Coherent Digital Demodulation of a Residual Carrier Signal Using IF Sampling

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Analysis is presented of an all-digital technique for the coherent demodulation of a residual carrier signal with a biphase modulated square-wave subcarrier. The processing technique, proposed for use in the DSN advanced receiver, employs the concept of IF sampling. It also uses an optimum Costas loop for subcarrier demodulation and data-aided carrier tracking, i.e., combined Costas and residual carrier tracking. It is shown that the loops perform essentially the same as the corresponding analog loops in terms of signal-to-noise ratio and loop bandwidth. Furthermore, the sampling does not introduce biases or other significant effects on the loops provided that the loop bandwidth is very small compared to the symbol rate, and that the number of samples per symbol is large compared to inverse loop bandwidth.

I. Introduction

The ever increasing advances in LSI and VLSI technology make possible the use of digital integrated circuits in real-time processing areas that were previously implemented with analog circuitry. This paper describes a digital implementation proposed for the DSN Advanced Receiver to avoid the inherent problems associated with analog systems such as dc offsets in the mixers and amplifiers, the need for calibration and adjustments, and less reliability, versatility, and flexibility than digital systems.

Two families of configurations have been considered for the design of the proposed advanced receiver: one associated with I-Q baseband sampling and another associated with IF sampling. In the first one, the sampled I and Q channels are generated first by demodulating by two in-quadrature reference signals and then sampling with two in-phase samplers; in the second, we first sample at IF and then demultiplex into baseband I and Q samples. The IF sampling configuration was chosen for three main reasons. First, it overcomes the dc offset problem of baseband systems, which is a serious problem in phase detection at low signal-to-noise ratios. Second, the hardware is simpler and dc amplifiers are eliminated. Third, the implementation concept is not proven, and is thus deserving of analysis and demonstration. One complexity and potential disadvantage of IF sampling is that the samples are offset from each other by one-half sampling period. It is shown that this I-Q offset inherent to IF sampling has no significant effect on the loops' performances.

The digital system employs synchronous bandpass sampling for the coherent demodulation of a residual carrier signal biphase modulated by a square-wave subcarrier. The receiver uses a PLL to track the residual carrier phase, an optimum Costas loop with integrate and dump filters for subcarrier demodulation, and a data-aided loop for improved tracking performance of the carrier phase.

II. Functional Block Diagram and Description

Figure 1 shows the functional block diagram of the proposed receiver. The received signal is at an intermediate frequency f_i and contains data that biphase modulates a square-wave subcarrier. The modulated subcarrier phase modulates the carrier with a modulation index Δ. The signal at the output of the bandpass filter is:

$$r(t) = \sqrt{2P} \sin(\omega_i t + \Delta D(t) + \theta_c) + n(t)$$

where

$r(t)$ is the received signal in volts (V)

P is the average signal power in V^2

Δ is the modulation index

$D(t) = d(t) \quad \mathbf{S}\text{in}(\omega_{sc} t + \theta_{sc})$ with $\mathbf{S}\text{in}\, x = \text{sgn}(\sin x)$

$d(t) = \sum_{\ell=-\infty}^{+\infty} a_\ell p(t - \ell T), a_\ell = \pm 1$ with equal probability

ω_i is the received IF frequency in rad/s

θ_c is the carrier phase in rad

ω_{sc} is the subcarrier frequency in rad/s

θ_{sc} is the subcarrier phase in rad

$n(t) = \sqrt{2}\, n_c(t) \cos(\omega_i t + \theta_c) - \sqrt{2}\, n_s(t) \sin(\omega_i t + \theta_c)$ is a narrowband white Gaussian noise process with $n_c(t)$ and $n_s(t)$ being statistically independent, stationary band-limited white Gaussian noise processes with one-sided spectral density N_0 in V^2/Hz and one-sided bandwidth W.

T is the symbol time

The bandpass filtering effect on the subcarrier is neglected, i.e., the subcarrier waveform is assumed to be an ideal square wave.

The reference signal for carrier lock is a combination of the sampling clock and the carrier-loop digitally controlled oscillator (DCO) output signal. The sampling frequency is derived from the symbol synchronization loop and is denoted by $4f_0$. The symbol synchronization loop, not considered here, is assumed to maintain perfect symbol sync, and f_0 is a multiple of the symbol rate.

The input signal, $r(t)$, is mixed to frequency f_0 using the carrier-loop DCO output signal, whose instantaneous phase is $\omega_1 t + \hat{\theta}_c$, with $\omega_1 = \omega_i - \omega_0$ and $\hat{\theta}_c$ denoting the estimate of θ_c. Thus the instantaneous residual carrier phase of $s(t)$ is denoted by $2\pi f_0 t + \phi_c$, where $\phi_c = \theta_c - \hat{\theta}_c$.

The IF signal $s(t)$ is sampled at frequency $f_s = 4f_0$, at times $t_n = n/4f_0$. The mth output of the sampler is multiplied by $\exp(jm\pi/2)$ and demultiplexed into real and imaginary parts with the real or m-even samples becoming the Q channel, and the imaginary or m-odd samples becoming the I channel.

The Q samples q_n are input to the residual carrier tracking loop and to the subcarrier Costas loop. The I samples i_n are input to the carrier Costas I arm. Note that the carrier Costas Q arm is the same as the subcarrier I arm. Sideband aided carrier tracking is accomplished by properly combining the outputs of the residual carrier and the carrier Costas phase detectors in order to maximize the total loop SNR.

III. Residual Carrier Phase Detection and Tracking

Residual carrier phase detection is accomplished using the Q-channel samples. The bandpass filtered signal $r(t)$ is mixed against the carrier DCO output signal $\sqrt{2} \cos((\omega_i - \omega_0)t + \hat{\theta}_c)$ and lowpass filtered to give:

$$s(t) = \sqrt{P} \sin(\omega_0 t + \Delta D(t) + \phi_c) + n_c(t) \cos(\omega_0 t + \phi_c) - n_s(t) \sin(\omega_0 t + \phi_c)$$

where $\phi_c = \theta_c - \hat{\theta}_c$ is the carrier phase error.

From the above equation, it follows that the quadrature sampled term is:

$$\begin{aligned} q_n &= Re\,\{s(m/(4f_0)) \cdot \exp(jm\,\pi/2)\}, \quad m = 2n \\ &= \sqrt{P} \sin(\Delta D(nT_s) + \phi_c(nT_s)) \\ &\quad + n_c(nT_s) \cos(\phi_c(nT_s)) + n_s(nT_s) \sin(\phi_c(nT_s)) \end{aligned} \tag{1}$$

and the in-phase sampled term is:

$$\begin{aligned} i_n &= Im\,\{s(m/(4f_0)) \cdot \exp(jm\,\pi/2)\} \quad m = 2n + 1 \\ &= \sqrt{P} \cos(\Delta D(nT_s + T_s/2) + \phi_c(nT_s + T_s/2)) \\ &\quad - n_c(nT_s + T_s/2) \sin(\phi_c(nT_s + T_s/2)) \\ &\quad - n_s(nT_s + T_s/2) \cos(\phi_c(nT_s + T_s/2)) \end{aligned} \tag{2}$$

where $T_s = 1/(2f_0)$.

The continuous counterpart of the noise portion of Eqs. (1) and (2) is essentially white noise with zero mean and one-sided spectral density N_0.

The S curve for the residual carrier-loop phase detector, i.e., the average output of the phase detector as a function of the phase, is obtained by plotting $C(\phi_c) = E\,[q_n\,|\phi_c]$ versus ϕ_c, where $E[\cdot]$ denotes the statistical expectation. From Eq. (1), and since the noise is zero mean,

$$C(\phi_c) = E\,[q_n\,|\phi_c] = \sqrt{P}\cos\Delta\,\sin\phi_c$$

which is the expected sinusoidal S curve, and is plotted in Fig. 2.

From Holmes (Ref. 1, Chapter 5), the rms phase error of a phase-locked loop due to random noise is

$$\sigma^2_{\phi_c} = N_0\,B_L/A^2$$

where N_0 is the one-sided spectral density at zero frequency at the phase detector output, B_L is the one-sided loop bandwidth, and A is the slope of the S curve at $\phi = 0$. It is also necessary to assume that the noise bandwidth is wide compared to B_L.

For the residual carrier phase detector, $A = \sqrt{P}\cos\Delta$ and the noise spectral density is the input noise spectral density or N_0. Thus

$$\sigma^2_{\phi_c} = N_0\,B_L/(P\cos^2\Delta) \tag{3}$$

This is the same as for an analog loop with the same bandwidth.

IV. Subcarrier Phase Detection and Tracking

The subcarrier phase detection and synchronization is done by means of an optimum Costas loop. The analysis assumes perfect symbol synchronization and that the residual carrier loop is locked so that $\phi_c = 0$.

The carrier Q samples q_n are multiplied in the two arms of the Costas loop by phase quadrature square-wave reference signals at the subcarrier frequency. The products in the two channels are summed over the symbol times to implement the matched filters of the optimum Costas loop. The channel with the reference in phase with the subcarrier is the data channel.

Let $I_q(i)$ and $Q_q(i)$ denote the subcarrier I- and Q-channel outputs for symbol i, with subscript q denoting the carrier Q channel. Then the outputs of the I and Q arm filters are respectively

$$\begin{aligned} I_q(i) &= \frac{1}{N}\sum_{n=1}^{N} q_n\,\mathrm{Sin}\,(\omega_{sc}nT_s + \hat{\theta}_{sc}) \\ &= \sqrt{P_d}\,d(i)\cdot F_I(\phi_{sc}(i))\cdot\cos(\phi_c(i)) + N_i \\ &\quad + \text{periodic terms.} \end{aligned} \tag{4}$$

$$\begin{aligned} Q_q(i) &= \frac{1}{N}\sum_{n=1}^{N} q_n\,\mathrm{Cos}\,(\omega_{sc}nT_s + \theta_{sc}) \\ &= \sqrt{P_d}\,d(i)\cdot F_q\,(\phi_{sc}(i))\cdot\cos(\phi_c(i)) + N_q \\ &\quad + \text{periodic terms} \end{aligned}$$

where the signal terms are:

$$\sqrt{P_d} = \sqrt{P}\sin\Delta = (\text{signal power})^{1/2}$$

$$d(i) = \frac{1}{N}\sum_{n=1}^{N} d(nT_s) \tag{5}$$

$$F_i(\phi_{sc}) = 1 - \frac{2}{\pi}\,|\phi_{sc}|, \qquad 0 \leqslant |\phi_{sc}| \leqslant \pi$$

$$F_q(\phi_{sc}) = \begin{cases} -\dfrac{2}{\pi}\,\phi_{sc}, & 0 \leqslant |\phi_{sc}| \leqslant \dfrac{\pi}{2} \\[2ex] -\dfrac{2}{\pi}\,(\pi - \phi_{sc}), & \dfrac{\pi}{2} \leqslant |\phi_{sc}| \leqslant \pi \end{cases}$$

and the noise terms are:

$$\begin{aligned} N_i &= \frac{1}{N}\sum_{n=1}^{N} [n_c(nT_s)\cos\phi_c - n_s(nT_s)\sin\phi_c] \\ &\quad \times \mathrm{Sin}\,(\omega_{sc}nT_s + \hat{\theta}_{sc}) \end{aligned}$$

$$\begin{aligned} N_q &= -\frac{1}{N}\sum_{n=1}^{N} [n_c(nT_s)\cos\phi_c - n_s(nT_s)\sin\phi_c] \\ &\quad \times \mathrm{Cos}\,(\omega_{sc}nT_s + \hat{\theta}_{sc}) \end{aligned}$$

with means:

$$E[N_i] = E[N_q] = E[N_i \cdot N_q] = 0$$

$$E[N_i^2] = E[N_q^2] = N_0/(2T)$$

Note that in the previous expressions we have dropped the variable i denoting the ith symbol.

The phase detector output signal is given by:

$$\begin{aligned} Z &= -I \cdot Q \\ &= P_d \cdot F_i(\phi_{sc}) \cdot F_q(\phi_{sc}) + N_i \cdot N_q \\ &\quad + \sqrt{P_d} \cdot F_i(\phi_{sc}) \cdot d \cdot N_q + \sqrt{P_d} \cdot F_q(\phi_{sc}) \cdot d \cdot N_i \\ &\quad + \text{periodic terms} \end{aligned}$$

The periodic terms arise from end effects when there is not an integer number of subcarrier cycles in a symbol time. Thus these terms have frequency depending on the relationship between symbol rate and subcarrier frequency. When the subcarrier frequency is close to a multiple of the symbol rate, the frequency of the periodic terms is low, but the amplitude is very small. When the subcarrier frequency is not close to a multiple of the symbol rate, the amplitude is higher, but the frequency is on the order of half the symbol rate. In this case, the frequency is assumed to be well outside the loop bandwidth. In either case, the effect of the periodic terms on the loop phase error is negligible. There are also small effects due to the sampling but they are negligible. The detailed analysis regarding these periodic terms will be published in a future report.

For ϕ_{sc} small, $F_q(\phi_{sc}) \approx 0$ and $F_i(\phi_{SC}) \approx 1$. With this approximation and neglecting the periodic terms, the phase detector output becomes

$$Z = P_d \cdot F_i(\phi_{sc}) \cdot F_q(\phi_{sc}) + N_{sc}$$

where $N_{sc} = \sqrt{P_d}\, d \cdot N_q + N_i \cdot N_q$ represents the signal X noise plus the noise X noise terms.

Now if we fix ϕ_{sc} and take the statistical expectation of Z we get the S curve, $S(\phi_{sc})$, of the subcarrier loop-phase detector. This is

$$S(\phi_{sc}) = E[Z \mid \phi_{sc}] = (2/\pi) \cdot P_d \cdot F_i(\phi_{sc}) \cdot F_q(\phi_{sc})$$

Notice that $S(\phi_{sc})$ is periodic in ϕ_{sc} with period π; in the interval $[-\pi/2, \pi/2]$, $S(\phi_{sc})$ has the following expression:

$$S(\phi_{sc}) = (2/\pi) \cdot P_d \cdot \phi_{sc} \cdot [1 - (2/\pi)|\phi_{sc}|], \quad |\phi_{sc}| \leqslant \pi/2$$

Figure 3 shows a plot of the normalized S curve $Sn(\phi_{sc})$ for $|\phi_{sc}| \leqslant \pi/2$; the S curve is defined as:

$$Sn(\phi_{sc}) = \pi/(2P_d) \cdot S(\phi_{sc})$$

Also notice that we have stable lock points at $\phi_{sc} = \pm k\pi$, which is to be expected in a Costas-loop S curve.

To get the phase error variance $\sigma^2_{\phi_{sc}}$, we assume that the noise term N_{sc} is white compared to the closed-loop bandwidth B_L. The one-sided spectral density of the above noise process, N_0', is given by

$$N_0'/(2T) = E[N_{sc}^2] = P_d \cdot N_0/(2T) + (N_0/2T)^2 \tag{6}$$

In deriving the above expression, $\phi_c = 0$ is used.

As for the residual carrier loop, the variance of the phase error can be expressed as (Ref. 1):

$$\sigma^2_{\phi_{sc}} = N_0' B_L / [S'(0)]^2$$

where B_L is now the one-sided bandwidth of the subcarrier loop and

$$S'(0) = \left. \frac{dS(\phi_{sc})}{d\phi_{sc}} \right|_{\phi_{sc} = 0}$$

Substituting N_0' and $S'(0)$ by their expressions we get:

$$\sigma^2_{\phi_{sc}} = \left(\frac{\pi}{2}\right)^2 \frac{N_0 B_L}{P_d} \left[1 + \frac{1/2}{E_s/N_0}\right]$$

Except for a factor of $(\pi/2)^2$, this is the same expression as for a Costas-loop tracking a sine-wave subcarrier. The factor $(\pi/2)^2$ is the same as that used for tracking unmodulated square waves with square-wave references. The factor $[1 + (1/2)/(E_s/N_0)]$ represents the squaring loss due to signal X noise and noise X noise terms, and is the same as for an optimum analog Costas loop, (Ref. 2). Thus, in this method of analysis, there is no theoretical degradation due to the digital implementation.

V. The Data-Aided Loop

A data-aided carrier tracking loop is a loop that combines residual carrier tracking and Costas-loop tracking to reduce carrier phase error. To accomplish this, a Costas data-aidingtype phase detector is implemented for the carrier by adding a third arm, with carrier I and subcarrier I. The output of the carrier Costas phase detector is added to the output of the residual carrier phase detector, with appropriate weighting. Minimum phase error for a given loop bandwidth is achieved by maximizing A^2/N_0 for the composite phase detector, where A is the slope of the S curve at zero phase and N_0 is the spectral density at zero frequency.

In the following analysis, it is assumed that the subcarrier loop is locked so that $\phi_{sc} = 0$, that ϕ_c is a slowly varying process, and that there is perfect symbol synchronization.

The Q arm of this loop is the same as the I arm of the subcarrier loop, and the output of its filter is given by Eq. (4):

$$I_q = \sqrt{P_d} \cdot d \cdot \cos(\phi_c) + N_i$$

The above expression neglects the periodic terms that get filtered out by the loop filter and the NCO; the variable i denoting the ith symbol is also omitted, and it is assumed that $\phi_{sc} = 0$; hence, $F_i(\phi_{sc}) = 1$.

Similarly, the output of the I-arm filter is given by:

$$I_i = \frac{1}{N} \sum_{n=1}^{N} i_n \ \mathrm{Sin}\left(\omega_{sc} nT_s + \omega_{sc} \frac{T_s}{2} + \hat{\theta}_{sc}\right)$$

$$= -\sqrt{P_d}\, d' \sin(\phi_c) + N_i'$$

where

$$d' = \frac{1}{N} \sum_{n=1}^{N} d(nT_s + T_s/2)$$

is also equal to d defined in Eq. (5) since we have a multiple of two samples per symbol and we have perfect symbol synchronization. The noise term is given by

$$N_i' = \frac{1}{N} \sum_{n=1}^{N} [n_c(nT_s + T_s/2) \sin\phi_c + n_s(nT_s + T_s/2) \cos\phi_c]$$

$$\times \quad \mathrm{Sin}(\omega_{sc} nT_s + \omega_{sc} T_s/2 + \hat{\theta}_{sc})$$

with $E[N_i'] = 0$, $E[N_i'^2] = N_0/(2T)$, and $E[N_i' \cdot N_i] = 0$ since N_i' and N_i are two orthogonal noise processes.

The carrier Costas phase detector output is:

$$U = -I_i \cdot I_q$$

$$= (1/2) P_d \sin(2\phi_c) + N_{cc}$$

where $N_{cc} = \sqrt{P_d} \cdot d \cdot [N_i \cdot \sin(\phi_c) + N_i' \cdot \cos(\phi_c)] + N_i \cdot N_i'$ represents the signal × noise and the noise × noise terms.

The S curve, $S1(\phi_c)$, of the carrier Costas-loop phase detector is found to be the regular Costas S curve, i.e., a sine wave in $2\phi_c$. Its expression is given by:

$$S1(\phi_c) = E[U \mid \phi_c] = (1/2) P_d \sin(2\phi_c)$$

The spectral density of N_{cc} is the same as that of N_{sc}, which was given by Eq. (6):

$$N_0' = P_d \cdot N_0 + N_0^2/(2T)$$

For only Costas tracking of the carrier, the phase-error variance is given by:

$$\sigma^2_{1\phi_c} = N_0' B_L / [S'1(0)]^2 = \frac{N_0 B_L}{P_d} \left[1 + \frac{1/2}{E_s/N_0}\right] \quad (7)$$

which is the same as that for any optimum Costas loop.

Finally, the output F_a of the residual carrier phase detector is combined with the output F_b of the carrier Costas phase detector. Let F_c be the combined phase-error signal with weights a and b; we have

$$F_c = a \cdot F_a + b \cdot F_b$$

If we want to maximize the total loop SNR then the weights a and b should be chosen such that:

$$a/b = (\sqrt{SNR_a} / \sqrt{SNR_b}) \cdot (\sigma_b/\sigma_a)$$

where

$SNR = 1/\sigma^2_{\phi_c}$ is the residual carrier loop SNR: Eq. (3)

$SNR = 1/\sigma^2_{1\phi_c}$ is the carrier Costas-loop SNR: Eq. (7)

$\sigma_a^2 = N_0 B_L$ is the variance of the noise process in the residual carrier loop

$\sigma_b^2 = N_0' B_L$ is the variance of the noise process in the carrier Costas loop

After substitution we get

$$a/b = \frac{\cos \Delta}{\sin \Delta} \sqrt{P_d + \frac{N_0}{2T}}$$

and the sideband aided loop SNR is: $SNR_c = SNR_a + SNR_b$

VI. Summary and Conclusions

For a telemetry processing system utilizing IF sampling, the random noise performances of residual carrier loops, optimum subcarrier Costas loops, and data-aided carrier tracking loops have been shown to be the same as the corresponding loops implemented by traditional analog means, or by digital means after analog demodulation to I-Q baseband. Some periodic effects are introduced by end effects when there is not an integer number of subcarrier cycles in a symbol time, but these effects are negligible for narrow-loop bandwidths because the frequency of the periodic effect is then outside the loop bandwidth. It is thus concluded that IF sampling is useful for avoiding the problems inherent with analog implementations, such as dc offsets in mixers and amplifiers, the need for calibration and adjustments, and less reliability, versatility, and flexibility than digital systems.

References

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2. Yuen, J. H., *Deep Space Telecommunications Systems Engineering*, Publication 82-76. Jet Propulsion Laboratory, Pasadena, California, July 1982.

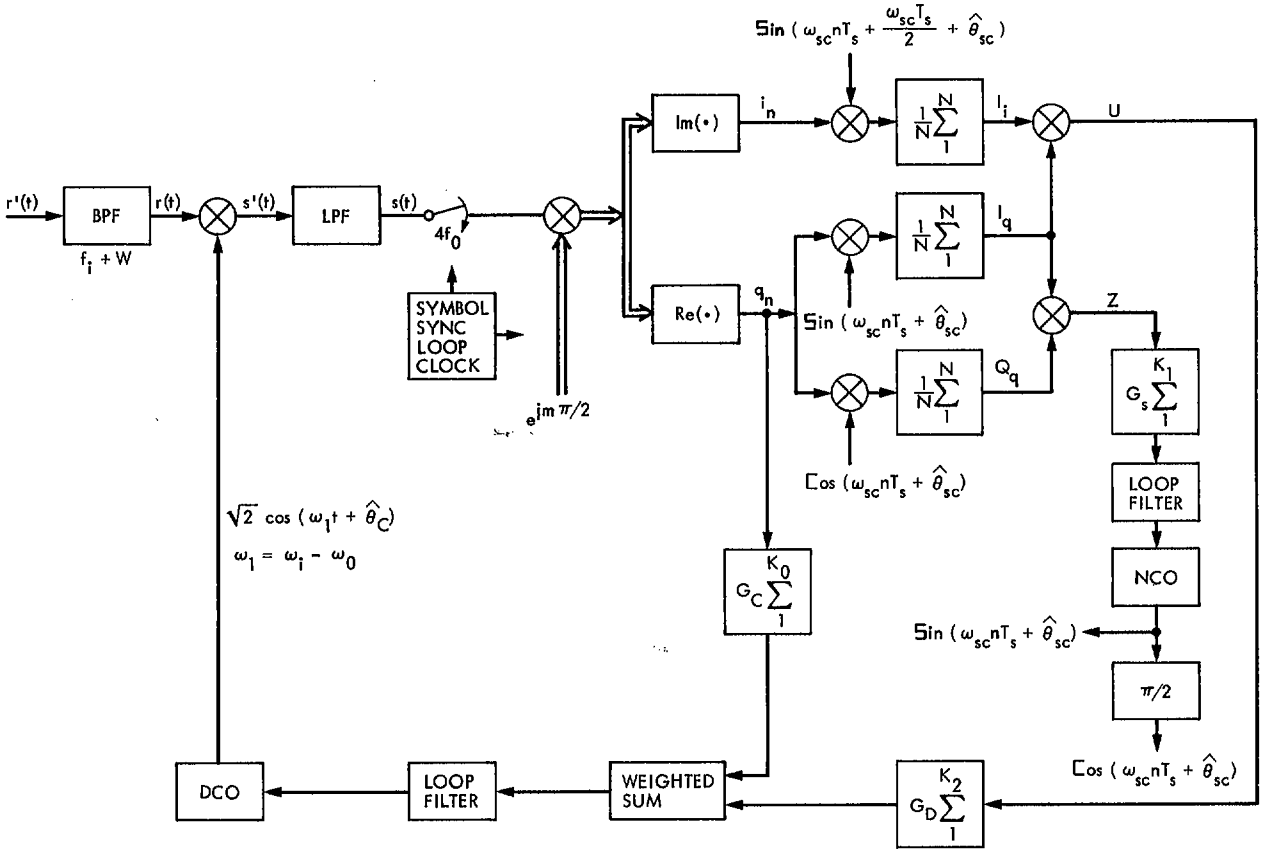

Fig. 1. Functional block diagram of proposed advanced receiver

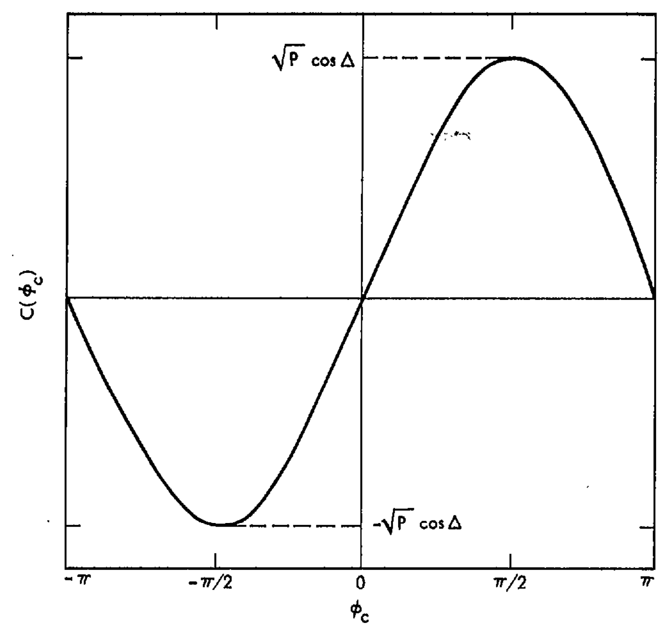

Fig. 2. S curve of residual carrier loop-phase detector; $C(\phi_c) = \sqrt{P}\cos\Delta \sin\phi_c$, $-\pi \leq \phi_c \leq \pi$

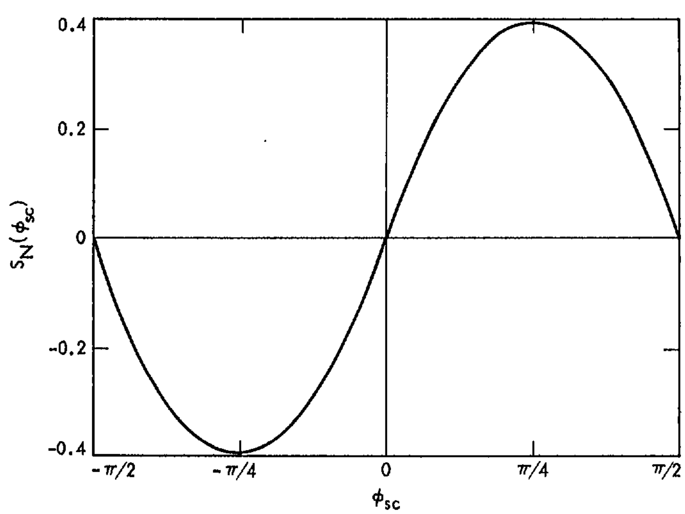

Fig. 3. S curve of subcarrier loop-phase detector; $S_N(\phi_{sc}) = \phi_{sc}(1 - 2/\pi|\phi_{sc}|)$, $-\pi/2 \leq \phi_{sc} \leq \pi/2$