Differential Range Validation: A New Technique for Near-Real-Time Validation of Multistation Ranging System Data

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Near-real-time validation of ranging system data is currently restricted to multiple range acquisitions during single passes (Pseudo-DRVID). This article describes a new technique ("Differential Range Validation") which utilizes predicted range and doppler pseudo-residuals to validate two-station, contiguous-pass range acquisitions down to the 10-meter level.

I. Introduction

During the first part of this decade, near-real-time validation of ranging data was only sporadically attempted and far less often successful. In March of 1975, this author introduced the "Pseudo-DRVID" Technique (see Ref. 1), which gave the Deep Space Network (DSN) the first viable near-real-time ranging validation capability. Since then, "Pseudo-DRVID" has enjoyed considerable success in validating multiple range acquisitions during single passes. However, there still remained a persistent and unfulfilled desire to be able to validate range acquisitions between two or more Deep Space Stations (DSSs). Responding to this need, this report presents a technique to validate two-station, contiguous-pass range acquisitions—hereafter to be referred to as "Differential Range Validation."

II. The Differential Concept

The reasons why Planetary Ranging Assembly (PRA) range acquisitions cannot be simply compared to predictions[1] are thoroughly explored in Ref. 1. Suffice it to say here that the range ambiguity values are (or should be!) frugally chosen to be only slightly larger than prediction (or orbit determination) errors, and this straightforwardly causes a direct comparison of PRA measurements to predictions to be without meaning in the conventional sense of a "residual." No matter that the *absolute* range error in predictions may be kilometers or tens of kilometers—it

[1]Here considered to be output from the PREDIK program. PREDIK is the Network Operations Control Center (NOCC) Sigma 5 tracking prediction program. Inputs to PREDIK are the Simulation Output Program (SOP) and Fast Phi-Factor Generation Program (FPGP).

is, in general, true that the *growth* of predicted-range error (excepting orbital and encounter phases!) is frequently of the magnitude:

$$\sim 0.1 \text{ Hz}$$

This would map into an increase in predicted-range error over (say) five hours of:

$$\frac{0.1 \text{ cycle}}{\text{second}}\left[\frac{1 \text{ meter}}{15.3 \text{ cycles}}\right][18{,}000 \text{ seconds}] = 118 \text{ meters}$$

At the same time, the *absolute* range change over the same 5-hour period may easily be 100,000 km or more. This then motivates the central concept of the Differential Range Validation Technique:

> PRA (or Mu II for that matter) range measurements between two different DSSs can be validated with a high degree of confidence by comparing the *differenced* range acquisitions to the *differenced* predicted range.

Even better, one is not constrained by the accuracy in predicted-range error change over several hours. In near-real-time, one is automatically given via the Network Operations Control Center (NOCC) Pseudo Residual Program a frequent measurement of the *rate of growth* of the predicted-range error—the *doppler psuedo-residual.* Obviously, one can easily and substantially correct differenced predicted range by simply adding a term computed directly from the observed doppler pseudo-residual. The specifics of the Differential Range Validation technique are presented in the following section.

III. The Differential Range Validation Algorithm

One starts by noting the relationship between range and the output of the PRA:

$$R(t) = K[M(t)] + RPRA(t)$$
$$0 \leq RPRA(t) < K$$

where

$R(t)$ = round trip range at time t

K = ambiguity resolution factor: a quantized input, in units of $R(t)$

$M(t)$ = integer, determined from independent orbital knowledge

$RPRA(t)$ = "scaled" output of the PRA, i.e., in the same units as $R(t)$

Additionally, the following parameters are required for a complete description of the algorithm:

X_i = parameter X applicable to DSS_i

TSF_i = track synthesizer frequency, Hz

N_i = number of components

$K_i = K_i(TSF_i, N_i) = \dfrac{c}{48(TSF_i)}[2^{(N_i+10)}]$

= ambiguity

c = speed of light, m/s

$R_{ai}(t)$ = actual round trip range, m

$R_{pi}(t)$ = predicted round trip range, m

$M_{ai}(t) = [R_{ai}(t) - R_{ai}(t) \text{ modulo } K_i]/K_i$

$M_{pi}(t) = [R_{pi}(t) - R_{pi}(t) \text{ modulo } K_i]/K_i$

$PRTR_i$ = PRA range measurement, range units (RU)

$RPRA_i(t) = \dfrac{c}{48(TSF_i)}[PRTR_i], \text{m}$

$T0$ = PRA acquisition time

$Bias_i$ = station range bias RU

Now consider a PRA acquisition at DSS 1 with a $T0 = t_1$ and a subsequent PRA acquisition at DSS 2 with a $T0 = t_2$:

$$R_{a1}(t_1) \cong K_1[M_{a1}(t_1)] + RPRA_1(t_1)$$
$$R_{a2}(t_2) \cong K_2[M_{a2}(t_2)] + RPRA_2(t_2)$$

and

$$\begin{aligned}\Delta R_a &\equiv R_{a2}(t_2) - R_{a1}(t_1)\\ &\cong RPRA_2(t_2) - RPRA_1(t_1)\\ &\quad + \{K_2[M_{a2}(t_2)] - K_1[M_{a1}(t_1)]\}\end{aligned}$$

Now in Pseudo-DRVID, one could reasonably assume the same number of components and the same track synthesizer frequency for each range acquisition during the same pass, and hence all relevant quantities would have the same ambiguity as a modulus. For range acquisitions at different DSSs, however, one might find different numbers of components:

$$N_2 \neq N_1$$

and one can certainly expect the track synthesizer frequencies to be different:

$$TSF_2 \neq TSF_1$$

If the number of components only were different, all quantities could easily be operated on by the smaller modulus (ambiguity), since

$$\frac{K_j}{K_i} = 2^{(N_j - N_i)} = \text{integer}, \qquad N_j > N_i$$

However, even minor changes in TSF have a dramatic effect on the actual PRA measurement. Consider the following example:

$$N_1 = N_2 = 10 \text{ components}$$

$$R_{a1} = R_{a2} = 3 \times 10^{11}\,\text{m}$$

Now let

$$TSF_1 = 22000000\,\text{Hz}$$

so that

$$K_1 = 297684.8679\,\text{m}$$

and

$$RPRA_1 = R_{a1} \text{ modulo } K_1 = 36905\,\text{m}$$

Similarly,

$$TSF_2 = 22000010\,\text{Hz}$$

$$K_2 = 297684.7325\,\text{m}$$

$$RPRA_2 = R_{a2} \text{ modulo } K_2 = 173289\,\text{m}$$

with a difference in PRA measurements of:

$$\Delta RPRA = RPRA_2 - RPRA_1 = 136384\,\text{m}$$

To compensate for the different ambiguities, the range acquisition performed with the larger ambiguity (say K_2) is "transformed" to an "equivalent" range acquisition at the smaller ambiguity (K_1). This is accomplished by writing:

$$K_2[M_{a2}(t_2)] = K_1 L + \epsilon; \qquad L = \text{integer}$$

or

$$\epsilon = (K_2[M_{a2}(t_2)]) \text{ modulo } K_1$$

One can now operate on ΔR_a with the ambiguity K_1 as follows:

$$\Delta R_a \cong RPRA_2(t_2) - RPRA_1(t_1) + \epsilon + \{K_1 L - K_1[M_{a1}(t_1)]\}$$

and

$$\Delta RPRA \cong (\Delta R_a) \text{ modulo } K_1 \cong RPRA_2(t_2) - RPRA_1(t_1) + \epsilon$$

with

$$\epsilon = (K_2[M_{a2}(t_2)]) \text{ modulo } K_1$$

Unfortunately, one does not have $M_{a2}(t_2)$; however if one assumes

$$M_{p2}(t_2) \approx M_{a2}(t_2)$$

then

$$\epsilon \approx (K_2[M_{p2}(t_2)]) \text{ modulo } K_1$$

One can easily see that even if predictions were in error by several times the ambiguity

$$M_{a2}(t_2) - M_{p2}(t_2) = J; \qquad J = \text{small integer}$$

the error in ϵ would still be extremely small:

$$\text{assume } \Delta TSF = TSF_2 - TSF_1$$

$$I = \text{arbitrary integer}$$

$$\begin{aligned}
\epsilon + \Delta\epsilon &= (K_2[M_{a2}(t_2) - J]) \text{ modulo } K_1 \\
\Delta\epsilon &= (K_2 M_{a2}(t_2) - JK_2) \text{ modulo } K_1 - \epsilon \\
&= \{[K_2 M_{a2}(t_2)] \text{ modulo } K_1 - \epsilon\} \\
&\quad + (-JK_2) \text{ modulo } K_1 + IK_1 \\
&= (-JK_2) \text{ modulo } K_1 + IK_1
\end{aligned}$$

Now

$$K_2 = K_1 2^{(N_2-N_1)}\left(1 - \frac{\Delta TSF}{TSF_2}\right)$$

so that

$$\Delta\epsilon = \left(-JK_1 2^{(N_2-N_1)}\left[1 - \frac{\Delta TSF}{TSF_2}\right]\right) \text{modulo } K_1 + IK_1$$

$$= (-JK_1 2^{(N_2-N_1)}) \text{ modulo } K_1$$

$$+ \left(JK_1 2^{(N_2-N_1)}\left[\frac{\Delta TSF}{TSF_2}\right]\right) \text{modulo } K_1 + IK_1$$

$$= \left(JK_1 2^{(N_2-N_1)}\left[\frac{\Delta TSF}{TSF_2}\right]\right) \text{modulo } K_1 + IK_1$$

but since

$$\left|JK_1 2^{(N_2-N_1)} \frac{\Delta TSF}{TSF_2}\right| <<< K_1$$

then

$$\Delta\epsilon = JK_1 2^{(N_2-N_1)} \frac{\Delta TSF}{TSF_2} + IK_1$$

and

$$(\Delta\epsilon) \text{ modulo } K_1 = JK_1 2^{(N_2-N_1)} \frac{\Delta TSF}{TSF_2}$$

Finally, one obtains the difference in predicted range:

$$\Delta R_p = R_{p2}(t_2) - R_{p1}(t_1)$$

and

$$\Delta RPRA_p = (\Delta R_p) \text{ modulo } K_1$$

$$= (R_{p2}(t_2) - R_{p1}(t_1)) \text{ modulo } K_1$$

One now has the differential quantity of interest:

$$\Delta RPRA - \Delta RPRA_p = RPRA_2(t_2) - RPRA_1(t_1) + \epsilon - (R_{p2}(t_2) - R_{p1}(t_1)) \text{ modulo } K_1$$

with

$$\epsilon \equiv (K_2[M_{p2}(t_2)]) \text{ modulo } K_1$$

or

$$\Delta RPRA - \Delta RPRA_p = \{RPRA_2(t_2) - RPRA_1(t_1) + K_2[M_{p2}(t_2)] + R_{p1}(t_1) - R_{p2}(t_2)\} \text{ modulo } K_1$$

IV. Correction of Predicted Range via Use of the Doppler Pseudo-Residual

As mentioned in Section II, the accuracy of the range prediction used in differential range validation can be substantially improved by merely utilizing the already automatically provided doppler pseudo-residual. To facilitate the discussion, define the notation

$$\Delta X(t) \equiv X(t_2) - X(t_1)$$

$$\delta X(t) \equiv X_a(t) - X_p(t)$$

where

$X_a(t)$ = "actual" quantity at time t

$X_p(t)$ = "predicted" quantity at time t

Thus,

$$\delta R(t) \equiv R_a(t) - R_p(t)$$

$$= \text{predicted-range error at time } t$$

Let one now assume a prediction error over a short time period (several hours) of the form:

$$\delta R(t) \approx A + Bt + Ct^2$$

and hence:

$$\frac{d}{dt}[\delta R(t)] \approx B + 2Ct$$

Now

$$D2(t) = 96\frac{240}{221}TSF_R - 96\frac{240}{221}TSF_T\left(1 - \frac{1}{c}\frac{dR}{dt}\right) + Bias_d$$

where

$D2(t)$ = two-way doppler

TSF_R = received track synthesizer frequency

TSF_T = transmitted track synthesizer frequency

$Bias_d$ = doppler bias

so that (with $TSF_T \equiv TSF$)

$$\delta D2(t) = \left(\frac{96}{c}\right)\frac{240}{221}\,TSF\left\{\frac{dR_a}{dt} - \frac{dR_p}{dt}\right\}$$

$$= \left(\frac{96}{c}\right)\frac{240}{221}\,TSF\left\{\frac{d}{dt}\,[\delta R]\right\}$$

$$= \text{value of doppler pseudo-residual}$$

Thus one has

$$\delta D2(t_2) = \left(\frac{96}{c}\right)\frac{240}{221}\,TSF\{B + 2Ct_2\}$$

$$\delta D2(t_1) = \left(\frac{96}{c}\right)\frac{240}{221}\,TSF\{B + 2Ct_1\}$$

and

$$\{\delta D2(t_1) + \delta D2(t_2)\}$$

$$= \left(\frac{96}{c}\right)\frac{240}{221}\,TSF\{B + 2Ct_1 + B + 2Ct_2\}$$

$$= \left(\frac{96}{c}\right)\frac{240}{221}\,TSF\{2(B + C\,[t_1 + t_2])\}$$

Now the quantity one is interested in is the range error *change*, $\Delta\delta R$, from t_1 to t_2:

$$\Delta\delta R = \delta R(t_2) - \delta R(t_1)$$

$$= A + Bt_2 + Ct_2^2 - (A + Bt_1 + Ct_1^2)$$

$$= B(t_2 - t_1) + C(t_2^2 - t_1^2)$$

$$= (t_2 - t_1)\,[B + C(t_2 + t_1)]$$

$$= (t_2 - t_1)\left[\frac{c}{96}\left(\frac{221}{240}\right)\frac{1}{TSF}\left\{\frac{\delta D2(t_1) + \delta D2(t_2)}{2}\right\}\right]$$

which is to say that one can incorporate almost exactly predicted-range error growth up to second order:

$$\delta R(t) = R_a(t) - R_p(t)$$

$$\approx A + Bt + Ct^2$$

by simply using the observed doppler pseudo residuals at t_1 and t_2:

$$\delta D2(t_1),\, \delta D2(t_2)$$

in a form as follows:

$$\Delta R_p' = \Delta R_p + \Delta\delta R$$

$$= R_{p2}(t_2) - R_{p1}(t_1)$$

$$+ (t_2 - t_1)\left[\frac{c}{96}\,\frac{221}{240}\,\frac{1}{TSF}\left\{\frac{\delta D2(t_1) + \delta D2(t_2)}{2}\right\}\right]$$

V. Final Expression for Differential Range Validation

Incorporating the station range delays and the doppler pseudo-residual correction, one arrives at the final expression:

$$\Delta RPRA - \Delta RPRA_p =$$

$$\Big\{ RPRA_2(t_2) - RPRA_1(t_1) + K_2\,[M_{p2}(t_2)]$$

$$- \frac{c}{48}\left[\frac{Bias_2}{TSF_2} - \frac{Bias_1}{TSF_1}\right] + R_{p1}(t_1) - R_{p2}(t_2)$$

$$- (t_2 - t_1)\left[\frac{c}{96}\,\frac{221}{240}\left(\frac{1}{2}\right)\left\{\frac{\delta D2(t_1)}{TSF_1} + \frac{\delta D2(t_2)}{TSF_2}\right\}\right]\Big\} \text{ modulo } K_1$$

VI. Preliminary Results of Differential Range Validation

Twelve cases of Viking two-station, contiguous-pass range acquisitions were compared with the differential range validation technique; the results are presented in Table I. The twelve cases presented in Table 1 produced the following composite result:

$$|\Delta RPRA - \Delta RPRA_p|_{\text{avg}} = 9.5\,\text{m}$$

It is noteworthy that these results were obtained via the exclusive use of routine tracking predictions.

An HP9810 program containing the differential range validation algorithm (see Ref. 2) has been delivered to the Network Analysis Team, Tracking (NAT Track), and the technique is considered operational.

VII. Summary

Attempts to validate ranging system data in near-real-time prior to 1975 were generally unsuccessful. In 1975, the Pseudo-DRVID technique was introduced, and it proved quite successful in the near-real-time validation of multiple range acquisitions during single passes. However, there still existed a need to be able to validate ranging data between separate DSSs. The differential range validation technique presented in this report answers this need by providing a practical method of validating two-station, contiguous-pass range acquisitions down to the 10-meter level during cruise phases.

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References

1. Berman, A. L., "Pseudo-DRVID: A New Technique For Near-Real-Time Validation of Ranging System Data," The Deep Space Network Progress Report 42-29, Jet Propulsion Laboratory, Pasadena, Calif., Oct. 15, 1975.
2. Bright, L. E., "Release of HP 9810 Calculator Program for Differential Range Validation" (to be published as a JPL internal document).

Table 1. Differential range validation results

Case	DOY, 1976	S/C	DSSs	$\Delta t = t_2 - t_1$, hr:min	$\Delta RPRA - \Delta RPRA_p$, m
1	152	27	61-11	17:45	13.1
2	153	27	61-11	1:40	5.8
3	156	27	63-11	12:08	−15.0
4	157	27	43-63	55	9.3
5	210	30	11-42	4:45	5.3
6	210	30	11-42	1:10	11.1
7	211	30	42-61	1:17	9.4
8	211	30	61-11	1:14	−5.2
9	213	30	61-11	4:21	−6.1
10	216	30	42-61	1:20	17.5
11	217	30	61-11	10:21	13.0
12	217	30	42-61	1:17	3.0