solutions.

## VI. Summary and Conclusions

Covariance analysis has been performed for the determination of baseline vectors observing the GPS satellites. Doubly differenced data were used to eliminate clock errors. If reasonably stable clocks are used at both ends of the baseline, singly differenced data can be used with periodic adjustments of clock parameters (Ref. 14). Such singly differenced data will possess better information content due to better geometry, because the need for simultaneous observation of at least two

[1] Because of being derived from RF carriers which have much shorter wavelength (19 cm) than the 30-m range-code wavelength.

[2] At least two GPS satellites need to be in view simultaneously to form the doubly differenced data.

GPS satellites is now removed. The frequency at which the clock parameters are adjusted depends on the stability of the clocks.

Even though the analysis has been performed using only the five GPS satellites currently in orbit, the results are believed to be not much inferior to the case using a full constellation of 18 satellites. This full constellation will provide world-wide, continuous 24-hour observations instead of the periodic U.S. coverage with the five satellites.

Precision determination of baselines ~1000 km in length is of particular interest for geodetic applications involving a world-wide station net. A sub-decimeter accuracy over such baselines can be achieved if the GPS ephemerides are known to 1 meter (RSS) and the zenith tropospheric delays to 2 centimeters. Further improvement in baseline vector determination will also require lower data noise and reference station location errors: For centimeter determination of 1000 km baselines, the data noise has to be kept to a 1 cm level, calibration of zenith troposphere to 0.1 cm, reference station location to 5 cm, and GPS ephemerides to 20 cm (rss). The sensitivities to these error sources may be reduced if measurements are made with more than one reference station, thus relaxing the requirements for their precision calibration.

## Acknowledgments

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**Table 1. GPS and baseline geometry**

| | | |
|---|---|---|
| GPS: | Semi-major axis | $a$ = 26,560 km |
| | Eccentricity | $e$ = 0 |
| | Inclination | $I$ = 63.4° |
| | Argument of perigee | $\omega$ = 0° |
| | Mean anomaly | $M$ = -66.97°<br>45.70°<br>-2.40°<br>-90.85°<br>1.42° |
| | Longitude of ascending node | $\Omega$ = 121.34°<br>0.05°<br>121.37°<br>121.65°<br>0.31° |
| | Epoch 1982/05/09 0000 hr UT | |
| Baselines: | Goldstone local | (150 m north-south) |
| | DSS 14 – DSS 13 | (22 km) |
| | JPL – Palos Verdes | (58 km) |
| | DSS 14 – JPL | (181 km) |
| | JPL – OVRO | (338 km) |
| | Haystack – OVRO | (4000 km) |

**Table 2. Error models**

| | |
|---|---|
| Data noise | DDGR 10 cm at 5-min intervals<br>DDGD 0.05 mm/s at 5-min count time |
| GPS ephemeris | 10 m rss over entire view periods |
| Zenith troposphere | $B < 80$ km,<br>5 cm at zenith common between stations<br>$0.2\ B^{0.7}$ cm difference between stations<br>$B > 100$ km,<br>5 cm at each end of the baseline (uncorrelated) |
| Reference station location | 1 m each component |

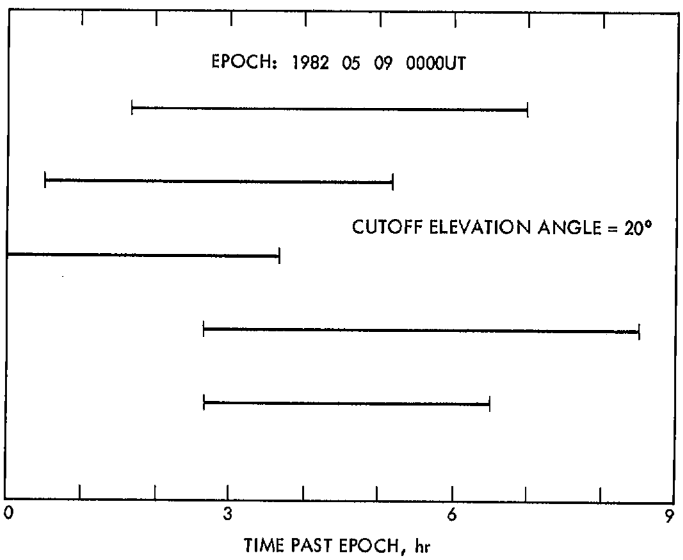


**Fig. 1. View periods of five GPS satellites from a station in California**

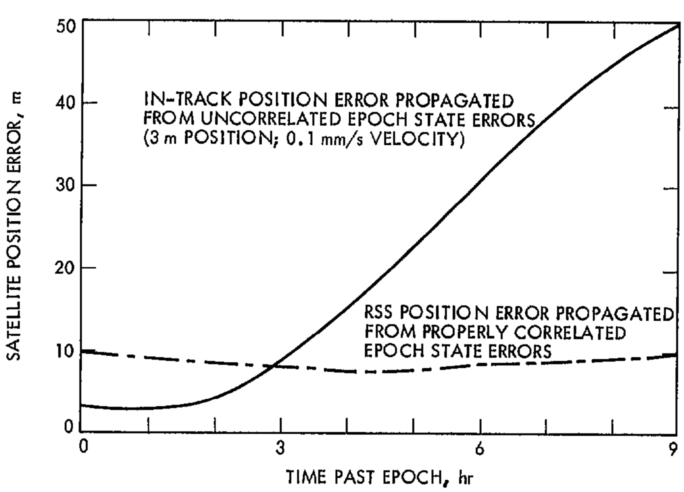


**Fig. 2. State error propagation of a GPS satellite**

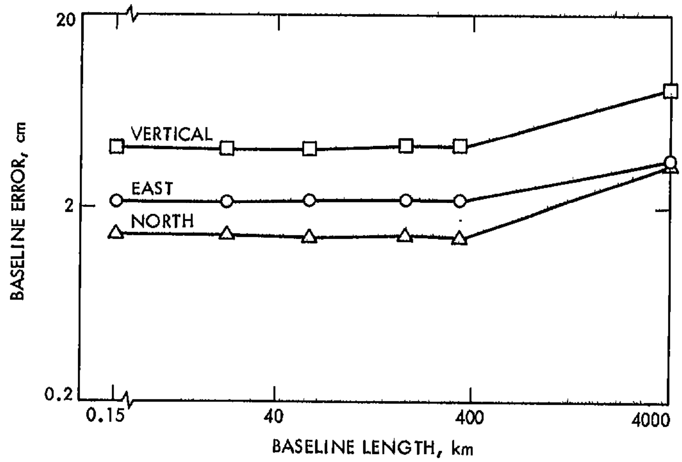


**Fig. 3. Effects of DDGR data noise on baseline determination**

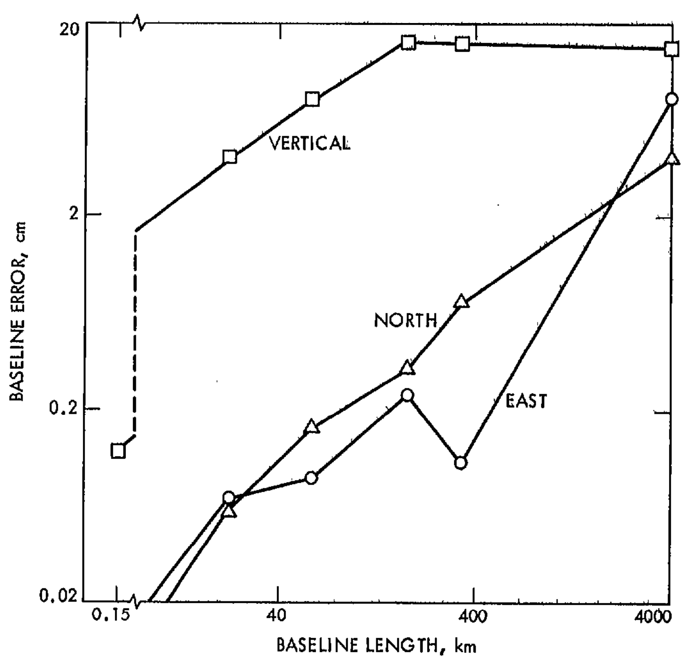


**Fig. 4. Effects of tropospheric delay error on baseline determination**

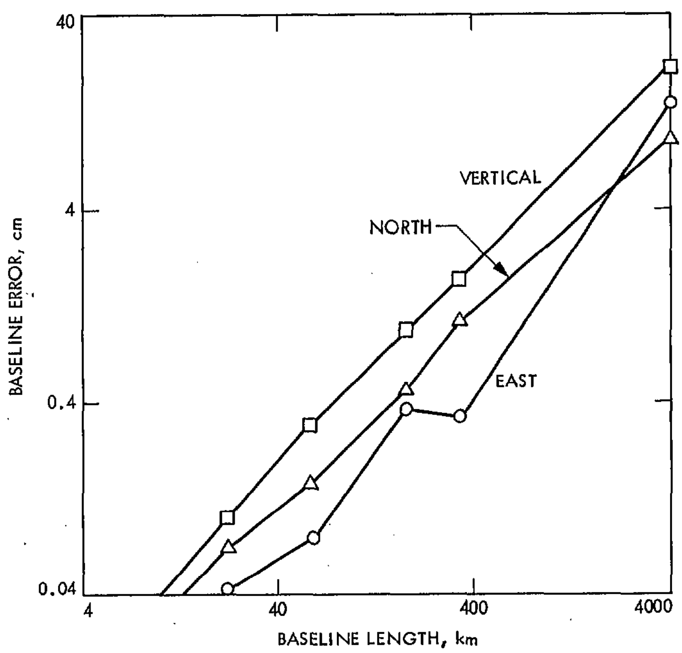


**Fig. 5. Effects of reference station location error on baseline determination**

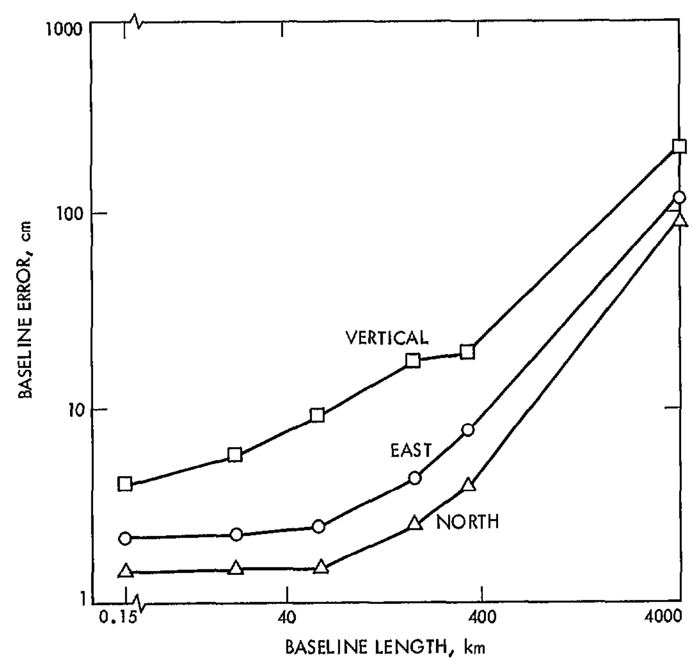


**Fig. 7. Total baseline determination error**

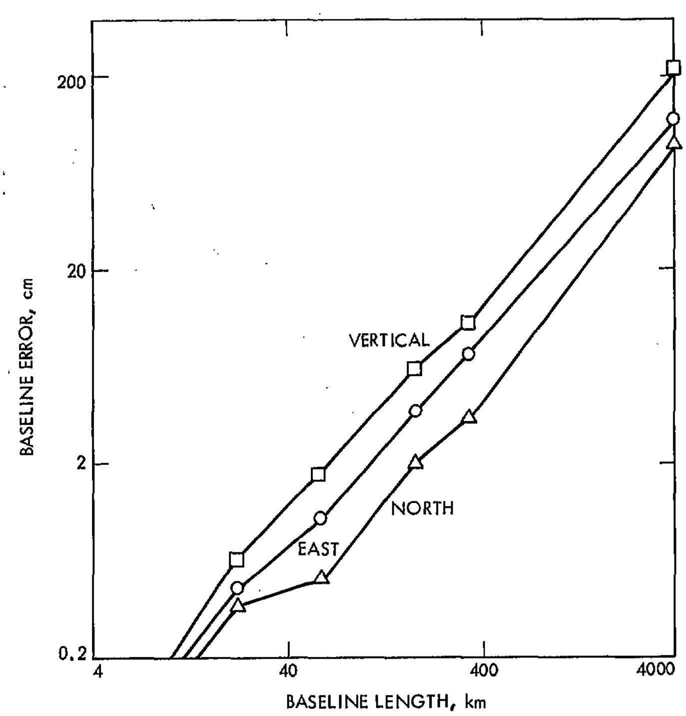


**Fig. 6. Effects of GPS ephemeris error on baseline determination**

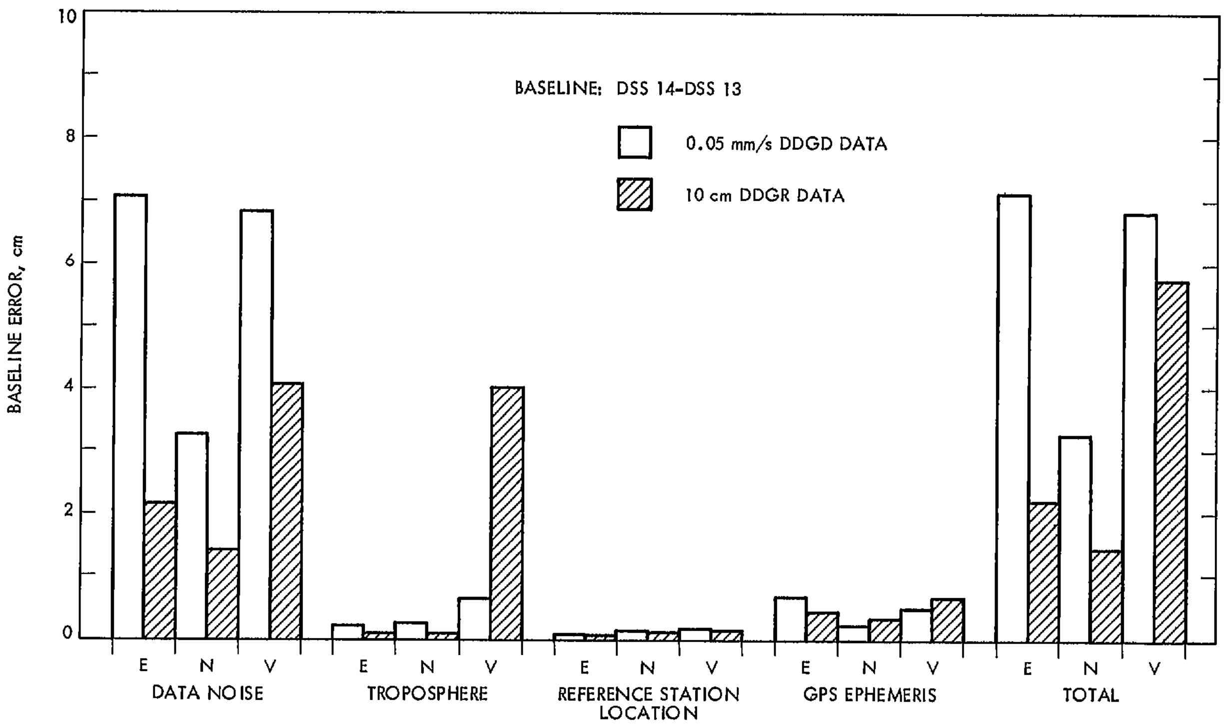


**Fig. 8. Comparison of DDGD and DDGR data for baseline determination**

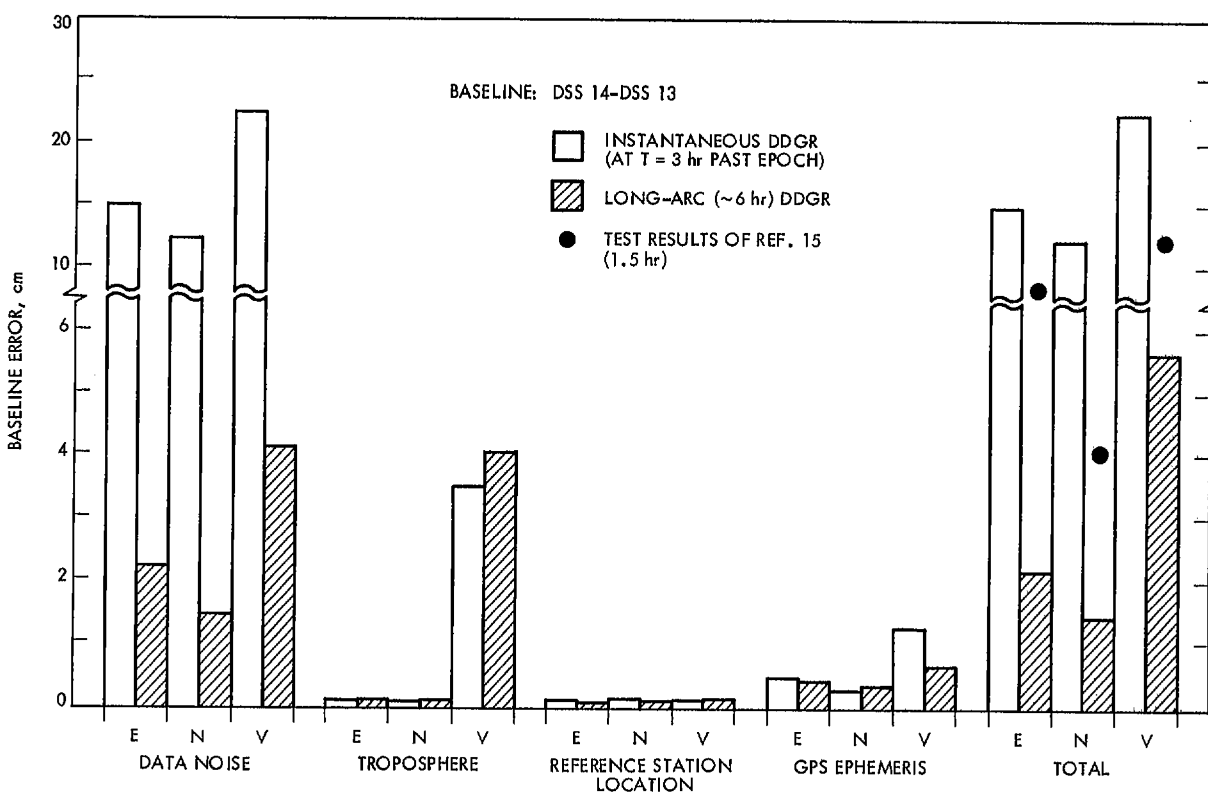


**Fig. 9. Comparison of instantaneous DDGR and long-arc DDGR for baseline determination**