Measurement and Analysis of Cryogenic Sapphire Dielectric Resonators and DROs

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This article presents the experimental and computational results of a study on a new kind of dielectric resonator oscillator (DRO). It consists of a cooled, cylindrically symmetric sapphire resonator surrounded by a metallic shield and is capable of higher Q's than any other dielectric resonator. Isolation of fields to the sapphire by the special nature of the electromagnetic mode allows the very low loss of the sapphire itself to be expressed. Calculations show that the plethora of modes in such resonators can be effectively reduced through the use of a ring resonator with appropriate dimensions. Experimental results show Q's ranging from 3×10^8 at 77 K to 10^9 at 4.2 K. Performance is estimated for several types of DROs incorporating these resonators. Phase noise reductions in X-band sources are indicated at values substantially lower than those previously available.

I. Introduction

A new kind of dielectric resonator promises to enable an important advance in the capability of dielectric resonator oscillators (DROs). This resonator, which consists of a cooled sapphire ring or cylinder surrounded by a metallic shield, is capable of higher Q's than any other dielectric resonator, equaling those of quartz crystals at temperatures which can be reached by means of thermoelectric cooling [1]–[4]. At 10 to 20 K, it rivals the performance of superconducting resonators that require temperatures 10 times lower. This article reports on the results of tests on such a sapphire resonator at 9 to 10 GHz (X-band), which show Q's ranging from 3×10^8 at 77 K to 10^9 at 4.2 K.

The high Q's of these resonators depend not only on a reduction of losses internal to the sapphire but also on isolation of the resonant energy from losses in the surrounding metallic shield. With a dielectric constant (~10) only a fraction of that of other dielectric resonator materials, sapphire resonators are at a substantial disadvantage in this regard. This is overcome in the resonators of the present study through a process similar to the optical phenomenon of total internal reflection.

This article presents the results of both experiment and calculation, which show that effective isolation can be obtained in modes with 5 to 10 full waves around their perimeters. New computations for mode Q's and frequencies for high-mode numbers are presented on the basis of previously published solutions to the wave equation for an isolated isotropic dielectric sphere [8], the only finite geometry for which, to the author's knowledge, closed-form solutions have been devel-

oped. An approximate method is developed to allow calculations for right cylinders and for rings with rectangular cross sections. This method is based on solutions (also approximate) for a rectangular dielectric waveguide [9]. The ring is assumed to be just such a waveguide bent around on itself. Losses in the metallic shield are explicitly considered. The plethora of modes in the cylinder and sphere has led us to consider the ring resonator for further analysis and study. We find that an appropriate choice of ring dimensions can greatly increase the mode spacing without sacrificing the isolating properties of the mode.

Analyses of several different types of oscillator applications are presented. Possible applications include low noise microwave oscillators using only thermoelectric cooling and oscillators with both extremely low noise and high stability at temperatures of 77 K and below.

II. Background

Cryogenic sapphire resonators have been studied experimentally by Blair in Australia [1], [2] and by V. B. Braginsky and coworkers in the USSR [3], [4] with the aim of developing ultra-stable microwave oscillators and discriminators. Previous work has included measurements of mode frequencies and evanescent field decay lengths; measurement and calculation of the temperature and frequency dependence of the Q's; measurement of the fractional thermal coefficient of the resonant frequency; and development and study of stabilized oscillator performance.

In these experimental studies, sapphire losses are found to drop dramatically as the temperature is reduced below ambient, showing an approximately T^5 dependence for temperatures down to about 50 K, where a Q of approximately 10^8 is attained (for X-band). The loss mechanism responsible for this behavior has been identified by Gurevich [10] as phonon generation due to lattice anharmonicity. The T^5 dependence of the losses is predicted by this theory, as is a linear dependence on frequency. Both are borne out in experimental data, indicating that this source of loss is inherent in the sapphire and probably cannot be improved upon by better sample preparation. It seems appropriate, then, to use the currently observed high-temperature behavior as a basis with which to engineer filters and DROs.

The temperature dependence of the frequency of sapphire dielectric resonators has also been studied by both Blair and Braginsky *et al.* [1]–[4]. The fractional frequency variation with temperature $\partial F/\partial T/F$ is found to saturate at about 6×10^{-5}/K at high temperatures (>300 K), dropping as the coefficient of expansion "freezes out" at lower temperatures [1], [2]. It decreases to 3×10^{-6} at 77 K and falls as T^3 at lower temperatures to a value estimated to be 10^{-12}/K at a temperature of 1 K [3], [4]. The values found at 77 K and below could allow very impressive oscillator stability equivalent to that of quartz crystals at 40 K. At 10 K, the readily attainable temperature variation of 10 microdegrees would cause a fractional frequency variation of only 10^{-14}.

The stability demonstrated by oscillators using sapphire and sapphire-filled resonators shows the efficacy of this reduction in expansion coefficient. A frequency stability of 10^{-12} was demonstrated by the Russian group [3], [4] using a Gunn-excited oscillator, and stability better than 10^{-13} has been reported by the Australian group [1], [2] using a frequency-locked Gunn oscillator at room temperature. Using a sapphire resonator coated with superconducting lead, we have demonstrated stability better than 10^{-14} at 100 seconds. In the last case, the higher stability is not attributable to the superconducting coating but rather to the use of a ruby maser as the source of excitation [5]–[7].

While all of the oscillators just mentioned operate at temperatures below 2 K, the prospect of both high stability (due to the low expansion coefficient) and extremely low phase noise (due to high Q) in the temperature range from 10 K to 77 K is perhaps the most exciting aspect of their performance. Of great significance here are the relatively small and inexpensive cryocoolers available in this temperature range. In addition, comparison to conventional DROs and to cavity-stabilized microwave oscillators also indicates a dramatic reduction in phase noise using a sapphire resonator at approximately 170 K, a temperature achievable using thermoelectric cooling. Here the Q of 2×10^6 compares with values of 1 to 3×10^4 available from other microwave resonators, indicating a corresponding reduction in phase noise of 36 to 46 dB.

III. Isolated Modes in Dielectric Resonators

Isolated modes in dielectric resonators achieve weak coupling to the surrounding space not primarily by an impedance mismatch due to the large dielectric constant but rather by isolating properties of the mode itself. These modes can be understood from Fig. 1 as consisting of a wave trapped and slowed by a circular dielectric waveguide. The wave equation

$$k_r^2 + k_\theta^2 + k_z^2 = \epsilon(2\pi/\lambda)^2$$

allows a large value of k_θ inside the dielectric if the thickness and width of the ring are large enough to allow only small values of k_z and k_r, respectively. Outside the dielectric, however, the dielectric constant ϵ is 1, and this large value of k_θ, still required by the symmetry of the mode for some distance

outside the dielectric, requires an imaginary part in one of the other components (found in k_r) to satisfy this same wave equation. This region of evanescent, decaying fields forms a buffer between the waves in the dielectric and allows traveling waves farther out. These modes have been misnamed "whispering gallery" modes [3], [4] but are more properly seen as analogous to the phenomenon of total internal reflection in optics.

To the author's knowledge, solutions in closed form for the modes of cylindrically symmetric dielectric resonators are available only for the isotropic sphere and the infinite cylinder. Of these, the sphere, being a finite structure, is appropriate for consideration here. Following solutions published by Gastine *et al.* for the modes TE_{nmr} [8], we have calculated frequencies and Q's for $m = 1$, for $r = 1, 2$, and for n ranging up to relatively large values. These values are plotted in Fig. 2 and show an exponential increase in Q as n and the frequency are increased. A shortcoming in this calculation is the inability to account for the effect of a metallic shield, which is necessary to allow a reasonably small overall size as well as desirable to increase the radiation-limited Q's as shown in Fig. 2. It seems apparent that replacing the completely absorbing space surrounding the sphere by only slightly absorbing copper should improve the Q, but by how much? An upper limit would seem to be the product of the Q's; e.g., for $n = 7$ and $r = 1$, a radiation-limited Q of 3×10^4 (from Fig. 2) combined with a copper-can Q of 10^4 would indicate that Q's up to 3×10^8 might be possible, an attractive prospect. It also seems clear from Fig. 1 that the can must be in the evanescent region and that there would be some trade-off between isolation from can losses and overall size.

In order to test these ideas, we mounted an uncoated sapphire cylinder whose length and diameter were both approximately 5 cm inside a copper can large enough to provide a 1-cm gap at the outside and on the ends. At liquid-nitrogen temperature and below, we found modes with high Q ($Q > 10^7$) for frequencies above approximately 7.5 GHz. This frequency corresponds to $n = 8$ or 9 from Fig. 2 with a corresponding free-space radiation-limited Q of 10^5 to 10^6. Since the measured Q is higher than these values, some enhancement of the Q results from the low-loss properties of the shielding can.

However, the plethora of modes which we found gave us no hope of successfully identifying the modes on the basis of the spherical solutions. Furthermore, the prospect of oscillator design is daunting, given the existence of strongly coupled low-Q modes very near in frequency to weakly coupled high-Q modes.

A simple application of the wave equation to the geometry of Fig. 1, forcing a correspondence of k_r and k_z to half-wave solutions in the r and z directions, respectively, indicated that the number of modes might be greatly reduced without great penalty by a resonator with the geometry shown in Fig. 3. As a next step, and in order to obtain a more complete picture of the modes, we constructed a mode picture based on solutions for the modes of a rectangular dielectric waveguide derived by Marcatili [9], who identifies modes E^x_{pq} and E^y_{pq} with electric polarization in the x and y directions, respectively, and with p and q half waves in the x and y directions. Identifying the x, y, and z coordinates of these solutions with the r, z, and theta directions indicated in Fig. 3, identifying mode indices p and q with the mode multiplicity in the r and z directions, and introducing a mode number n corresponding to the number of full waves around the perimeter of the ring, we identify modes E^r_{npq} and E^z_{npq} for the ring.

Following Marcatili [9], we find E^y_{pq} modes for the rectangular dielectric waveguide as the solutions of $p^2 X + q^2 Y = 1$ where $X = (\pi/a)^2 (1 + 2A/\pi a)^{-2} (k_1^2 - k_\theta^2)^{-1}$ and $Y = (\pi/b)^2 (1 + 2A/\pi n^2 b)^{-2} (k_1^2 - k_\theta^2)^{-1}$ (where, in turn, $A = \lambda/2\sqrt{\epsilon - 1}$, $k_1 = 2\pi n/\lambda$, $n = \sqrt{(\epsilon)}$, $\lambda = c/f$, and a and b are the height and width of the ring cross section).

Explicitly accounting for the ring geometry by constraining the solution to exhibit n full waves around an effective ring perimeter r_{eff}, we define $k_\theta = 2\pi n/r_{\text{eff}}$, where r_{eff} is defined in terms of the inner and outer ring radii as $r^2_{\text{eff}} = (r_i^2 + r_o^2)/2$.

Solution of the wave equation outside the dielectric, matching the very large k_θ allowed inside, requires an imaginary part to at least one of the components of the wave vector k.

Decaying fields (imaginary components to the wave vector k) are required in the space just outside the dielectric by the wave equation as a result of the large value of k_θ allowed by the dielectric. A lower limit to the decay rate is obtained by identifying the decay length l_d as

$$l_d = 1/\pi \sqrt{(\pi/k_\theta)^2 - (2/\lambda)^2}$$

Assuming that the gap is much smaller than the radius, we identify the Q enhancement factor as the square of the field decay to the metallic wall a distance l_{gap} away:

$$\text{Q ratio} = \exp(2 \times l_{\text{gap}}/l_d)$$

We have calculated modes for a solid cylinder 5 cm in diameter and 5 cm long, identifying parameters $r_i = 0$, $r_o = 2.5$ cm, $r_{\text{gap}} = 1$ cm, $a = 2$ cm, and $b = r_o - r_i = 2.5$ cm, and for the ring in Fig. 3 with parameters given by $r_i = 1.5$ cm, $r_o = 2.5$ cm, $r_{\text{gap}} = 1$ cm, $a = 2$ cm, and $b = r_o - r_i = 1$ cm.

The results of these calculations are shown in Figs. 4 and 5. The predictions shown in Fig. 4 are in excellent qualitative agreement with the results of our measurements on the cylinder, confirming the validity of our approach. The efficacy of the ring geometry in reducing mode density is dramatically shown in a comparison of the two figures. The actual number of modes is larger than the number shown because modes with poor or no isolation are not shown. The calculations found 398 modes below 9 GHz for the cylinder and 60 for the ring. These modes are all doubly degenerate, a fact which was noted for many of them during the measurement process. Typical splitting of the modes was observed to be 10^{-5} to 10^{-6} fractional frequency deviation.

An inherent problem in the use of these resonators in active oscillators, and an important reason for choosing the ring for further study, is that the coupling of any mode to the external electronics will tend to scale in direct proportion to the coupling to the wall. This means that even though two modes may have very different Q's, if they are near to each other in frequency, mode selection may very well be a difficult problem. For example, if one of the modes is critically coupled to the active electronic elements, the other is likely to be nearly critically coupled as well.

IV. Q Measurements

Figure 6 shows the results of measurements of the Q of two of the modes of the 5 cm by 5 cm cylinder for temperatures below 77 K. Also plotted are higher temperature results reported by Braginsky *et al.* [3]. Good agreement is found with the results of these higher temperature data, confirming that these losses are inherent in the sapphire itself and are not due to impurities, surface treatment, etc. The leveling off of the loss reduction at about 10^{-8} is characteristic of the results reported by others and is probably due to impurities. The further Q improvement at the lowest temperatures is also typical, with the lowest point being marginally better than any others reported to date.

A consideration for resonator design is the requirement for surface finish and dimensional uniformity for the shaped dielectric cavity. Braginsky *et al.* [3] have used methods developed for optical fibers to estimate the losses caused by scattering from surface roughness. They find that for a resonator of centimeter dimensions, a roughness of 3-micrometer characteristic height will cause losses on the order of 10^{-10}. Although this value is smaller than any losses measured so far, the resonator used in the measurements reported here was fabricated with an optical quality polish on all surfaces to assure no loss contributions from this source. Precautions such as acid etch and purified alcohol rinse were taken to assure that no foreign material adhered to the surface.

V. Predictions of Oscillator Noise Performance

The reduction in phase noise over that in conventional DRO and cavity oscillators which would result from the high Q of a cooled sapphire resonator is shown in Fig. 7. Q's of 10,000 and 30,000 are assumed for the conventional oscillators, respectively, along with values from Fig. 6 for the sapphire DROs. Also shown is a further reduction which would result from the application of ruby maser technology to such oscillators.

Multiplicative $1/f$ noise $S_\phi(f)$ in the active device is assumed to be −100 dBc/f (/Hz) [11], [12] and −130 dBc/f (/Hz) for the curves indicating maser excitation.[1] The latter value corresponds to an upper limit obtained in tests of a low-Q S-band ruby maser oscillator [5], a value substantially quieter than that reported for any other active microwave device. It has been well documented that multiplicative $1/f$ noise in semiconducting devices can be reduced by operating devices in parallel or, similarly, by large gate dimensions. Thus it seems likely that the low $1/f$ noise in the ruby is due to its very large volume (~1 cm^3). Ruby masers have been operated at temperatures as high as 90 K and at frequencies up to 42 GHz [11], [12]. Their application to low noise oscillators could open a new window in low noise oscillator capability.

[1]This compares to the best X-band GaAs FET multiplicative noise (−109 dBc/f [Hz]) thus far discovered by the author.

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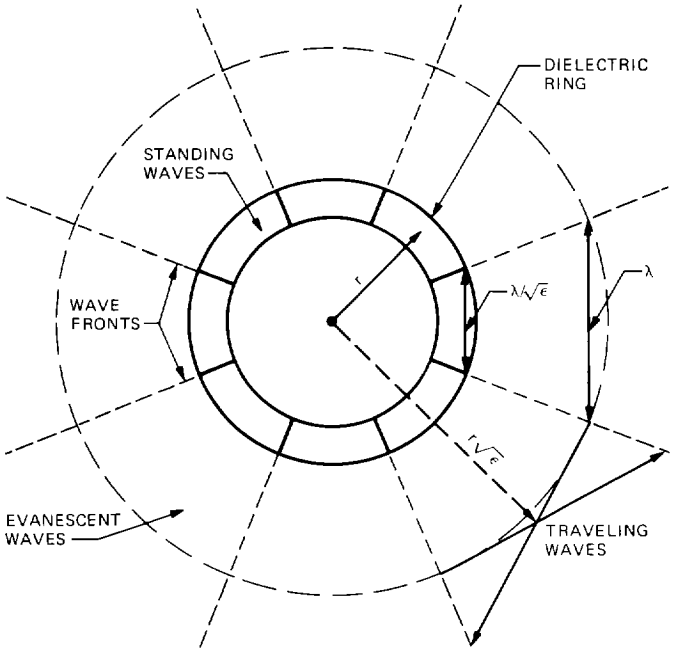

Fig. 1. Diagram showing the character of the electromagnetic field in the vicinity of a dielectric ring for an eight-fold cylindrically symmetric mode

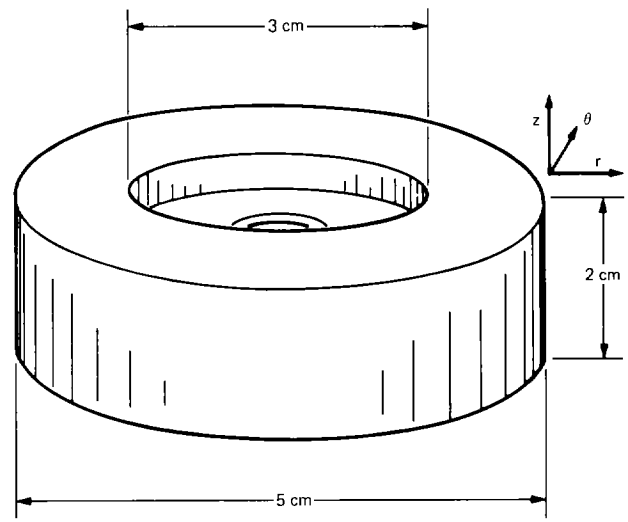

Fig. 3. Sapphire ring construction showing directional axis identification at ring perimeter

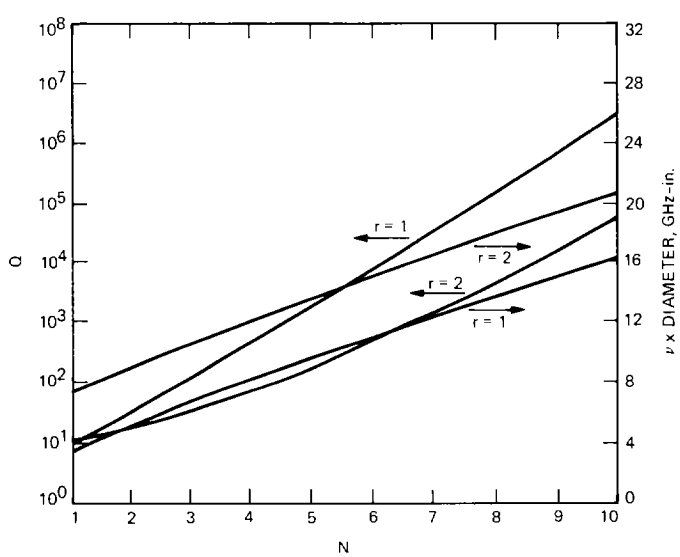

Fig. 2. Radiation-limited Q and frequency for TE_{nmr} modes of an isolated sphere with epsilon = 10, m = 1, and r = 1, 2

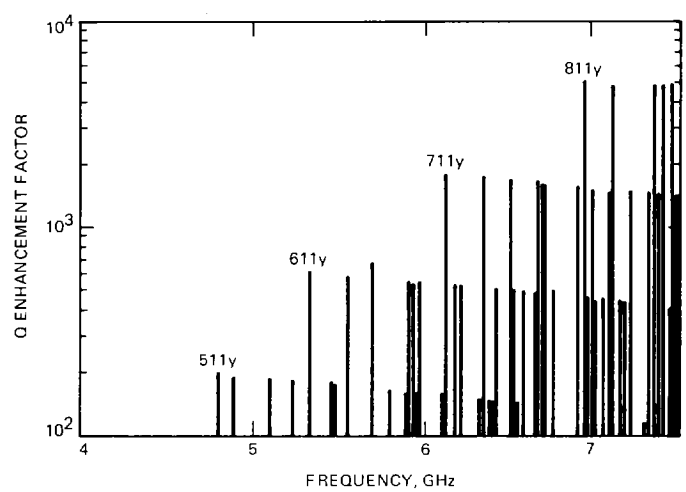

Fig. 4. Calculated mode frequencies and Q enhancement factors for a dielectric sapphire cylinder 5 cm in diameter and 5 cm high surrounded by a lossy shield 1 cm from the surface

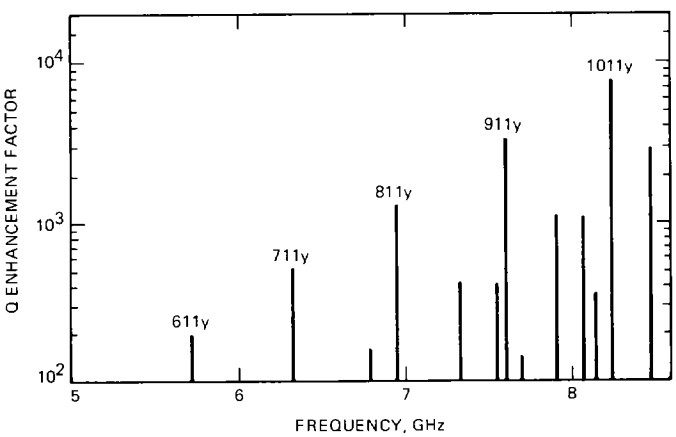

Fig. 5. Calculated mode frequencies and Q enhancement factors for the ring shown in Fig. 3 surrounded by a lossy shield 1 cm from the surface

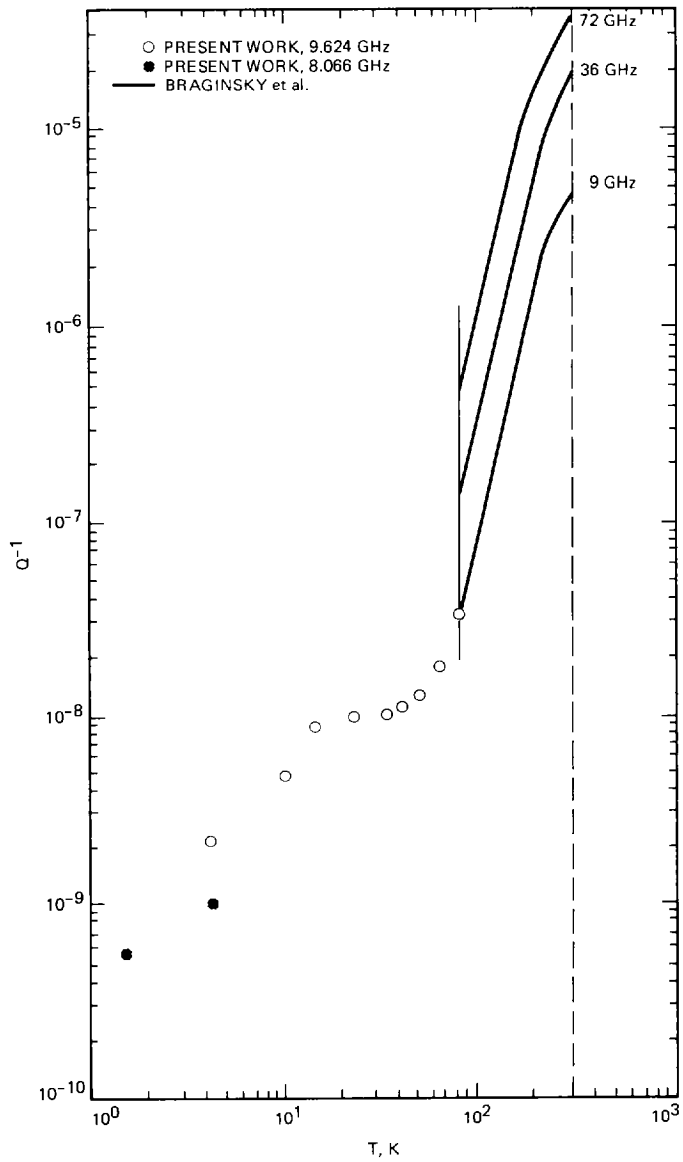

Fig. 6. Q measurements for a sapphire cylinder 5 cm in diameter and 5 cm high contained in a lead-plated shielding can approximately 1 cm away (also shown are higher-temperature data by Braginsky *et al.* [2]

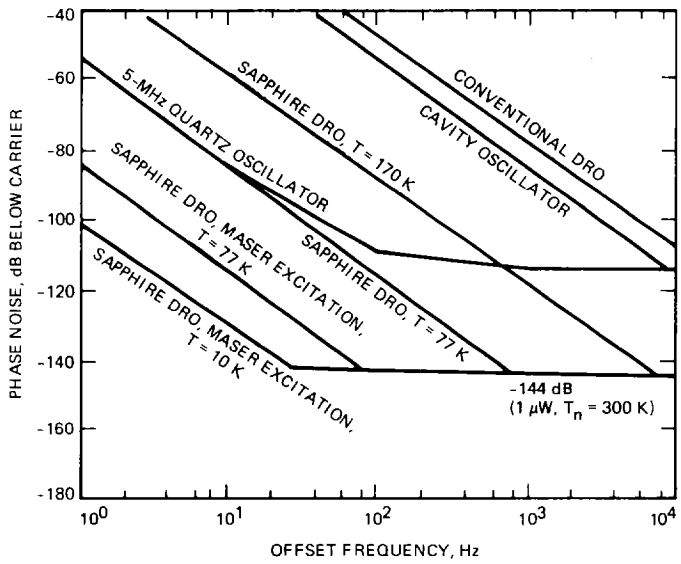

Fig. 7. Phase noise for various X-band sources, including conventional DRO and cavity oscillators, a state-of-the-art quartz crystal oscillator referenced to 10 GHz, and predictions for several sapphire DROs