Tau Ranging Revisited

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The advent of reasonably fast and inexpensive Very Large Scale Integration (VLSI) components offers an opportunity to reconsider the merits of composite-code-uplink ranging systems, abandoned years ago. It is shown in this article that a ranging receiver with a sufficient and reasonable number of correlators is competitive with the current sequential component ranging system and may outperform that system by some 1.5–2.5 dB. The optimum transmitter code, the optimum receiver, and a near-maximum-likelihood range-estimation algorithm are presented.

I. Introduction and Background

Prior to 1973, planetary and lunar spacecraft ranging systems at JPL utilized a transmitted uplink code made by combining binary clock and pseudonoise sequences in a majority-vote logic [1].[1] The "composite-code-uplink" ranging receiver consisted of one or two channels that correlated the transponded signal with combinations of the clock and each separate component sequentially through each successive symbol-delay to determine the precise delay on each of the components. Since transmitted power was distributed among the various clock and pseudonoise code components but the receiver was sensitive to only one component at one phase at a time, acquisition time was longer by about a factor of 16 than if the receiver could have processed all the received power during the acquisition time.

Consequently, once sufficient analysis and precautions had been taken to ensure that uplink ranging sidebands would not interfere with the spacecraft command system, a "sequential-component-uplink" ranging method [2] was devised that is still being used today. The term "sequential," in this case, refers to the transmitted code, which is a time series of square waves of successively shorter wavelengths. The receiver, still a few-correlator device, is programmed to acquire the sequentially transmitted components one by one. The newer method had a 16:1 acquisition-time advantage over the older scheme because it could utilize all the transponded power for each component during the acquisition time. The necessity to program the uplink code and to program the receiver to switch components at the proper round-trip-light-time interval was a disadvantage compensated by the signal-to-noise advantage.

During a transition period, both ranging techniques were used. The older, composite transmitted code method was referred to as the "τ" system, and the newer, sequential component code method was called the "μ" system. These names were dubbed by Robertson Stevens, now the Chief Engineer

[1] R. W. Tappan and R. C. Tausworthe, "DSIF Technical Description, Planetary Ranging Equipment, Mariner-Mars 1971 Configuration," JPL Document TD505943A (internal document), Jet Propulsion Laboratory, Pasadena, California, February 4, 1972.

of the Deep Space Network, one day while strategizing at the blackboard in a design meeting, groping for a notation to distinguish the two. They derive from the initials of the then-purveyors of the two systems: the author (guess which one) and Warren Martin. Usage of these designations has decayed over the years because the τ system is no longer extant. The designations are reinstituted in this article for brevity in referencing the two schemes.

The τ planetary composite transmitter code was generated by combining a clock square wave with a majority-vote logic of 5 pseudonoise sequences in an exclusive-OR fashion. The components had symbol-periods of 2, 7, 11, 15, 19, and 23, for a total code period of N = 1,009,470. This, when clocked at a symbol-period t_0 of about 1 μsec, gave a repetition period of about 1 sec, yielding a 2-way range ambiguity interval of approximately 150,000 km.

The majority-vote combining logic in the old-τ system was chosen [3], [4] because it evenly (and optimally, for that strategy) distributed the power among all components for sequential detection. All component delay measurements were thus made with times approximately proportional to the component periods. The clock-code (period 2) phase was acquired first, and then the other 75 code phase correlations were made sequentially. Two correlator channels were time-shared during range acquisition in a way that balanced the channel gains and removed any residual dc bias voltage in the baseband detection process.

The reason for utilizing only a few correlators in ranging receivers until now has been that each correlator channel consisted of relatively expensive analog and unit-logic digital hardware. However, with the advent of reasonably high speed digital analog-to-digital devices and very large scale integrated (VLSI) digital devices, correlators may now be made at much more modest cost [7].

It is now economically feasible under VLSI technology to put the needed number of correlators into a receiver to build a detector for each of the components at each symbol-delay of the composite code. (It still may be impractical to build a full matched filter for the overall transmitted code, however.) It is therefore appropriate to reevaluate the relative merits of the two ranging methods under the removed constraint that previously fixed the number of correlator channels.

This article shows that a new-τ composite-component uplink code (or "$\nu\tau$," for brevity) which utilizes a new combining logic for the transmitter code and a 77-correlator receiver is again favorable in performance. In fact, it is shown that the $\nu\tau$ method is only about 0.25 dB below the performance of a matched filter for the optimal transmitter code. As the μ system is now configured, about half of the range-measurement time is spent in correlating with the highest-frequency component (the clock), the other half being spent determining the range-cells of the lower-frequency components. The $\nu\tau$ system thus outperforms it by some 2.5 dB in signal-to-noise ratio.

A companion paper by the author and J. R. Smith [7] discusses the requirements and conceptual design of the correlator channels and VLSI devices required.

II. The Transmitted Code

The code component periods remain the same as in the earlier τ system: 2, 7, 11, 15, 19, and 23. The combining logic for the transmitted code, however, is now taken to be

$$x(\mathrm{c}) = c_1 \text{ XOR AND } (\mathrm{c}) \tag{1}$$

in which $\mathrm{c} = (c_1, \ldots, c_6)$, c_1 is the clock (period 2) sequence, XOR is the exclusive-OR function, and AND ( ) is the logical-AND of all 6 component sequences. The logical-AND, or "unanimous-vote" logic, is the limiting case of the majority-vote logic used previously.

Since AND (c) contains only one "1" in its truth table of 64 entries, the in-phase cross-correlation of $x(\mathrm{c})$ with c_1 will be [3]

$$R_{x1} = \frac{(64-1)-1}{64} = 0.97 \tag{2}$$

The in-phase cross-correlation of $x(\mathrm{c})$ with a code of the form $(c_1 \text{ XOR } c_i)$, for $i = 2, \ldots, 6$, is only

$$R_{xi} = \frac{1+1}{64} = 0.031 \tag{3}$$

(These figures are only approximate, being influenced slightly by the sense of imbalances between 0's and 1's in each of the pseudonoise sequences. These imbalances can be chosen to further optimize the reception, but this is left as an exercise for the implementer.)

The cross-correlations as functions of ranging delay are shown in Fig. 1.

III. Performance

The $\nu\tau$ receiver achieves range measurement precision by clock-component correlation, just as did both predecessors. But since the clock component of the transmitted code contains 94 percent (i.e., 0.97^2) of the total ranging power, there

is only a 0.27 dB degradation in acquisition time from transmitting the clock component alone (such a code would not, however, remove the ambiguity of the range).

The requirements for range accuracy demand that the standard deviation of the range measurement due to noise be about 1/1000 of the "chip" time, t_0, or symbol-rate of the clock. The relative clock variance σ_Δ^2 of $\Delta = \tau/t_0$ is thus about 10^{-6}. In order to achieve this accuracy, the received signal-energy/noise-density ratio must accordingly be high [5].

$$\frac{ST}{N_0} = \frac{1}{16 R_{xi}^2 \sigma_\Delta^2} = 6.7 \times 10^4 \tag{4}$$

where S is the total signal power, T is the correlator integration time, and N_0 is the received noise (single-sided) spectral density.

Detection of the pseudonoise component phase is accomplished by correlating the received signal with each of the separate phases of (c_1 XOR c_i). The power in each component is only about $0.001 = 0.31^2$ of the total, so the component detection-energy/noise-density ratio is about

$$\frac{S_i T}{N_0} = \frac{R_{xi}^2 ST}{N_0} = 0.001 \times 6.7 \times 10^4 = 67 \tag{5}$$

The maximum required pseudonoise component-energy/noise-density ratio for an error probability of 0.01 was about 10 for the old-τ system. Since detection of the pseudonoise range cell (see below) involves summing channel values for *pairs* of correlators, the additional noise may degrade the required $S_i T/N_0$ to about 20 (a full analysis has not yet been made). The better-than-a-factor-of-three margin ensures, however, that unerring full-range acquisition is almost certain.

IV. Maximum-Likelihood Receiver

The maximum-likelihood estimator of the clock component phase is well known and will not be repeated here. The remaining pseudonoise code delays, however, are to be estimated from measurements made in parallel with the clock delay determination. This approach differs significantly from the old-τ method, where the receiver pseudonoise sequences were acquired *after* the clock so that the receiver codes could be adjusted to then be in step with the received signal. Thus, whereas the old-τ system enjoyed full component correlation in only one integration bin per component, the $\nu\tau$ scheme must make do with partial component correlation in two adjacent correlation channels for each component.

The derivation of the maximum-likelihood detector is straightforward: We presume that we receive the transmitted binary ranging signal $x(t) = x(\mathbf{c}[t])$, normalized here to unit power, immersed in wideband Gaussian noise $n(t)$, as

$$y(t) = \alpha x(t - \tau) + n(t) \tag{6}$$

where $\alpha = S^{1/2}$. The time delay τ is to be estimated as that value $\hat{\tau}$ maximizing the conditional probability (density) function

$$p\{\tau \mid y(t), 0 \leqslant t \leqslant T\} \tag{7}$$

Under the usual assumption that τ is uniformly distributed over the unambiguous-range interval, the likelihood ratio, by Bayes' rule, is

$$\lambda = \frac{p\{\hat{\tau} \mid y(t)\}}{p\{\tau \mid y(t)\}} = \frac{p\{y(t) \mid \hat{\tau}\}}{p\{y(t) \mid \tau\}} \geqslant 1 \tag{8}$$

where the interval $(0, T)$ dependency has been suppressed for notational convenience.

The probability of receiving $y(t)$, given τ, is the probability that the noise in $n(t) = y(t) - \alpha x(t - \tau)$. Because of the wideband Gaussian character of the noise, the likelihood ratio becomes [6]

$$\lambda = \frac{\exp\left(-\frac{1}{N_0}\int_0^T \left[y(t) - \alpha x(t - \hat{\tau})\right]^2 dt\right)}{\exp\left(-\frac{1}{N_0}\int_0^T \left[y(t) - \alpha x(t - \tau)\right]^2 dt\right)} \geqslant 1 \tag{9}$$

where $\exp(x)$ is the exponential function e^x.

By noting $x^2(t) = 1$, canceling like terms in the numerator and denominator, and taking logarithms, we find that the condition on $\hat{\tau}$ is that

$$\int_0^T y(t) x(t - \hat{\tau})\, dt \geqslant \int_0^T y(t) x(t - \tau)\, dt \tag{10}$$

That is, $\hat{\tau}$ will be the maximum-likelihood estimator of τ provided that it maximizes the correlation between the observed $y(t)$ and the delayed transmitted code. However, a continuum of correlators is infeasible, so we must infer $\hat{\tau}$ from the finite number of measurements we do make. It has been shown [3] that the maximum-likelihood value can be inferred using correlations of the incoming signal and various delays of

the transmitted codes, in the form $x_1 = c_1$ and $x_i = c_1$ XOR c_i for $i = 2, \ldots, 6$.

$$I_{ij} = \int_0^T y(t)x_i(t - \hat{\tau}_j)\,dt \tag{11}$$

To decrease ranging inaccuracy caused by waveform distortion within the communication system, estimation of the clock component phase in the current μ system is performed by maximum-likelihood methods applied only to the fundamental harmonic of the received clock component. This results in a modest, justifiable increase in required integration time. The $\nu\tau$ method would presumably have the same requirement for this clock estimation scheme.

Since almost all of the transmitted power is in the clock component of the code, the contribution of the 75 other correlators in improving the accuracy of the clock phase estimate will be negligible. Hence, the value of $(\hat{\tau} \bmod 2t_0)$ may be determined from the clock-channel correlators alone. (Combined, weighted estimation of the clock phase from all channels can be done, however, with only slightly more complexity in the estimation program, if desired.) Since 94 percent of the transmitted power is in the clock component, and since maximum-likelihood estimation is performed on this component, the clock phase estimate is very nearly the same as the maximum-likelihood estimate of a pure clock signal.

Thus, any method that with high likelihood selects the proper range-cell delays of the remaining components will measure the range within 0.27 dB of the performance of a maximum-likelihood device, insofar as ranging accuracy is concerned.

The clock-channel measurement of $(\hat{\tau} \bmod 2t_0)$ is required to be very close to the actual value of $(\tau \bmod 2t_0)$ for system accuracy. From this value, $(\hat{\tau} \bmod t_0)$ may be determined, as well as the ± 1 sense of the in-step correlation (Fig. 1). Therefore, determination of $(\hat{\tau} \bmod Nt_0)$ additionally requires only the estimation of the integer values k_i such that

$$\hat{\tau} - k_i t_0 = (\hat{\tau} \bmod t_0) \tag{12}$$

for each of the remaining pseudonoise components (period $N_i t_0$). We may estimate these values from the correlator outputs of each pseudonoise component and combine them to form the overall range using the Chinese remainder theorem, just as did the previous τ system.

Only two of the correlator integration values I_{ij}, $j = 1, \ldots, N_i$ for the ith code component may derive from partial correlation with the true delay. The other I_{ij} values correspond to out-of-phase correlation levels. Let z_{ij} denote the normalized sum of adjacent correlators at the jth position of the ith code component:

$$\begin{aligned} z_{ij} &= \frac{(I_{ij} + I_{i,j+1})}{\alpha} \\ &= R_{xi}(\tau - jt_0) + R_{xi}(\tau - (j+1)t_0) + n_{ij} \\ &= \bar{z}_{ij} + n_{ij} \end{aligned} \tag{13}$$

for an appropriately defined noise term n_{ij}. The signal portion of z_{ij}, denoted $\bar{z}_{ij}$ in the equation above (see Fig. 2),

$$\bar{z}_{ij} = \begin{cases} R_{xi} \times \left(1 + \dfrac{1}{N_i}\right)\left(\dfrac{\tau}{t_0} - j + 1\right) & \text{if } (j-1)t_0 \leqslant \tau < jt_0 \\ R_{xi} \times \left(1 + \dfrac{1}{N_i}\right) & \text{if } jt_0 \leqslant \tau < (j+1)t_0 \\ R_{xi} \times \left(1 + \dfrac{1}{N_i}\right)\left(j - \dfrac{\tau}{t_0}\right) & \text{if } (j+1)t_0 \leqslant \tau < (j+2)t_0 \\ 0 & \text{elsewhere} \end{cases} \tag{14}$$

If k is the correct range cell, i.e., $kt_0 \leqslant \tau < (k+1)t_0$, then the geometry of the correlation function (Fig. 1) leads to the estimator

$$\hat{\tau}_i - kt_0 = \frac{N_i I_{i,k+1} - I_{ik}}{(N_i - 1)(I_{ik} + I_{i,k+1})} = (\hat{\tau}_i \bmod t_0) \tag{15}$$

The right-hand side of this equation should be $(\hat{\tau}_i \bmod t_0)$, as already estimated accurately by clock-channel computations, within the expected noise deviation.

If there were no noise, the value of j yielding the maximum z_{ij} would be the correct range cell. But with noise, the cells on either side must also be scrutinized, as the spillover correlation and noise may cause us otherwise to choose the index of the wrong range cell.

Hence, let $\hat{k}_i$ be that j-index for which z_{ij} is maximum, and let k_i be $\hat{k}_i - 1$, $\hat{k}_i$, or $\hat{k}_i + 1$, whichever minimizes the difference between $\hat{\tau}_i$ and $\hat{\tau}_1$. This k_i is a high-likelihood range-cell estimate for the ith component because the maximum-likelihood range cell is certainly one of the three candidates, and the comparison of the symbol-fraction range offset, calculated as above, against the accurate determination of the clock

symbol fraction, rules out the other two candidates with high probability.

Analysis indicates that the probability of making the wrong choice would be very remote. As may be seen in the geometry of the cross-correlation function, shown in Fig. 1, the wrong pair of correlator values inserted into the symbol fraction formula above produces a significantly different estimate for $\hat{\tau}_i$ than for $\hat{\tau}_1$.

V. Range Calculation Algorithm

The procedure used by the $\nu\tau$ receiver to reconstruct the range is therefore basically the same as in the old-τ system, except for the way the component range cell determinations are made. Having read the 77 correlation values all at once after an integration interval T, the normal calculations on the two clock values determine the clock component range delay (mod $2t_0$) with high accuracy. The remaining 75 correlation values are grouped by component, and then a value $\hat{k}_i$ is chosen for each component. This $\hat{k}_i$ is the range-cell index j that maximizes the sum of adjacent correlation values $I_{i,j} + I_{i,j+1}$. Each of the three range-cell indices $\hat{k}_i - 1$, $\hat{k}_i$, and $\hat{k}_i + 1$ in turn is hypothesized to be the correct range-cell index. The appropriate correlator values for each of these indices are then used to compute a symbol-fraction offset using the estimator formula above. The candidate that comes closest to the clock-channel measurement wins.

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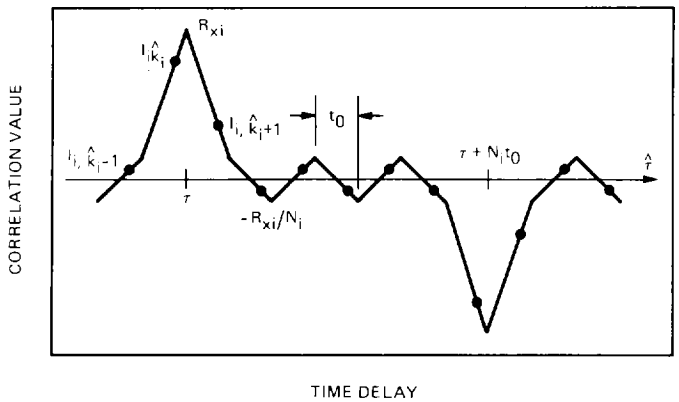

Fig. 1. Composite-code cross-correlation function $R_{xi}(\hat{\tau})$

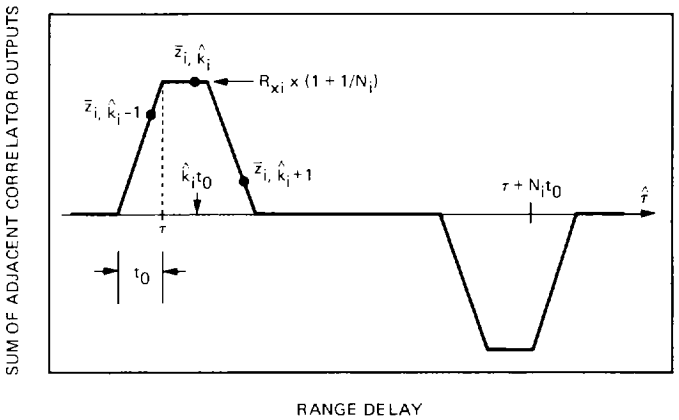

Fig. 2. Sum of adjacent correlator outputs versus range delay