Three-Frequency Ranging Systems and Their Applications to Ionospheric Delay Calibration

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In this article, results of an analytical study of a three-frequency ranging system are presented. In particular, the effects of both receiver noise and the errors introduced by inter-frequency biases on the determination of pseudorange and ionospheric parameters are discussed. It is shown that by properly choosing the three frequencies, the effect of receiver noise can be minimized to a degree that the first- and second-order ionospheric contributions to the signal delay, which vary as the inverse square and inverse cube of the frequency, respectively, can be calibrated for precise pseudorange measurements. Also, one can solve for the ionospheric parameters, obtaining values for the line-of-sight total electron content (TEC) and for the magnitude of Faraday rotation, which, in turn, can be exploited to extract information about the Earth's magnetic field. Moreover, it is shown that simultaneous measurements of three signal phases can be used to obtain absolute TEC and the geometric range, provided that the phase bias can be calibrated to acceptable levels. In particular, the phase of the Global Positioning System (GPS) L3 signal can be used with the L1 and L2 GPS phases in order to find absolute TEC. Once absolute TEC is obtained at a given instant, its value at later times can be updated by adding to it the relative TEC changes as determined from continuous L1 and L2 phase measurements. The simple procedure outlined here can be implemented in codeless GPS receivers in order to find the ionospheric group delay and the absolute TEC. Public GPS users can also take advantage of this method at times when precise pseudorange observables are not available. Although the case of the GPS L3 signal is used for numerical calculations, the results presented are of a general nature and can be used in designing future three-frequency ranging systems.

I. Introduction

The Global Positioning System (GPS) transmits two primary signals: at L1 = 1575.42 MHz modulated with precision (P) and clear/acquisition (C/A) codes, and at L2 = 1227.6 MHz modulated with P-code. Use of these signals for ionospheric calibration is well known [1]. GPS satellites also transmit a third signal at L3 = 1381.05 MHz, which is used by the control segment to monitor the health of the nuclear-detonation detection devices on board GPS satellites. Four block I satellites transmit P-code on L3 for five minutes each day. Six block II satellites transmit L3 modulated with C/A code. There are two burst modes of transmission for block II satellites: 36 sec maximum and

1.5 sec.[1] The change from the P-code modulation on L3 for block I satellites to C/A modulation for block II satellites was due to recommendations from radio astronomers desiring less contamination of the radio spectrum by the L3 signal. L3 modulated with C/A has a spectrum that is narrower than that of L3 modulated with P-code. Table 1 summarizes some characteristics of the L1, L2, and L3 signals.

II. Determination of Absolute Total Electron Content From Phase Measurements

L1 and L2 carrier phase data can be used to measure relative total electron content (TEC) changes. The absolute TEC has to be determined from pseudorange measurements. The pseudorange is a measurement of the range to GPS satellites and is made by the user with an imprecise clock. However, when the L3 phase is used with the L1 and L2 phases, the absolute value of TEC can be obtained by using the second difference in carrier phase. When this method was first proposed [2], the ionospheric group time delay was removed by using the second difference in the carrier's phase and its upper and lower sidebands. The method is used here in order to find the absolute TEC from the second difference in phase between L1, L2, and L3. The time delay of a signal due to the ionosphere, to the first order, is given by

$$\Delta \tau_i = \frac{q}{f_i^2} \ \text{(sec)} \qquad (i = 1, 2, 3) \qquad (1)$$

where $q = 1.34 \times 10^{-7} \times$ TEC and where the f_i's ($i = 1, 2, 3$) are the three frequencies of operation. Assume that the phase shifts ϕ_1, ϕ_2, and ϕ_3 correspond to L1, L2, and L3 signals. From the definition of signal phase and Eq. (1), it follows that

$$\phi_i = \frac{q}{f_i} \ \text{(cycles)} \qquad (i = 1, 2, 3) \qquad (2)$$

The second difference in the phase shift is defined by

$$\Delta_2 \phi = (\phi_1 - \phi_3) - (\phi_3 - \phi_2) \qquad (3)$$

Using Eq. (2) for the phase and substituting it into Eq. (3), it is seen that

$$\Delta_2 \phi = q \times \left[\frac{1}{f_1} + \frac{1}{f_2} - \frac{2}{f_3} \right] \qquad (4)$$

When $\Delta_2 \phi = 1$ cycle, it is found from Eq. (4) that at L1, L2, and L3 frequencies, TEC $= 6.35 \times 10^{18}$ electrons/m^2. Therefore, for the second difference of phase to be ambiguous, TEC would have to be greater than 6.35×10^{18}, a value that is almost never exceeded for Earth's ionosphere. Hence, there is no ambiguity if TEC is obtained by using the second difference of phase at GPS frequencies.

In practice, random data noise is associated with the measurement of the carrier phases. If it is assumed that data noises have equal standard deviation, σ_{n_ϕ}, and that they are uncorrelated, it is easy to show that the uncertainty in the second phase difference can be expressed as $\sigma_{\Delta_2\phi} = \sqrt{6} \ \sigma_{n_\phi}$. Using Eq. (4), it can be seen that the uncertainty in TEC measurements, σ_{TEC}, can be written as

$$\sigma_{\text{TEC}} = \left[\frac{1}{f_1} + \frac{1}{f_2} - \frac{2}{f_3} \right]^{-1} \times \frac{\sqrt{6} \ \sigma_{n_\phi}}{1.34 \times 10^{-7}} \qquad (5)$$

At GPS frequencies, $\sigma_{\text{TEC}} = 1.6 \times 10^{19} \ \sigma_{n_\phi}$. When $\sigma_{n_\phi} = 0.001$ cycle, then $\sigma_{\text{TEC}} = 1.6 \times 10^{16}$ electrons/m^2, which roughly equals the uncertainty in the absolute TEC measurements when pseudorange observables at L1 and L2 frequencies are used. Inter-frequency phase bias is another source of error that has to be taken into consideration. If ϕ_3 is taken as the reference phase and ϕ_{01} and ϕ_{02} are taken as the phase offsets of ϕ_1 and ϕ_2 with respect to ϕ_3, then the error in TEC, δTEC, in terms of the phase bias $\delta \Delta_2 \phi = \phi_{01} + \phi_{02}$ can be expressed as

$$\delta \text{TEC} = \left[\frac{1}{f_1} + \frac{1}{f_2} - \frac{2}{f_3} \right]^{-1} \times \frac{\delta \Delta_2 \phi}{1.34 \times 10^{-7}} \qquad (6)$$

If an accuracy of 5×10^{16} electrons/m^2 for TEC measurements at GPS frequencies is desired, the phase bias must be known to within a hundredth of a cycle. This requirement puts a stringent demand on the calibration of phase biases. The absolute time delay at L1 can be written, using Eqs. (1) and (4), as

$$\Delta \tau_1 = \left[f_1 + \frac{f_1^2}{f_2} - \frac{2f_1^2}{f_3} \right]^{-1} \times \Delta_2 \phi \qquad (7)$$

Currently, block I satellites transmit L3 for five minutes each day. Assuming one phase measurement per sec-

ond, 300 simultaneous measurements of L1, L2, and L3 phases can be obtained. This is a large enough collection of data for use in Eqs. (4) and (7). Once absolute TEC is obtained at a given time, its value at later times can be updated by adding to it the relative TEC changes determined from continuous L1 and L2 phase measurements. The 36-sec burst mode of the block II satellites can also be used to obtain an initial absolute TEC that is less precise due to the short integration time. The simple procedure outlined above can be implemented in codeless GPS receivers in order to find the ionospheric group delay and the absolute TEC. Public users can also take advantage of this method at times when anti-spoofing is turned on and precise pseudorange observables are not available.

III. Effect of the Receiver Noise in a Three-Frequency Pseudorange System

In the previous section it was shown that three-phase measurements can enable the determination of absolute TEC as well as geometric delay by removing the first-order ionospheric effect. In order to remove the second-order effect, which varies as the inverse cube of the frequency, a third pseudorange measurement is needed. In this section, the effect of the receiver noise on the determination of pseudorange and ionospheric parameters in a three-frequency system is discussed. This analysis extends the work in [3], which studied the effect of the receiver noise on the pseudorange measurement error for a two-frequency system. The pseudorange equations for a three-frequency system can be written as

$$\tau_i = \tau_g + \frac{q}{f_i^2} + \frac{p}{f_i^3} + n_i \qquad (i = 1, 2, 3) \qquad (8)$$

where τ_i is the measured pseudorange at frequency f_i in channel i; τ_g is the geometric delay. The second and third terms on the right-hand side of Eq. (8) are part of the excess delay caused by the ionosphere: q is proportional to TEC along the line of sight, and p is proportional to the integral of the product of electron density, Earth's magnetic field intensity, and the cosine of the angle between the direction of the magnetic field and the line of sight. The fourth term, n_i, is the receiver noise in channel i. Other nondispersive pseudorange uncertainties that are independent of the receiver noise are ignored in this analysis. When measurements are made at three frequencies, one can solve for τ_g, q, and p. These quantities are given by the expressions

$$\tau_g = a \, \Delta\tau_{1n} + b \, \Delta\tau_{2n} + c \, \Delta\tau_{3n} \qquad (9)$$

$$q = a'\Delta\tau_{1n} + b'\Delta\tau_{2n} + c'\Delta\tau_{3n} \qquad (10)$$

$$p = a''\Delta\tau_{1n} + b''\Delta\tau_{2n} + c''\Delta\tau_{3n} \qquad (11)$$

where $\Delta\tau_{in} = \tau_i - n_i$ $(i = 1, 2, 3)$ is the difference between the measured pseudorange at frequency f_i in channel i and the noise in channel i, and where

$$a = f_1^3(f_2 - f_3)/D \qquad (12)$$

$$b = f_2^3(f_3 - f_1)/D \qquad (13)$$

$$c = f_3^3(f_1 - f_2)/D \qquad (14)$$

and

$$a' = f_1^3(f_3^3 - f_2^3)/D \qquad (15)$$

$$b' = f_2^3(f_1^3 - f_3^3)/D \qquad (16)$$

$$c' = f_3^3(f_2^3 - f_1^3)/D \qquad (17)$$

and

$$a'' = f_1^3 f_2 f_3(f_2^2 - f_3^2)/D \qquad (18)$$

$$b'' = f_1 f_2^3 f_3(f_3^2 - f_1^2)/D \qquad (19)$$

$$c'' = f_1 f_2 f_3^3(f_1^2 - f_2^2)/D \qquad (20)$$

and

$$D = (f_1 - f_2)(f_2 - f_3)(f_1 - f_3)(f_1 + f_2 + f_3) \qquad (21)$$

Since noise n_i is unknown to the user, the estimates of τ_g, q, and p are written by letting n_i $(i = 1, 2, 3)$ be zero in Eqs. (9)-(11). These estimates can therefore be written as

$$\tilde{\tau}_g = a \, \tau_1 + b \, \tau_2 + c \, \tau_3 \qquad (22)$$

$$\tilde{q} = a'\tau_1 + b'\tau_2 + c'\tau_3 \qquad (23)$$

$$\tilde{p} = a''\tau_1 + b''\tau_2 + c''\tau_3 \qquad (24)$$

The errors in the user estimates of τ_g, q, and p are

$$\delta\tau_g = \tilde{\tau}_g - \tau_g = a\,n_1 + b\,n_2 + c\,n_3 \qquad (25)$$

$$\delta q = \tilde{q} - q = a'n_1 + b'n_2 + c'n_3 \qquad (26)$$

$$\delta p = \tilde{p} - p = a''n_1 + b''n_2 + c''n_3 \qquad (27)$$

It is assumed that n_1, n_2, and n_3 have the same variance σ_n^2 and that their correlation coefficients ρ_{12}, ρ_{23}, and ρ_{13} are nonzero and different from each other. The standard deviations of $\delta\tau_g$, δq, and δp are denoted by σ_{τ_g}, σ_q, and σ_p, respectively, and their normalized value with respect to the standard deviation of a single-channel measurement at f_1 can be written as

$$\frac{\sigma_{\tau_g}}{\sigma_n} = \left\{ 1 - 2\big[ab(1 - \rho_{12}) + bc(1 - \rho_{23})\right.$$

$$\left. + ac(1 - \rho_{13})\big] \right\}^{\frac{1}{2}} \qquad (28)$$

$$\frac{\sigma_q}{f_1^2\sigma_n} = \left\{ -\frac{2}{f_1^4}\big[a'b'(1 - \rho_{12}) + b'c'(1 - \rho_{23})\right.$$

$$\left. + a'c'(1 - \rho_{13})\big] \right\}^{\frac{1}{2}} \qquad (29)$$

$$\frac{\sigma_p}{f_1^3\sigma_n} = \left\{ -\frac{2}{f_1^6}\big[a''b''(1 - \rho_{12}) + b''c''(1 - \rho_{23})\right.$$

$$\left. + a''c''(1 - \rho_{13})\big] \right\}^{\frac{1}{2}} \qquad (30)$$

An examination of Eqs. (28)–(30) shows that σ_{τ_g}/σ_n, $\sigma_q/(f_1^2\sigma_n)$, and $\sigma_p/(f_1^3\sigma_n)$ are only functions of the ratios f_1/f_2 and f_1/f_3. The first term in the braces of Eq. (28) is the contribution of the single-channel measurement error to pseudorange, and the second term is the error due to the ionospheric delay calibration. Figures 1(a) and 1(b) refer to Eq. (28): The solid line in these figures is the plot of Eq. (28), and the dashed line shows the contribution of the ionospheric calibration to the error in pseudorange measurements. Figure 1(b) is an enlargement of part of Fig. 1(a). In these graphs, two of the frequencies are fixed at L1 and L2, and the third frequency is allowed to vary. Equations (29) and (30) are plotted in Figs. 1(c) and 1(d), respectively. In plotting these graphs, correlation coefficients are assumed to be zero.

As can be seen from Eq. (21) and Eqs. (28)–(30), whenever any two of the three frequencies become equal to each other, the uncertainty in the solution for pseudorange and ionospheric parameters becomes infinite. In Fig. 1, the locations of these singularities are at $f_3 = $ L1 and $f_3 = $ L2. The smaller values for the uncertainties occur as one gets further away from the singularities, although there is always a local minimum between the two singularities. It can be seen from Fig. 1 that the GPS L3 signal happens to be very close to this minimum. Even so, the separation of the three GPS frequencies is not large enough to bring the contribution of the receiver noise to an acceptable level. In the case of pseudorange, for example, the uncertainty introduced by the three-channel receiver noise is about 26 times that of a single-channel receiver. Hence, the present precision of Rogue—the GPS receiver located at the three Deep Space Network (DSN) tracking sites—pseudorange measurements, which is about 14 cm for 1 sec of integration time, introduces an error of approximately 4 m.

Table 2 shows the normalized uncertainties in pseudorange and ionospheric parameters for five different sets of frequencies. The primary, f_1, and secondary, f_2, frequencies are chosen first, and f_3, which is between f_1 and f_2, is the frequency corresponding to the local minimum discussed above. It is seen from this table that by properly choosing the three frequencies, the receiver noise contribution can be brought to acceptable levels for use in precise orbit determination and ionospheric delay calibration. It should be noted that the ratio of primary to secondary frequency is the important factor in the determination of the uncertainties and not the individual magnitude of the frequencies.

IV. Effect of Inter-Frequency Biases in a Three-Frequency Pseudorange System

The biases that exist between different frequency channels when pseudorange measurements are made can be accounted for. The proper equation for the analysis is Eq. (8), with n_i's replaced by biases (B_i's). The errors contributed by these biases to the measurements of pseudorange τ_g and ionospheric parameters q and p are given by

$$\delta\tau_g = a\,B_1 + b\,B_2 + c\,B_3 \qquad (31)$$

$$\delta q = a'B_1 + b'B_2 + c'B_3 \qquad (32)$$

$$\delta p = a''B_1 + b''B_2 + c''B_3 \qquad (33)$$

Assume that frequency channel one is the reference so that $B_1 = 0$, and let $B_2 = B_3 = B$ nsec. The error in TEC, δTEC, due to bias B can be written as

$$\delta\text{TEC} = -\frac{a'}{1.34 \times 10^{-7}} \times B \qquad (34)$$

Table 3 shows the errors introduced by these biases in the measurements of pseudorange and ionospheric parameters for $B = 1$ nsec. Errors due to biases with magnitudes other than 1 nsec can be obtained by scaling the results of Table 3.

V. Conclusions

In the work described in this article, three-frequency ranging systems and their applications to ionospheric delay calibration were studied. Specifically, the effects of biases and random data noises on phase and pseudorange measurements were considered, and general expressions for the computation of errors due to the biases and data noises were given. It was shown that simultaneous measurements of three phases can be used to obtain absolute TEC and the geometric range, provided that the phase bias can be calibrated to acceptable levels. In order to remove the ionospheric first- and second-order effects, which vary as the inverse square and inverse cube of frequency, and thus obtain the geometric range, three pseudorange measurements have to be made. Receiver noise and bias errors are magnified through three-frequency channel measurements. The effects of these error sources on the ionospheric parameters, which are proportional to TEC and to the magnitude of Faraday rotation, and on the geometric range were discussed in detail and were placed in evidence by providing numerical examples. The results presented here can be used as a guide in designing future three-frequency precision ranging systems.

Acknowledgment

References

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Table 1. GPS signal characteristics

Signal	Frequency	Ellipticity	RF signal levels	P_{LCP}[a]$/P_{total}$, %
L1	154×10.23 MHz	-1.2 dB	-163 dBW P-code	0.5
L2	120×10.23 MHz	-3.2 dB	-166 dBW P-code	3.2
L3	135×10.23 MHz	-2.0 dB	-165.2 dBW P and C/A	1.3

[a] LCP = left-hand circular polarization.

Table 2. Normalized uncertainties in pseudorange and ionospheric parameters

f_1, GHz	f_2, GHz	f_3, GHz	σ_{τ_g}/σ_n	$\sigma_q/(f_1^2 \sigma_n)$	$\sigma_p/(f_1^3 \sigma_n)$
1.6	1.2	1.36	19.9	44.7	25.3
1.4	1	1.15	14.6	31.1	17.1
3	2	2.36	10.1	20.1	10.6
0.4	0.15	0.2	2.1	2	0.67
1	0.1	0.14	1.04	0.09	0.01

Table 3. Inter-frequency bias errors

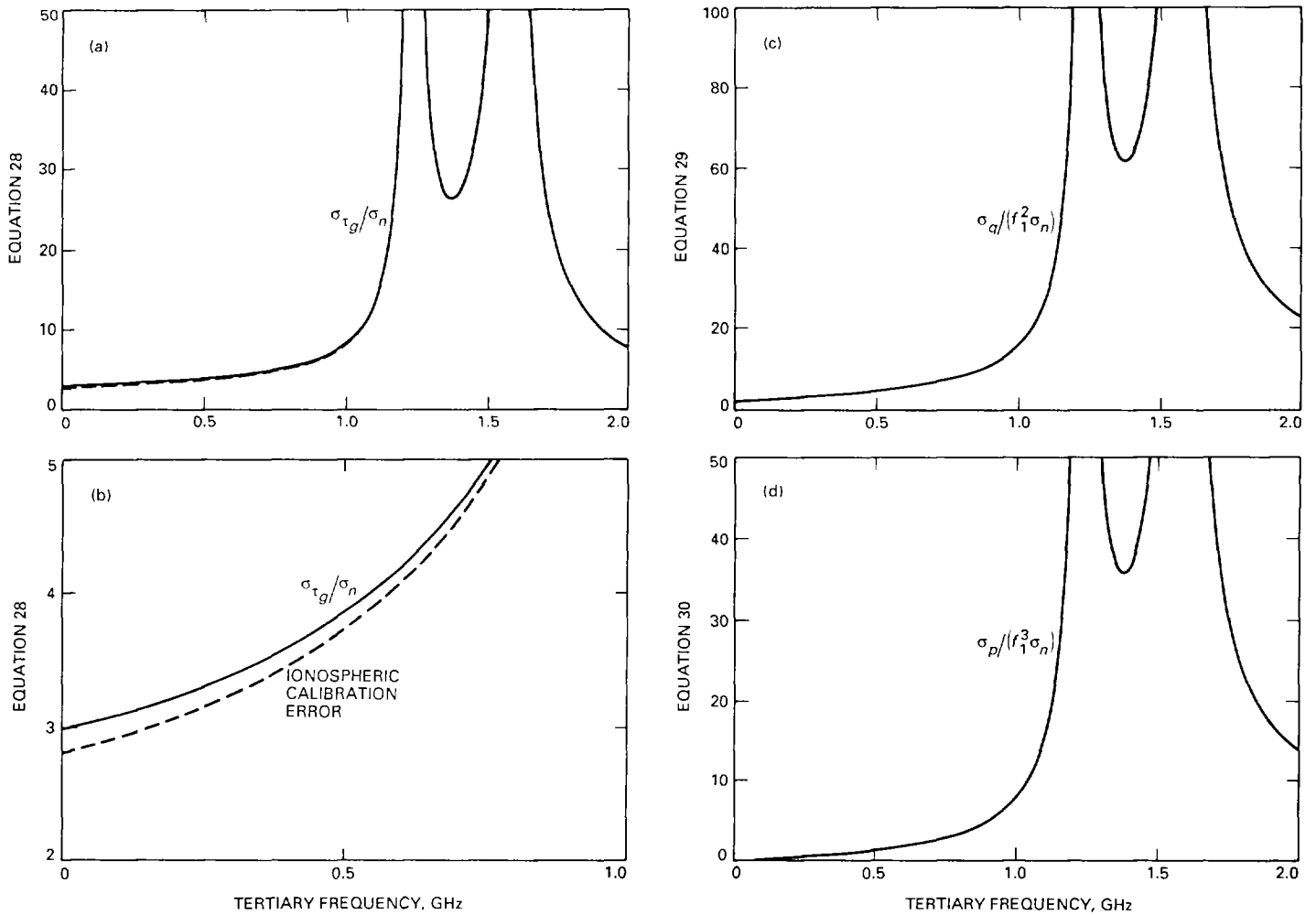| f_1, GHz	f_2, GHz	f_3, GHz	$\bar{\delta}\tau_g$, nsec	$\bar{\delta}$TEC, electrons/m^2	$(1/f_1^3)\,\bar{\delta}p$, nsec
1.6	1.2	1.36	-9.2	3.7×10^{17}	-10.4
1.4	1	1.15	-6.8	2.0×10^{17}	-7.1
3	2	2.36	-4.7	6.1×10^{17}	-4.3
0.4	0.15	0.2	-0.71	1.2×10^{15}	-0.28
1	0.1	0.14	-0.04	3.3×10^{14}	-0.003

Fig. 1. Plots of Eqs. (28), (29), and (30) versus tertiary frequency f_3 for primary frequency $f_1 = 1.57542$ GHz and secondary frequency $f_2 = 1.2276$ GHz: (a) and (b) refer to Eq. (28), with the solid line as the plot of the equation and the dashed line as the contribution of the ionospheric calibration to the error in pseudorange measurements—(b) is an enlargement of a portion of (a); (c) shows the plot of Eq. (29); and (d) shows the plot of Eq. (30).