The Accuracy of Radio Interferometric Measurements of Earth Rotation

T. M. Eubanks, J. A. Steppe, and M. A. Spieth
Tracking Systems and Applications Section

The accuracy of VLBI earth rotation (UT1) measurements is examined by intercomparing TEMPO and POLARIS data for 1982 and the first half of 1983. None of these data are simultaneous, and so a proper intercomparison requires accounting for the scatter introduced by the rapid, unpredictable, UT1 variations driven by exchanges of angular momentum with the atmosphere. A statistical model of these variations, based on meteorological estimates of the Atmospheric Angular Momentum (AAM) is derived, and the optimal linear (Kalman) smoother for this model is constructed. The scatter between smoothed and independent raw data is consistent with the residual formal errors, which do not depend upon the actual scatter of the UT1 data. This represents the first time that an accurate prediction of the scatter between UT1 data sets has been possible.

I. Introduction

Observations of extragalactic radio sources with VLBI between widely separated antennas can be used to make highly accurate measurements of the orientation and length of the vector between each pair of antennas (the baseline vector). Changes in baseline lengths are currently being monitored by several groups in the hope of eventually detecting tectonic motions over transcontinental or intercontinental distances. Changes in the baseline orientation are dominated by rotations of the earth as a whole. Independent Very Long Baseline Interferometry (VLBI) earth orientation estimates are now produced routinely by the POLARIS project of the National Geodetic Survey (NGS) (Ref. 1) and by the TEMPO program of the Deep Space Network (DSN) at the Jet Propulsion Laboratory (JPL) (Ref. 2). The VLBI data provide a substantial reduction in the Root Mean Square (rms) scatter seen in previous UT1 intercomparisons (Refs. 3, 4, and 5).

Knowledge of the earth orientation is essential for accurate spacecraft navigation, and is of intrinsic geophysical interest (Refs. 6, 7, and 8). An understanding of the true errors in these VLBI measurements is essential to realizing their full potential. It is generally not possible to verify the full accuracy of baseline length measurements because of the lack of independent length determinations of comparable accuracy over the same baseline. Baseline orientation changes are dominated by changes in the total earth orientation, and it is thus possible to intercompare earth orientation results from widely separated baselines. The intercomparisons described in this article are thus of interest to all geodetic users of VLBI, as well as to those particularly interested in the UT1.

II. VLBI Measurements of the UT1

The JPL TEMPO program monitors changes in earth orientation, together with station clock behavior, in support of interplanetary navigation by the DSN. TEMPO analyzes the results from two VLBI observing sessions per week, one each on the Spain–California (SC) and Australia–California (AC) DSN baselines. The SC baseline, with a latitude of only 3°, best determines the UT1, and only the TEMPO results from that baseline will be used in this article. The accuracy and density of these data have improved considerably since the lengthening of each observing session to a total of 3 hours in February 1982. Since some useful measurements are also available from January 1982, we started our intercomparison at the beginning of that year. Thirty TEMPO UT1 measurements are available for the period between 1982.0 and 1983.4. Two of these have formal errors > 2.0 ms and comparable scatters; these had a negligible effect on our analysis and were excluded from further consideration. The remaining measurements have formal UT1 errors from 0.22 to 1.43 ms.

The POLARIS project was specifically established to monitor earth orientation changes with state-of-the-art VLBI equipment (Ref. 5). This project has conducted 24-hour observing sessions every week since June 1981, over the single baseline between the Westford Observatory in Massachusetts and the George R. Agassiz Station (GRAS) at Fort Davis, Texas. The latitude of this baseline is 20°. Some of the POLARIS experiments also involved simultaneous observations at the Onsala Space Observatory in Sweden. A total of 68 POLARIS UT1 measurements are available for the period between 1982.0 and 1983.4, with formal UT1 errors of 0.05 to 0.63 ms.

Any single-baseline VLBI experiment is sensitive to only two components of earth orientation. Since neither the TEMPO SC baseline nor the POLARIS Westford-GRAS baseline is exactly equatorial, neither can measure the UT1 directly. Both the TEMPO data and the POLARIS single-baseline UT1 estimates thus have a minor contribution from the Bureau International de L'Heure (BIH) smoothed Circular-D Polar Motion estimates. We assumed that these have a 10 milliarcsecond random error, found the induced UT1 error for each baseline, and found the rss of this error and the quoted formal error to better estimate the true formal error of each single-baseline UT1 estimate. The resulting formal errors are called the raw formal errors. We did not adjust the formal errors from the POLARIS multiple baseline experiments in any way.

Our null hypothesis is that the true measurement errors are independent random variables with standard deviations equal to the raw formal errors. The only way to test this hypothesis rigorously is by an intercomparison of independent measurements. Since there are no simultaneous TEMPO and POLARIS measurements, any intercomparison will involve averaging or interpolating the data in some fashion. The scatter between a Kalman smoothing of one data set and independent raw data provides a useful test of both the data accuracy and the statistical model of the UT1. The various smoothings discussed in this analysis use data taken before 1982.0 to minimize any "edge effect" at the beginning of the intercomparison. The filter rapidly down-weights any old information, however, and the effect of including the pre-1982 data is small.

III. Kalman Filtering and Smoothing of the UT1

The UT1 Kalman filter and smoother were developed at JPL as part of an effort to smooth and predict earth orientation changes for spacecraft navigation. The UT1 filter uses a statistical model of the unpredictable high-frequency UT1 fluctuations driven by exchanges of angular momentum with the atmosphere (Ref. 7). Meteorologically derived AAM data indicate that the UT1 power spectrum should be proportional to the frequency^{-4} at periods < 100 days (Ref. 8). A random process that obeys such a power law at all frequencies is an integrated random walk,

$$\frac{d^2\, UT1}{dt^2} = W(t) \tag{1}$$

where $W(t)$ is a white noise with a constant power spectral density, Q. (We are ignoring problems of existence of derivatives, etc., that can be handled rigorously, at the cost of loss of clarity (Ref. 9)). This model thus assumes that the torques on the solid earth can be described as a white noise, and that in the absence of these torques the rate of rotation of the solid earth would be a constant. For the filter to work properly, the UT1 model must provide realistic estimates of the UT1 signal-to-measurement-noise ratio (SNR), especially at frequencies where the true SNR is near unity. At frequencies where the true SNR is very large or very small, it is generally sufficient if the model SNR is also very large or very small, respectively. With modern measurement techniques and typical measurement densities, the SNR will be near unity for frequencies between 0.02 and 0.05 day^{-1}. The seasonal variations in the UT1 are well above the noise, and the model SNR is also large at those frequencies, so we do not include these explicitly in the UT1 model. There are also predictable monthly and fortnightly UT1 oscillations of tidal origin. These are estimated from an a priori model (Ref. 10) and removed from all of the UT1 data before any further analysis.

Given a linear stochastic model, it is relatively straightforward to derive the corresponding Kalman filter (Refs. 9, 11, and 12). The forward filter, which estimates the UT1 based on past and present data, is best described recursively in the state space formalism. The state vector corresponding to Eq. (1) is

$$\mathbf{X} = \begin{bmatrix} UT1 \\ \\ \dfrac{dUT1}{dt} \end{bmatrix} \tag{2}$$

Given estimates of the state $\widehat{\mathbf{X}}$, and the covariance matrix $\mathbf{P}$ at time T, the state estimate at a later time $T + \Delta t$, in the absence of measurements, is given by the state propagation equations:

$$\widehat{\mathbf{X}}_{T+\Delta t} = \Phi(\Delta t)\,\widehat{\mathbf{X}}(T) \tag{3a}$$

and

$$\mathbf{P}_{T+\Delta t} = \Phi(\Delta t) \quad \mathbf{P}_T\,\Phi^T(\Delta t) \tag{3b}$$

$$+ \int_T^{T+\Delta t} \Phi(T + \Delta t - t') \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix} \Phi^T(T + \Delta t - t')\; dt'$$

where

$$\Phi(\Delta t) = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix}$$

is the state transition matrix. The integral in Eq. (3b) describes the increasing uncertainty induced by the unpredictable rapid fluctuations in UT1, and Q is thus a measure of the strength of these fluctuations. Based on our analysis of the AAM data, we adopted a value of 0.0036 $(\text{ms/day})^2/\text{day}$ for Q. Note that if Q is set to zero, the filter will simply fit a straight line to the data. When a new measurement is available, the optimum state estimate consists of the vector weighted average of the propagated state and the new measurement. This weighted average takes advantage of the correlation between the components of the state vector to determine both the UT1 and its derivative from a series of UT1 measurements. The optimal smoothing is the vector weighted average of the forward filter and a similar filter run backwards on the same data (Refs. 11 and 13).

IV. Results

In our preliminary data analysis, we noticed a bias of 0.35 to 0.40 ms between the TEMPO and POLARIS data in all intercomparisons, including those with the pre-1982 data. Such biases arise from slight differences in the orientation of the reference frames used in the reduction of the two data sets. We subtracted 0.375 ms from all of the TEMPO UT1 data used in this intercomparison. Figure 1 shows the raw TEMPO and POLARIS data, together with the Kalman smoothing of the combined data set, after subtracting the BIH smoothed Circular-D UT1, the tidal effects, and the TEMPO-POLARIS bias. One-sigma error bars are shown with the raw VLBI data. It is apparent that the TEMPO and POLARIS data agree at the ms level, and that there are UT1 fluctuations of up to 5 ms ignored by the Circular-D smoothing. The agreement between the two data sets is especially impressive considering that these are derived from completely independent efforts, using different equipment observing schedules, software, and procedures (Refs. 1 and 2).

Figure 2 shows our Kalman smoothing of the POLARIS data together with the raw TEMPO data, again after subtracting the BIH smoothed Circular-D data, the tidal model, and the TEMPO-POLARIS bias. The smoothing residuals (the difference between the raw and smoothed data in Fig. 2) are shown in Fig. 3, together with one-sigma error bars. Each residual formal error is the rss of the raw TEMPO formal error and the formal-error estimate of the POLARIS Kalman smoothing at that epoch. The rms scatter of these residuals is 0.57 ms, and the ensemble χ^2 is 35.4, for 28 degrees of freedom, which is consistent with the null hypothesis. The results of the χ^2 test should be regarded as an approximation of the true significance of the observed scatter since smoothing reduces the independence of the smoothing residuals. The rapid down-weighting of old information by the filter means that smoothing errors should have correlation times of a few weeks at most. The long-term correlation between the residuals in Fig. 3, if real, is thus probably not induced by the smoothing.

Any small random error not in the VLBI error budget would be most noticeable in the scatter of residuals with small formal errors. There are 17 residuals in Fig. 3 with formal error estimates < 0.5 ms. These differences have an rms of 0.50 ms and a χ^2 of 26.5 with, of course, 17 degrees of freedom, again consistent with the null hypothesis. There do not seem to be any obvious outliers in this data set. In fact, none of the residuals in Fig. 3 are as much as three formal errors from zero. A similar analysis, with similar results, was conducted by comparing the smoothed TEMPO and the raw POLARIS data. The lower density of the TEMPO data introduces more correlation into the smoothing residuals, which complicates the interpretation of these results.

Since we implemented the Kalman smoother by a weighted average of forward and backward filters, it is easy to estimate the UT1 at the epoch of any measurement from an independent smoothing of all the other measurements. This "excluded" smoothing thus provides a statistically independent estimate of the UT1 at the time of each measurement. We examined the scatter between each measurement, except the last, and the excluded smoothing of the combined VLBI data set. The residual formal error was again the rss of the raw data and smoothing formal errors. There are 95 such residuals, with an rms of 0.62 ms and a χ^2 of 123.9 (with 95 degrees of freedom). Of these differences, 44 have residual formal errors <0.5 ms, with a rms of 0.40 ms and a χ^2 of 44.1 with 44 degrees of freedom. Only one residual is more than three formal errors from zero. Although this point (at 4.6 formal errors from zero) is not likely to have occurred by chance, it is unclear if this is a true outlier or merely a symptom of unusually rapid changes in the UT1. There are enough excluded smoothing residuals to examine the scatter as a function of the residual formal errors for formal errors between 0.4 and 1.0 ms. Except for the possible outlier, the scatter is consistent with the formal error throughout this range. We attempted to estimate the variance (assumed to be a constant) of any unmodeled error in these residuals. The estimated variance was consistent with zero, and we think that it is unlikely that any such unmodeled random error could have a standard deviation as large as 0.4 ms.

An analysis of all three components of the earth orientation (Ref. 14), conducted after this paper was originally written, indicates that the POLARIS Westford Fort Davis single-baseline data have a substantial systematic error in one component. This error, which is partly seasonal in nature, is not present in the multiple-baseline POLARIS data. The systematic error in the single-baseline data is probably due to tropospheric modeling errors and induces a systematic UT1 error with an rms scatter of ~ 0.34 ms. The intercomparisons discussed in this article did not distinguish between multiple- and single-baseline results, and the temporal distribution of the available TEMPO data was not well suited for detecting seasonal systematic errors. It is not surprising that this error was not detected in the analysis presented in this paper.

V. Conclusions

Our results show that VLBI determinations of the earth rotation from two independent efforts are consistent to within 0.4 to 0.5 ms (20 to 25 cm), and that the observed scatter is explained by the measurement formal errors and a statistical model for the rapid UT1 fluctuations driven by the atmosphere. These fluctuations will make it impossible to validate the subdecimeter VLBI formal errors without either a denser series of measurements or a set of exactly simultaneous data. The periods of intense activity planned during the MERIT campaign (Ref. 15) may provide the necessary raw material for such validations. Since this intercomparison uses only 1.3 years of data, it is insensitive to any long-period systematic errors. The accumulation and analysis of more VLBI data, as well as data from other techniques, will be necessary to fully understand the role of systematic errors in the earth rotation data. We plan to report on our analysis of all three components of the earth orientation in the future.

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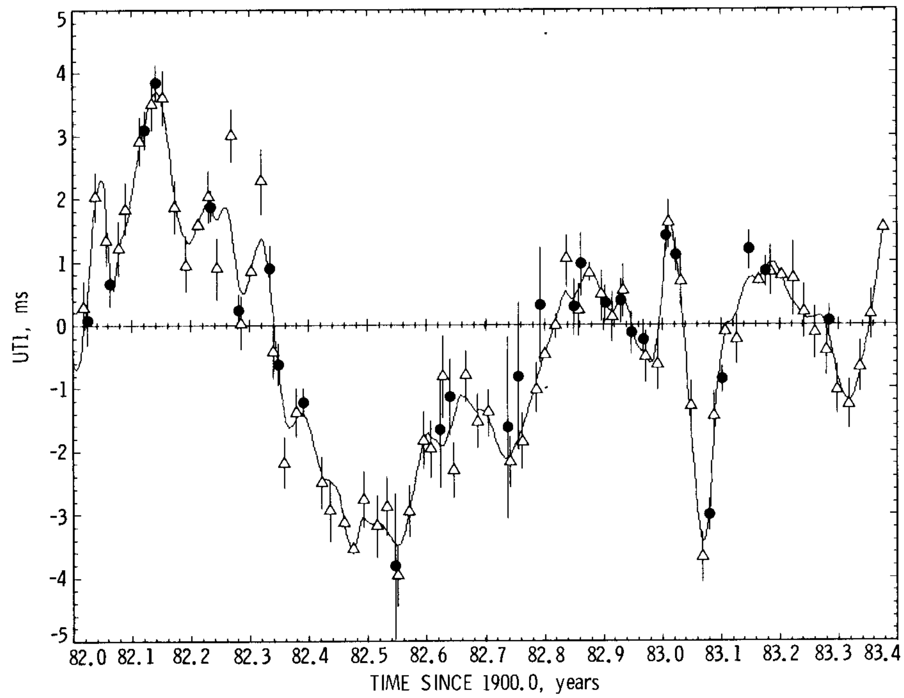

Fig. 1. The TEMPO (closed circles) and POLARIS (open triangles) UT1 estimates after subtracting the short-period tidal effects, the BIH smoothed Circular-D UT1, and the relative bias. One-sigma error bars are presented with each measurement. The solid line is the Kalman smoothing of the combined VLBI data set, also after subtraction of the Circular-D smoothing.

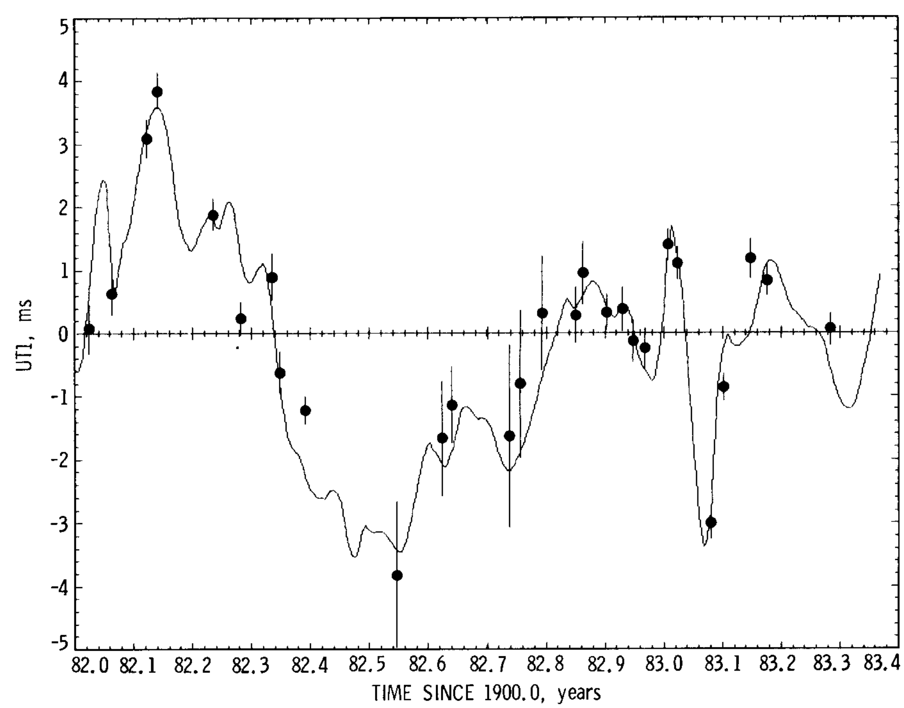

Fig. 2. The raw TEMPO data as given in Fig. 1, together with the Kalman smoothing of the POLARIS data, again after subtraction of the BIH smoothed Circular-D UT1 estimate, the tidal model, and the relative bias.

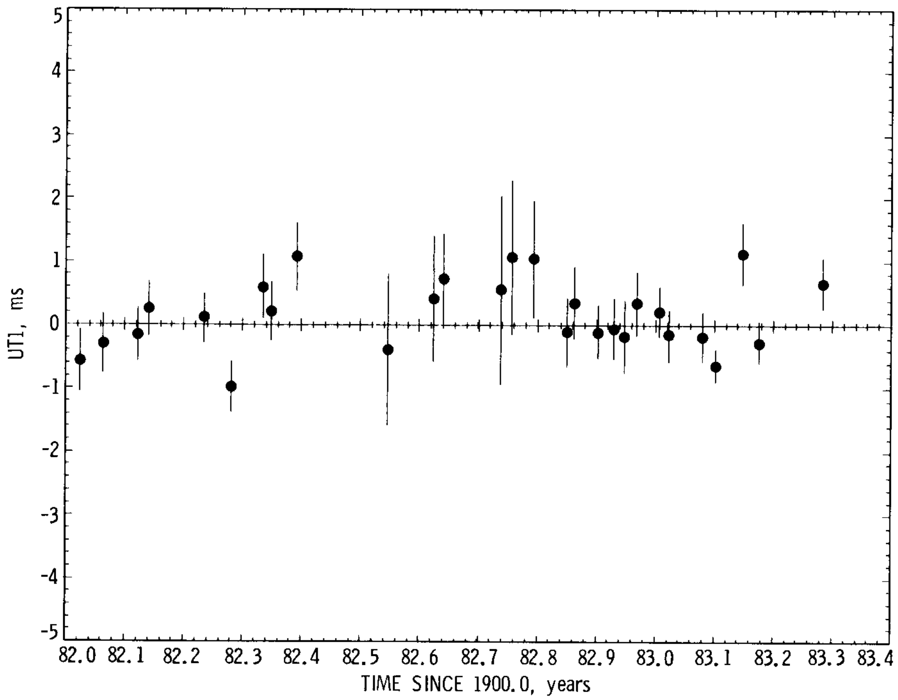

Fig. 3. The difference between the raw TEMPO data and the Kalman smoothed POLARIS data shown in Fig. 2. The error bars on these differences are the rss of the raw TEMPO formal errors and the error estimated by the POLARIS Kalman smoothing at the epoch of each TEMPO measurement.