Deep-Space Navigation With Differenced Data Types Part I: Differenced Range Information Content

S. W. Thurman
Navigation Systems Section

This article is the first in a series of articles investigating the use of differenced data types for deep-space navigation. Subsequent articles will address the primary error sources affecting these data types and other important issues regarding their implementation. At the present time, radio interferometric measurements such as Delta-Differenced One-Way Range (ΔDOR) are formed by differencing near-simultaneous spacecraft and quasar time-delay measurements, in order to eliminate the effects of station clock offset errors and other error sources. This article presents a simple error covariance analysis of an alternative tracking scheme, in which differenced ("two-way" minus "three-way") range data are used directly to estimate the declination and right ascension of a distant spacecraft. The tracking scheme also estimates a measurement bias representing station clock offset and other calibration errors. The results of the analysis indicate that, with clock offset and other calibration data derived from the Global Positioning System (GPS), coupled with improvements in Deep Space Network (DSN) ranging system calibration accuracy, it is theoretically possible to determine spacecraft angular coordinates to accuracies of 30 to 90 nrad using about four hours of differenced range data. This level of accuracy should enable differenced range to support medium-accuracy missions on its own, or serve as a backup to ΔDOR for missions requiring greater navigation accuracy. Additional work will need to be performed to establish estimates of the complete spacecraft navigation accuracies that may be achieved operationally with combinations of differenced range data and other data types.

I. Introduction

The idea of using Very Long Baseline Interferometry (VLBI) in the form of differenced Doppler and range measurements for deep-space navigation originated approximately twenty years ago as a means for improving navigation accuracies over those obtained with single-station Doppler and range measurements [1–3]. In addition to providing increased navigation performance, differenced Doppler and ranging measurements were also found to be less sensitive to station location errors and poorly modeled spacecraft nongravitational forces, error sources that can have a significant effect on conventional Doppler and range data.

There were actually five different interferometric angular measurement schemes originally considered for use in spacecraft navigation:

(1) Two-Way Minus Three-Way Doppler,

(2) Delta-Differenced One-Way Doppler (ΔDOD),

(3) Two-Way Minus Three-Way Range (differenced range),

(4) Differenced Near-Simultaneous Two-Way Range (DTR), and

(5) Delta-Differenced One-Way Range (ΔDOR).

Of these, the differenced Doppler data types have been used primarily in the navigation of planetary orbiters and will not be discussed further; this article focuses on the use of differenced ranging techniques for spacecraft navigation in interplanetary space.

Delta-Differenced One-Way Range measurements are formed by differencing spacecraft VLBI time-delay measurements with similar observations of a stable extragalactic radio source such as a quasar (located nearby on the celestial sphere). The ΔDOR technique originated as a means to greatly reduce or eliminate the effects of station location and clock offset errors, which afflict the other differenced ranging schemes mentioned above [4]. During the Voyager Saturn encounter in 1980, both DTR and ΔDOR data were taken by the Deep Space Network (DSN) and used for orbit determination; DTR was serving as an operational data type, while ΔDOR was considered an experimental data type. Based on the superior performance of ΔDOR during this encounter, it was adopted as the operational angular data type for the remainder of the Voyager mission [5]. The ΔDOR technique will be used extensively for the Galileo, Ulysses, and Mars Observer missions during the next several years. Previous studies have indicated that the ΔDOR system that will be used by Galileo is theoretically capable of determining angular coordinates to accuracies of 20 to 25 nrad [6].

Two-Way Minus Three-Way Range, which is referred to here as differenced range, has never been used operationally, primarily due to a lack of capability within the DSN to accurately determine clock offsets between stations. These clock offsets must be calibrated to accuracies of 1 to 2 nsec in order to reduce the biases introduced into differenced range measurements by timing errors to acceptable levels. Although differenced range data have not actually been used, the basic system elements needed to implement this data type—two-way and three-way ranging systems and a highly accurate clock offset calibration system—are now either operational or within reach. Studies indicate that the Global Positioning System (GPS) can potentially deliver clock calibration information of 1 nsec accuracy over intercontinental baselines [7]. While two-way range has been in operational use for many years, three-way ranging was used operationally for the first time during the Voyager Neptune encounter, making use of GPS-based clock offset calibration data with accuracies of 10 to 20 nsec [8,9]. Another potentially significant error source in differenced ranging measurements, that of Earth-orientation uncertainty, can also be determined using GPS and VLBI measurements to accuracies of better than 10 nrad, which is sufficient to ensure that the effects of these errors on differenced range data are minimal [10].

If differenced ranging could deliver angular accuracies of 50 to 100 nrad with the aid of GPS-based clock offset and Earth-orientation calibrations, it could serve as a supplementary or backup data type to ΔDOR, which is very accurate but operationally cumbersome. Differenced ranging can be accomplished without interruption of spacecraft command and telemetry exchanges, since it does not require quasar tracking data as does ΔDOR. Differenced range also would not require the correlation process needed to form ΔDOR measurements; two-way and three-way range data can be sent from DSN sites to the Space Flight Operations Facility (SFOF) at the Jet Propulsion Laboratory (JPL) in near-real time, where the differenced range data could be computed and calibrated directly within the existing navigation software system. Thus, differenced range data are easier to acquire than ΔDOR data and can be made available for orbit determination use in near-real time, although it must be remembered that the quality of differenced range data can depend substantially upon the quality of externally supplied calibrations.

What follows is a simple, analytic error covariance analysis of the DSN's theoretical ability to determine spacecraft angular coordinates using about four hours of differenced ranging data that encompass differenced ranging passes from one or two DSN intercontinental baselines. A measurement bias assumed to represent the clock offset and station signal-path calibration errors that affect the data is also included in the analysis, in order to determine if these errors can be estimated using the differenced range data signature itself. In particular, the sensitivity of angular accuracies to the a priori measurement bias uncertainty is investigated to determine whether or not 50–100 nrad angular accuracy can be achieved with projected GPS-based calibrations. Further work is needed to establish realistic estimates of the complete spacecraft navigation accuracies that may be achieved using differenced range in concert with other data types.

II. Error Covariance Analysis

The differenced range measurement scheme is illustrated in Fig. 1. The observable consists simply of the difference of the downlink pathlengths between the two participating tracking stations. The geometry associated with differenced range measurements is depicted in Fig. 2. Using Fig. 2 as a guide, it has been shown by previous researchers [1,2,4] that the differenced range observable, denoted here as $\Delta\rho$, is

$$\Delta\rho = \vec{B} \cdot (\vec{r}/r) = r_B \cos\delta \cos H_B + z_B \sin\delta \qquad (1)$$

where

$r_B =$ baseline component normal to the spin axis of Earth

$z_B =$ baseline component parallel to the spin axis of Earth

$H_B =$ baseline hour angle, $\alpha_B - \alpha$

$\alpha =$ spacecraft right ascension

$\alpha_B =$ baseline right ascension, $\alpha_g + \lambda_B$

$\alpha_g =$ right ascension of the Greenwich meridian

$\lambda_B =$ baseline longitude

$\delta =$ spacecraft declination

In Eq. (1), the baseline vector components are expressed as a function of cylindrical coordinates which are, in turn, functions of the cylindrical coordinates of the two stations comprising the baseline. Figure 3 illustrates the station and spacecraft position-coordinate definitions used in this analysis. Using the station coordinates shown in Fig. 3, the baseline coordinates are

$$r_B = \left\{ \left(r_{s_1} + r_{s_2}\right)^2 - 2r_{s_1}r_{s_2}\left[1 + \cos(\lambda_1 - \lambda_2)\right] \right\}^{1/2}$$

$$z_B = z_{s_1} - z_{s_2} \qquad (2)$$

$$\lambda_B = \tan^{-1}\left(\frac{r_{s_1}\sin\lambda_1 - r_{s_2}\sin\lambda_2}{r_{s_1}\cos\lambda_1 - r_{s_2}\cos\lambda_2} \right)$$

where

$r_{s_1}, r_{s_2} =$ station distances from the spin axis of Earth

$z_{s_1}, z_{s_2} =$ station distances from the equator of Earth

$\lambda_1, \lambda_2 =$ station longitudes

The differenced range observable model, Eq. (1), serves as the basis for the development of analytic expressions for the error covariance matrix associated with a weighted least-squares estimate of the spacecraft coordinates δ and α. The measurement equation assumed for the estimation process is

$$z = \Delta\rho + b + \nu \qquad (3)$$

where

$z =$ observed value of differenced range measurement

$\Delta\rho =$ actual differenced range value

$b =$ measurement bias

$\nu =$ random variable representing measurement noise

A series of N independent measurements can be combined into a linear matrix equation relating small perturbations of the measurements from their predicted values to small perturbations of the spacecraft coordinates and the measurement bias from their a priori values as

$$\Delta\vec{z} = A\Delta\vec{x} + \vec{\nu} \qquad (4)$$

where

$\Delta\vec{z} =$ vector of measurement residuals (actual–predicted)

$\Delta\vec{x} = [\Delta\delta, \Delta\alpha, \Delta b]^T$

$\vec{\nu} = [\nu_1, \nu_2, \cdots, \nu_N]^T$

The A matrix is the differential correction matrix containing the partial derivatives of the measurements with respect to the estimated parameters:

$$A = \begin{bmatrix} (\partial z_1/\partial\vec{x}) \\ (\partial z_2/\partial\vec{x}) \\ \cdot \\ \cdot \\ (\partial z_N/\partial\vec{x}) \end{bmatrix} \qquad (5)$$

In this analysis, the spacecraft angular coordinates δ and α are assumed to be constant over the duration of the tracking pass, a reasonable assumption for spacecraft at interplanetary distances and the time periods (about

12 hours) of interest here. For a series of differenced range measurements of constant variance $\sigma^2_{\Delta\rho}$, the information array J, which is the product A^TA scaled by the inverse of the measurement variance, can be written as

$$J = A^T A \cdot (1/\sigma^2_{\Delta\rho}) = (N/\sigma^2_{\Delta\rho}) \sum_{i=1}^{N} (\partial z_i/\partial \vec{x})^T (\partial z_i/\partial \vec{x}) \quad (6)$$

It has been shown in previous work of this nature [4,11, 12] that if the time interval between measurements is both constant and small, then the summation in Eq. (6) can be described approximately by the integral expression

$$J \approx \left(\frac{1}{\sigma^2_{\Delta\rho}\Delta t}\right) \int_{t_1}^{t_2} (\partial z/\partial \vec{x})^T (\partial z/\partial \vec{x}) dt \quad (7)$$

where

$t_1, t_2 =$ tracking pass start and stop times, respectively

$\Delta t =$ time interval between measurements

Using Eqs. (1), (3), and (7), the error covariance matrix associated with the estimates of δ, α, and b can be written as

$$\Lambda = E\left[(\vec{x} - \hat{\vec{x}})(\vec{x} - \hat{\vec{x}})^T\right] = \begin{bmatrix} \sigma^2_\delta & \sigma^2_{\delta\alpha} & \sigma^2_{\delta b} \\ \sigma^2_{\delta\alpha} & \sigma^2_\alpha & \sigma^2_{\alpha b} \\ \sigma^2_{\delta b} & \sigma^2_{\alpha b} & \sigma^2_b \end{bmatrix}$$

$$= \left[\Lambda_o^{-1} + J\right]^{-1} \quad (8)$$

In Eq. (8), Λ_o is the a priori error covariance matrix. The integration specified in Eq. (7) can be carried out with a change of variable; the baseline hour angle can be written as

$$H_B = \omega(t - t_o) \quad (9)$$

where

$\omega t_o = \alpha$

$\omega =$ Earth rotation rate

The variable of integration and its associated limits—assuming a symmetric tracking pass about H_B = 90 deg (the reasoning behind this choice is discussed immediately below)—become

$$dt = dH_B/\omega \quad (10)$$

$$H_{B_1}, H_{B_2} = \frac{\pi}{2} - \Psi, \frac{\pi}{2} + \Psi \quad (11)$$

where

$\Psi \triangleq$ tracking pass half-width

From the illustration of differenced range measurement geometry in Fig. 2, it can be shown that a spacecraft near the Earth's equator (in an angular sense) is generally near its highest point in the sky, as seen by both stations comprising the baseline, when the baseline hour angle is at or near 90 deg or 270 deg (depending upon how the direction in which the baseline vector points is defined). The tracking pass width, which is the angle 2Ψ, represents the angle that the Earth can rotate through (centered about $H_B \approx$ 90 deg or 270 deg) with the spacecraft in view from both of the stations participating in the differenced ranging pass. It will be seen subsequently that the value of Ψ for a particular baseline depends upon the longitudinal separation of the stations forming the baseline and the declination of the spacecraft being tracked.

The information array is calculated by using the partial derivatives of the observable model, which is specified by Eqs. (1) and (3), to integrate Eq. (7) with the variable of integration and limits given in Eqs. (10) and (11). The partial derivatives are

$$\partial z/\partial(\delta, \alpha, b) = [-r_B \sin\delta \cos H_B + z_B \cos\delta,$$
$$r_B \cos\delta \sin H_B, 1] \quad (12)$$

The information array is found to be

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{12} & J_{22} & J_{23} \\ J_{13} & J_{23} & J_{33} \end{bmatrix} \left(\frac{1}{\sigma^2_{\Delta\rho}\omega\Delta t}\right) \quad (13)$$

where

$J_{11} = (r_B \sin\delta)^2(\Psi - 1/2 \sin 2\Psi) + 2(Z_B \cos\delta)^2 \Psi$

$J_{12} = 2(r_B z_B \cos^2\delta) \sin\Psi$

$$J_{13} = 2(z_B \cos\delta)\Psi$$
$$J_{22} = (r_B \cos\delta)^2(\Psi + 1/2 \sin 2\Psi)$$
$$J_{23} = 2(r_B \cos\delta)\sin\Psi$$
$$J_{33} = 2\Psi$$

To complete the development of the error covariance matrix, Eq. (8), the a priori error covariance matrix Λ_o must be specified. In this analysis, it is assumed that some a priori estimate of the differenced range measurement bias b is available, representing the result of clock offset and station signal path calibrations performed prior to the tracking pass. The inverse of the a priori error covariance matrix can then be written as

$$\Lambda_o^{-1} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & (\frac{1}{\sigma_{b_0}})^2 \end{bmatrix} \tag{14}$$

where

$\sigma_{b_0} = 1\sigma$ uncertainty of a priori measurement bias calibration

Substituting Eq. (14) into the error covariance matrix expression, Eq. (8), along with the information array elements given in Eq. (13), the diagonal elements of the error covariance matrix are found to be

$$\sigma_\delta^2 = \left(\frac{\omega\sigma_{\Delta\rho}^2\Delta t}{D}\right)\{(\Psi + 1/2\sin 2\Psi) \times [2\Psi + \omega\Delta t(\sigma_{\Delta\rho}/\sigma_{b_0})^2] - d_3\} \tag{15}$$

$$\sigma_\alpha^2 = \left(\frac{\omega\sigma_{\Delta\rho}^2\Delta t}{D}\right)\{\tan^2\delta(\Psi - 1/2\sin^2\Psi) \times [2\Psi + \omega\Delta t(\sigma_{\Delta\rho}/\sigma_{b_0})^2] + 2\Psi(z_B/r_B)^2(\sigma_{\Delta\rho}/\sigma_{b_0})^2\omega\Delta t\} \tag{16}$$

$$\sigma_b^2 = \left(\frac{\omega\sigma_{\Delta\rho}^2\Delta t}{D}\right)[r_B^2\sin^2\delta(d_1) + z_B^2\cos^2\delta(d_2 - d_3)] \tag{17}$$

where

$$D = r_B^2\sin^2\delta\{d_1[2\Psi + \omega\Delta t(\sigma_{\Delta\rho}/\sigma_{b_0})^2] - d_3(\Psi - 1/2\sin 2\Psi)\} + z_B^2\cos^2\delta[(\omega\Delta t)(\sigma_{\Delta\rho}/\sigma_{b_0})^2(d_2 - d_3)] \tag{18}$$

and

$$\left.\begin{aligned} d_1 &= \Psi^2 - 1/4\sin^2 2\Psi \\ d_2 &= 2\Psi^2 + \Psi\sin 2\Psi \\ d_3 &= 4\sin^2\Psi \end{aligned}\right\} \tag{19}$$

Inspection of Eq. (17) reveals that when $\delta = 0$, the variance of the measurement bias estimate is equal to its a priori value. Thus, to the level of the approximations made here, the differenced range data signature does not contain any information regarding the clock offset/signal path delay for spacecraft at zero declination. This fact can be further illustrated by noting that the information array, Eq. (13), becomes singular when $\delta = 0$. For spacecraft at declinations at or near zero, differenced range navigation accuracy is limited by the accuracy of the a priori measurement bias calibration.

III. Navigation Accuracy Results

In order to compute estimation accuracies for δ, α, and b using Eqs. (15)–(19), suitable values must be chosen for the differenced range data accuracy $\sigma_{\Delta\rho}$, the data rate Δt, and the a priori measurement bias uncertainty σ_{b_0}. The two-way S-band (2.3-GHz) range data currently being acquired from the Galileo spacecraft typically exhibit an intrinsic noise level (1σ) of about 15 cm over the course of a single ranging pass.[1] If a similar noise level is assumed for three-way range data (which when used operationally are acquired with the same ranging system hardware used for two-way data), then the differenced range noise level, assuming that the two-way and three-way measurement noise processes are independent, would be at worst $(15^2 + 15^2)^{1/2}$, or 21.2 cm. The value of $\sigma_{\Delta\rho}$ used in this study was rounded to 20 cm. For comparison, ΔDOR random measurement noise has been predicted to be about

[1] V. M. Pollmeier, Galileo Navigation Team, and G. S. Johnson, Sequential Ranging Assembly (SRA) System Engineer, private communication, Jet Propulsion Laboratory, Pasadena, California.

14 cm for ΔDOR tracking of Galileo with the DSN Narrow Channel Bandwidth VLBI System.[2] The chosen value of Δt was 240 sec, which should be within the capabilities of the present DSN ranging system.[3] Values of σ_{b_0} were chosen ranging from 10 nsec, which is the best accuracy obtainable using present DSN ranging system calibrations,[4] to 1.7 nsec, which is representative of the accuracy that could be obtained with GPS-based transmission media and station clock offset calibrations [7,13].

The final parameter that needs to be addressed is the tracking pass width attainable from DSN baselines. Figure 4 illustrates the overlap regions in which a spacecraft would be simultaneously visible from two Deep Space Communications Complexes (DSCCs) as a function of spacecraft declination and the longitude of the subspacecraft point (the longitude line that has the same right ascension as the spacecraft), assuming a 10-deg elevation cutoff at each DSCC. The angular width of each overlap region shown in Fig. 4 corresponds to the tracking pass width, 2Ψ, which can be obtained from a specified baseline for a given spacecraft declination. In looking at Fig. 4, it is evident that tracking pass width can vary greatly with declination, especially for the Madrid–Goldstone baseline, since both of the DSN sites making up this baseline are located in the northern hemisphere. It is also evident that the tracking pass width for the Madrid–Canberra baseline is too small to be effectively utilized over the entire declination range shown. Over the declination range of -25 deg to $+25$ deg, which is roughly spanned by the ecliptic plane (where most interplanetary spacecraft trajectories lie), the tracking pass width for the Canberra–Goldstone baseline is very nearly constant, having a value of about 60 deg. The tracking pass width for the Madrid–Goldstone baseline, on the other hand, varies from about 90 deg at a declination of 25 deg to essentially zero at -25 deg.

Using Eqs. (15)–(19), the theoretical angular accuracies for a single differenced ranging pass from the Canberra–Goldstone baseline and the Madrid–Goldstone baseline were computed and are shown in Figs. 5(a) and 5(b) and Figs. 6(a) and 6(b), respectively, as a function of declination. Table 1 contains current values of station location and baseline coordinate data for three representative DSN stations and the baselines formed by them,[5] which were used to generate the results shown in Figs. 5 and 6, and all subsequent figures. The tracking pass width values used were derived from Fig. 4. The value of Ψ used for the Canberra–Goldstone baseline was 30 deg over the declination range considered (-20 deg to $+20$ deg). In the case of the Madrid–Goldstone baseline, Ψ varied from about 6 deg at a declination of -20 deg to a value of 30 deg for declinations of 5 deg and greater (at high declinations, the maximum possible value of Ψ for this baseline actually exceeds 30 deg, but the maximum value used in this study was fixed at 30 deg). Note that the Canberra–Goldstone baseline is potentially capable of determining both δ and α to an accuracy of about 80 nrad, except when δ is within a few degrees of zero. Clearly, the Madrid–Goldstone baseline cannot determine δ very well at all in the near-zero declination regime; declination accuracy in this region depends primarily upon the magnitude of the baseline z-height coordinate, which for this baseline is very small.

Measurement bias estimation accuracies obtained for an a priori measurement bias uncertainty of 10 nsec are shown in Figs. 7(a) and 7(b). In contrast to the accuracy statistics shown in Figs. 5 and 6, in which it was assumed that no a priori information was available for δ and α, the bias uncertainties shown in Fig. 7 were computed assuming a priori uncertainties (1σ) of 100, 200, and 500 nrad for both δ and α, in order to determine how well the measurement bias may be estimated in the near-zero declination regime. (Recall that the measurement bias cannot be estimated if $\delta = 0$ and no a priori knowledge of δ and α is assumed.)

Figure 7(b), which shows the bias estimation accuracy for the Madrid–Goldstone baseline, indicates that bias estimation capability is very sensitive to the tracking pass width, as shown by the accuracy improvement which takes place at higher declinations, where Ψ is larger. The statistics shown in Fig. 7 suggest that for spacecraft at near-zero declinations, a series of differenced ranging passes could be used to achieve sub-100-nrad declination accuracies, even for 10-nsec a priori bias uncertainties, since the information gained from each pass would serve as the a priori information needed to estimate the bias during the next pass. (This tracking strategy, of course, assumes that estimates of the rate of change of both δ and α are also obtained from the differenced range data or other data types.)

[2] J. B. Thomas, "An Error Analysis for Galileo Angular Position Measurements With the Block I ΔDOR System," JPL Engineering Memorandum 335-26 (internal document), Jet Propulsion Laboratory, Pasadena, California, November 11, 1981.

[3] G. S. Johnson, private communication, Jet Propulsion Laboratory, Pasadena, California.

[4] With present DSN clock offset, signal path, and transmission media calibration systems, differenced range measurement bias errors would be dominated by clock offset calibration errors; the JPL Frequency and Timing Systems (FTS) Group currently issues weekly DSN clock offset estimates derived from GPS tracking data which are accurate to 10–20 nsec.

[5] T. D. Moyer, "Station Location Sets Referred to the Radio Frame," JPL Interoffice Memorandum 314.5-1334 (internal document), Jet Propulsion Laboratory, Pasadena, California, February 24, 1989.

Figures 8(a) and 8(b) show the angular accuracies obtained by combining data from both Canberra–Goldstone and Madrid–Goldstone differenced ranging passes. These computations were performed by substituting the information array expressions in Eq. (13) and appropriate a priori error covariances into the general error covariance matrix given in Eq. (8). For each declination value considered, the error covariance for δ and α obtained from the first tracking pass was used as the a priori error covariance for the second tracking pass. In these cases, the tracking pass duration for each baseline was 2 hours ($\Psi = 15$ deg), for a total of 4 hours of data between the two. The measurement biases for the two baselines were assumed to be separate and independent of each other and to have identical a priori uncertainties. No results are shown in Fig. 8 for declinations below −15 deg because the mutual visibility time period for the Madrid–Goldstone baseline drops below 2 hours at this point; spacecraft at very low declinations must be tracked primarily by the Canberra–Goldstone baseline.

It can be seen in Fig. 8 that both σ_δ and σ_α are highly dependent upon the magnitude of the a priori measurement bias uncertainty, except perhaps when σ_{b_0} is assumed to be 1.7 nsec. Note that when the a priori bias uncertainty is 1.7 nsec, the dual-baseline accuracies shown in Fig. 8 are superior to the single-baseline accuracies shown in Figs. 5 and 6, but that when the a priori bias uncertainty is greater, the single-baseline accuracies are sometimes better than those of the dual-baseline data set. Remember that the total tracking time was the same in all cases: 4 hours on one baseline in the single-baseline cases, and 2 hours on each of two different baselines in the dual-baseline cases. The sole exception to this rule occurred for the Madrid–Goldstone single-baseline cases with declinations of less than about 5 deg, where tracking-pass durations of less than 4 hours had to be used (see Fig. 4). Based on the accuracy curves shown in Figs. 5, 6, and 8, it appears that if the a priori DSN differenced range bias uncertainties are large, it is generally preferable to acquire a longer arc of differenced range data from a single baseline so that the bias can be estimated, whereas if the a priori biases are well known, it is generally better to acquire shorter data arcs from different baselines.

IV. Discussion

The results presented in this analysis should be interpreted cautiously, as they are dependent upon the assumptions made and the values chosen for parameters such as the a priori bias uncertainty and the differenced range measurement accuracy. It must also be remembered that this type of analysis establishes only theoretical navigation accuracies for an idealistic model of spacecraft motion. In computing the accuracy statistics presented above, the effects of Earth orientation errors and baseline coordinate errors on the differenced range data were not considered. Through the use of extensive DSN VLBI quasar observations, DSN baseline coordinates have already been determined to an accuracy of 5 to 10 cm, corresponding to baseline orientation uncertainties of about 5 to 10 nrad.[6] With a combination of GPS tracking data and VLBI quasar observations, Earth orientation uncertainty, which affects DSN Doppler, range, and ΔDOR measurements as well as differenced range measurements, should be determined on a regular basis to an accuracy of about 10 nrad in the near future (2 to 5 years), as opposed to the current level of about 30 nrad [10]. If independent Earth orientation and baseline orientation errors of 10 nrad (1σ) were included in this analysis, the navigation accuracies shown in Figs. 5, 6, and 8 would be degraded by about 10 percent. Another differenced range error source which was not considered in this study is troposphere calibration error. The effects of this error source, though negligible for data acquired at higher elevation angles, may be significant for elevations of about 15 deg or less, given current troposphere calibration error uncertainties. Fortunately, tropospheric signal delays may also be predicted using GPS-based measurements to a level of accuracy that would reduce the effects of these calibration errors to negligible levels [13].

As pointed out earlier, differenced range has some operational advantages over ΔDOR, in that ranging data can be acquired without interrupting spacecraft command and telemetry activities. This aspect of differenced range may prove to be invaluable during periods in which it is desirable to obtain continuous spacecraft telemetry, such as the approach phase preceding a planetary encounter or spacecraft maneuver. Even though differenced range can eliminate some of the operational problems associated with ΔDOR, it has its own set of problems. One of ΔDOR's virtues is that it is largely a self-calibrating data type; as seen in the differenced range accuracy results presented here, the angular coordinate accuracies obtained from differenced range data can be highly dependent upon the quality of externally supplied station clock offset, signal path, and transmission media calibrations.

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[6] J. S. Border, "An Analysis of ΔDOR and ΔDOD Measurement Errors for Mars Observer Using the DSN Narrow Channel Bandwidth VLBI System," JPL Interoffice Memorandum 335.1-90-026 (internal document), Jet Propulsion Laboratory, Pasadena, California, May 15, 1990.

V. Conclusions

The theoretical angular navigation accuracy that can be achieved with differenced range data was explored, using a simple analytic model for the error covariance matrix obtained from one or two differenced range tracking passes made from DSN intercontinental baselines. The analysis took into account the effects of station clock offset and signal-path calibration errors by including a differenced range measurement bias as an estimated parameter in addition to the declination and right ascension of the spacecraft being tracked. The a priori values assumed for the measurement bias uncertainty were based upon the current and projected accuracy of calibrations derived from GPS satellites and DSN VLBI data. The differenced range data accuracy was chosen to be representative of current DSN ranging system performance in the Galileo mission.

For an assumed a priori measurement bias uncertainty (1σ) of 1.7 nsec (51 cm), the spacecraft angular coordinates could be determined to an accuracy of 30 to 90 nrad with about 4 hours of data. If the a priori measurement bias uncertainty was 10 nsec (150 cm), which is representative of current DSN calibration capabilities, the resulting navigation accuracies ranged from 30 nrad to about 400 nrad. The analysis indicated that it is not possible to estimate the measurement bias for spacecraft at zero declination unless some a priori knowledge of the spacecraft angular coordinates exists; hence, differenced range angular accuracies were found to be poorest in the near-zero declination regime. Earth orientation and baseline orientation errors were not explicitly treated, but if these errors can be kept to 10 nrad or less, which should be possible with GPS and VLBI-based calibration data, their effect on differenced range navigation accuracy should be small, causing the accuracy statistics given herein to be degraded by about 10 percent. Further work needs to be done in order to determine the effects of transmission media calibration errors on differenced range data and to establish the complete spacecraft navigation accuracies that may be achieved in realistic circumstances using differenced range data in combination with other data types.

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[3] V. J. Ondrasik and K. H. Rourke, "An Analytical Study of the Advantages Which Differenced Tracking Data May Offer for Ameliorating the Effects of Unknown Spacecraft Accelerations," *DSN Progress Report*, JPL Technical Report 32-1526, vol. IV, May–June 1971, Jet Propulsion Laboratory, Pasadena, California, pp. 61–70, August 15, 1971.

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Table 1. DSN station and baseline coordinates

Station	Location	r_s, km	z_s, km	λ, deg
DSS 14	Goldstone	5203.997	3677.052	243.1105
DSS 43	Canberra	5205.251	−3674.749	148.9813
DSS 63	Madrid	4862.451	4115.109	355.7520
Baseline	**Length, km**	r_B**, km**	z_B**, km**	λ_B**, deg**
DSS 43–14	10,588.966	7620.841	7351.801	286.0523
DSS 63–14	8390.430	8378.986	−438.057	210.7265

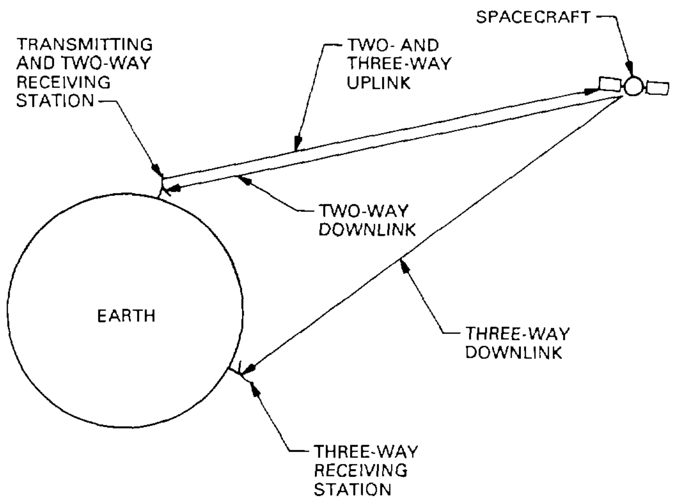

Fig. 1. Two-way and three-way data configuration.

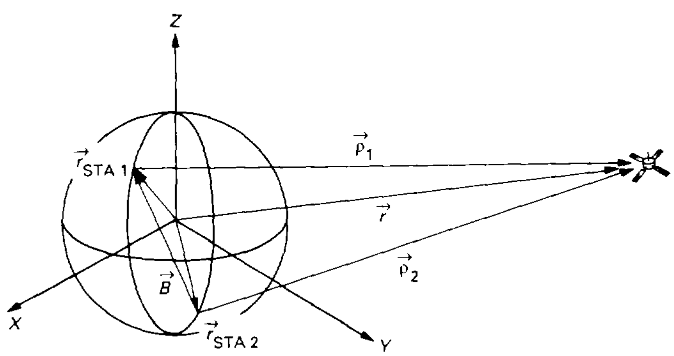

Fig. 2. Differenced range measurement geometry.

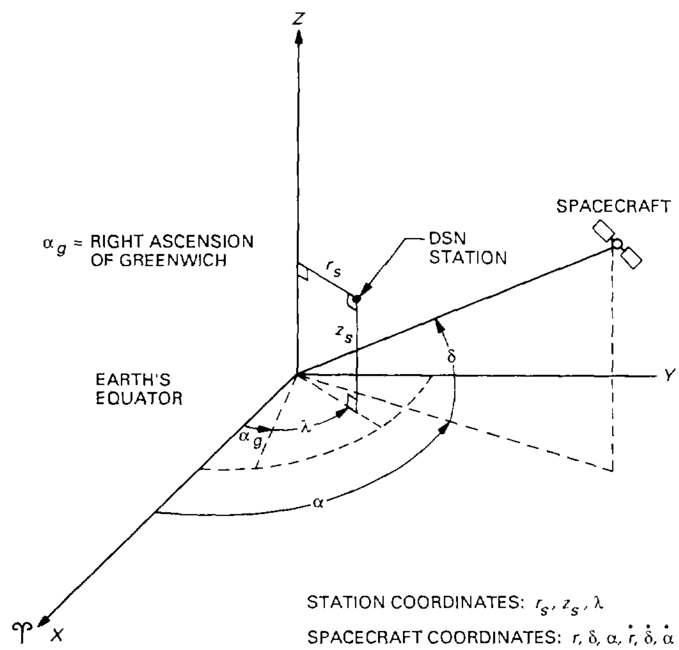

Fig. 3. Spacecraft and DSN station coordinates.

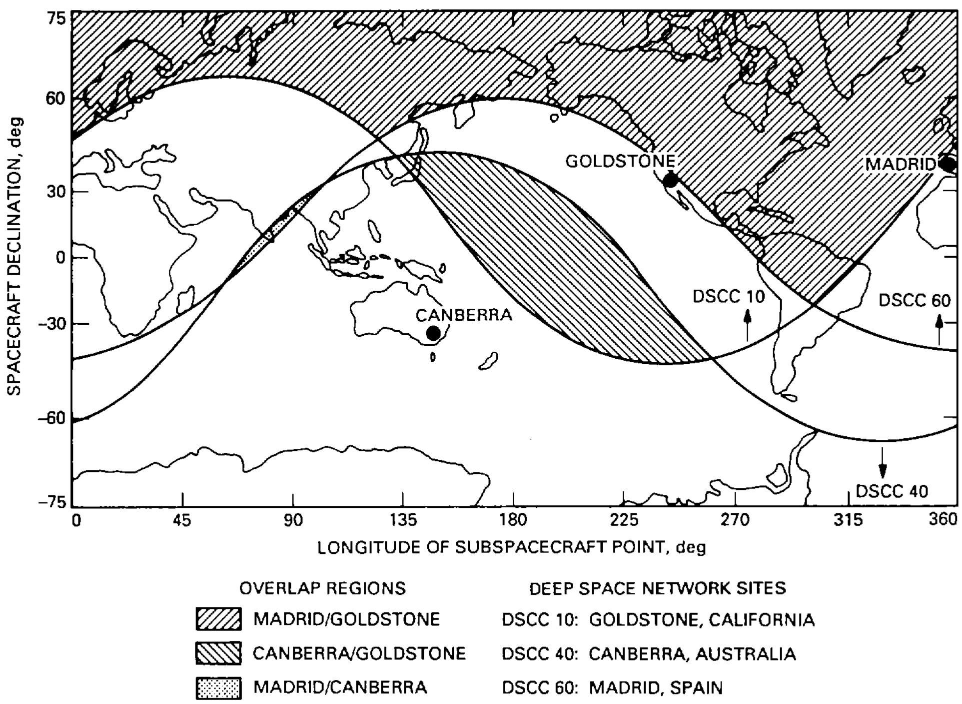

Fig. 4. DSN baseline visibility regions.

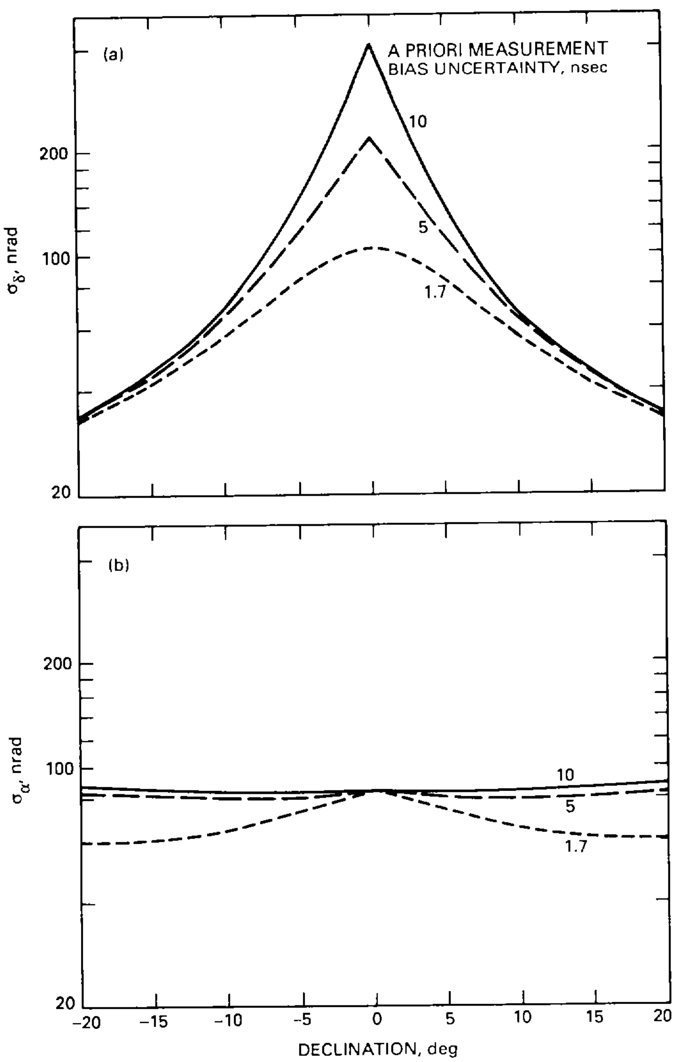

Fig. 5. Canberra–Goldstone baseline navigation accuracy: (a) declination, and (b) right ascension.

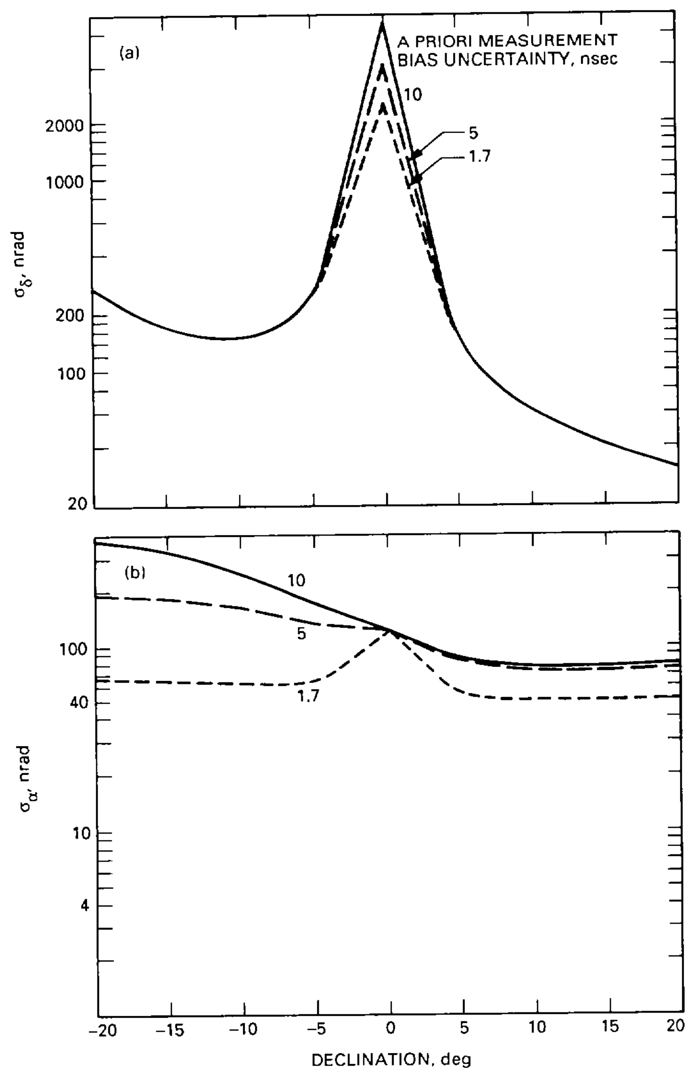

Fig. 6. Madrid–Goldstone baseline navigation accuracy: (a) declination, and (b) right ascension.

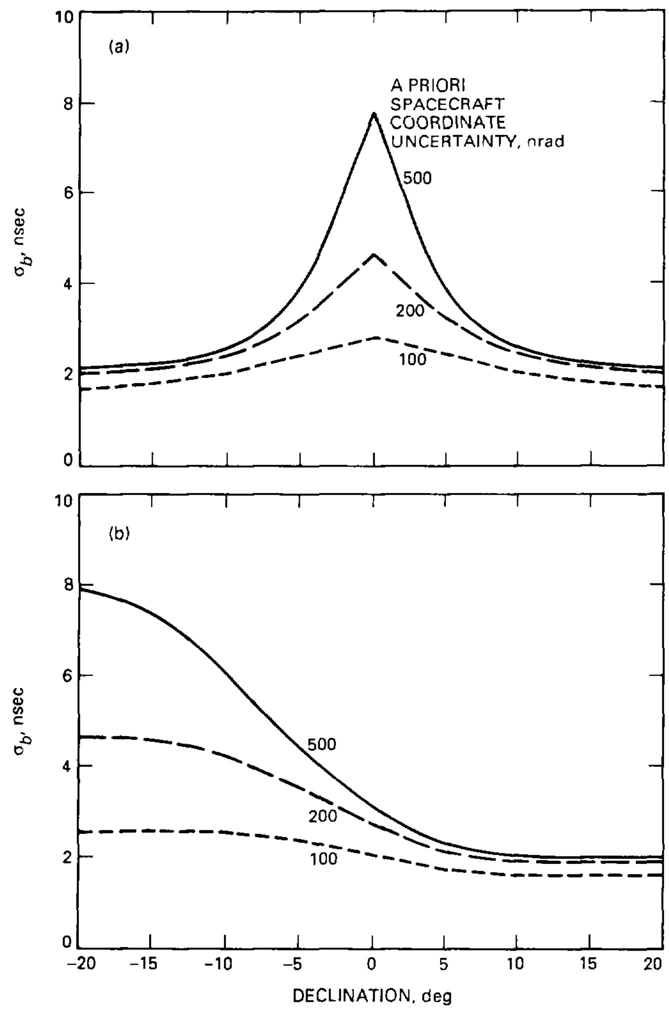

Fig. 7. Measurement bias estimation accuracy, with a priori bias uncertainty = 10 nsec: (a) Canberra–Goldstone, and (b) Madrid–Goldstone.

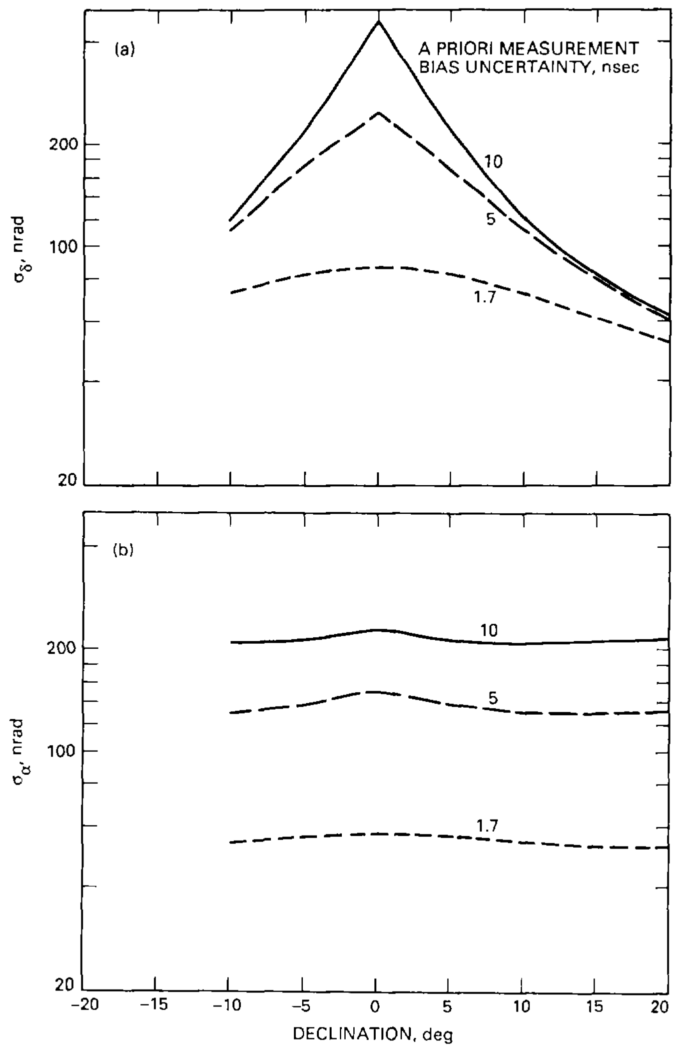

Fig. 8. Dual-baseline navigation accuracy: (a) declination, and (b) right ascension.