Optical Self-Crossing Measurements for Ida Ephemeris Improvement

G. W. Null and P. W. Chodas
Navigation Systems Section

One promising method to minimize asteroid ephemeris errors involves "self-crossing" observations: A single asteroid is observed relative to the same star field at two different times so that the star-catalog error will essentially cancel out. This technique can potentially provide improvement of about a factor of 2.5 in time-of-flight accuracy for the proposed Galileo spacecraft flyby of asteroid 243 Ida in 1993. This would enable improved spacecraft-instrument sequencing for the Ida encounter. Self-crossing techniques may also be useful for late 1990s asteroid encounters by the Cassini and Comet Rendezvous Asteroid Flyby (CRAF) spacecraft.

I. Introduction

This article investigates some potential navigation applications of Earth-based (Hubble Space Telescope and/ or ground-based) optical angular tracking of asteroid targets to the Galileo mission. The main emphasis is on minimizing the effect of star-catalog errors by using "self-crossing" techniques [1]. The analysis indicates that significant ephemeris accuracy improvements may be possible for Galileo target asteroid 243 Ida.

Astrometric accuracy (one standard error) of 100 nanoradians (nrad) has been assumed for most of the simulated crossing-point data. This accuracy will probably require Hubble Space Telescope (HST) data, although ground-based data may also be valuable in some cases.

In contrast with the goal of 100-nrad observational accuracy, the root-mean-square accuracy of existing ground-based data for the Galileo target asteroids is about 6000 nrad; the difference primarily results from star-catalog errors [2]. Thus, even if the observational accuracy goal is not completely achieved, it may still be possible to make useful improvements in asteroid-encounter navigation.

Previous analyses [3] based on preflight estimates of nominal HST performance indicated that astrometric target-location accuracy of 25–50 nrad might be possible (in the absence of star-catalog errors), but recognized the possibility that this accuracy might not be achieved. Although the Space Telescope Science Institute expects that post-launch problems with mirror defects will have a relatively minor effect on astrometric performance [4],[1] a more conservative 100-nrad HST observational accuracy goal has been adopted here.

Actual HST astrometric performance still must be demonstrated by additional new HST data and anal-

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[1] H. E. Bond, Documentation Scientist, Space Telescope Science Institute (STScI) User Support Branch, and H. S. Stockman, Deputy Director, STScI, "Special Report on HST's Imaging Performance," disseminated by the STScI User Support Branch, Baltimore, Maryland, via the STScI Electronic Information Service, June 29, 1990.

yses, as provided by the HST Astrometry and Wide-Field/Planetary Camera (WF/PC) Science Teams. If this process indicates favorable accuracy, HST astrometric data for asteroid 243 Ida will be requested.

This article first gives the expected improvements to the time-of-flight accuracy due to the use of crossing-point data, leaving supporting details until later sections of the article. Section VII reviews the most important topics encountered during the analysis.

Seven major sections are included: Introduction, Improvement in Time-of-Flight Accuracy, Overview of Navigation for Galileo–Asteroid Encounters, Crossing-Point Observation Methods, Crossing-Point Opportunities, Crossing-Point Information Content, and Summary.

II. Improvement in Time-of-Flight Accuracy

Crossing-point observations substantially improve the ability to predict the time-of-arrival of the Galileo spacecraft at asteroid Ida. Table 1 shows how various combinations of available Ida self-crossing points, added to the baseline data set of other types of observations, reduce the error (σ_S) in the Galileo spacecraft time-of-flight position component.

The crossing-point observation of August 1990/December 1990 is ground based, so its assumed accuracy is 300 nrad; the other crossing-point observations are made by the HST and have an expected accuracy of 100 nrad. The baseline set of other data types includes conventional ground-based observations of Ida, Deep Space Network (DSN) radio metric and Delta Very Long Baseline Interferometry (ΔVLBI) observations of the spacecraft, and onboard optical observations by the spacecraft. The baseline data pattern and accuracy is from [2], with minor variations that will be discussed later. The last baseline observation (an onboard optical image of Ida) is assumed to occur at Ida encounter minus one day.

As can be seen from the first line of Table 1, the result for the baseline case (no crossing-point data) is $\sigma_S =$ 139 km. Three Ida self-crossing opportunities are considered individually and in combination. Check marks indicate the inclusion of a crossing-point observation. A baseline set of traditional astrometric data is included in all cases. Adding individual HST crossing points reduces the error to 64 and 56 km, while adding the ground-based crossing points reduces the error to 122 km. Combinations of crossing points provide further improvement to the 40- to 50-km level, more accurate than the baseline case by about a factor of 2.5.

Improved time-of-arrival prediction enables improved pointing predictions, which, in turn, provide the opportunity to acquire additional onboard images of Ida and thus improve the Galileo mission science return. As can be seen, several different combinations of crossing points give good accuracy, thus providing a good check of the ephemeris solution prior to computation of the final Galileo instrument-sequencing load, which is done one day before encounter.

III. Overview of Navigation for Galileo–Asteroid Encounters

Navigation for the Galileo–asteroid encounters is briefly described in this section.

A. Galileo Target Selection

The Galileo spacecraft was launched on October 18, 1989, and will arrive at its primary target (Jupiter and its Galilean satellites) in December 1995. Current Galileo Project plans include a spacecraft encounter with asteroid 951 Gaspra in October 1991, and may also include an August 1993 encounter with asteroid 243 Ida. The Gaspra encounter is definitely planned, since this maximizes the probability of at least one successful encounter.

A decision to target asteroid Ida is more difficult, and depends partly on a post-Gaspra assessment of the additional science value of a second encounter. Also, there are competing demands (between asteroid and Jupiter-system science) for the remaining maneuver-fuel margin. The Galileo Asteroid Ida Decision Plan calls for a definite decision in July 1992, after thorough analysis of the Gaspra results and consultation with the scientific community.[2] There may be roughly a 50-percent probability that the Ida encounter will actually be executed.[3]

B. Determining the Geocentric Spacecraft Position

The National Aeronautics and Space Administration/Jet Propulsion Laboratory (NASA/JPL) DSN Navigation System for the Galileo mission provides Earth-based quasar-relative VLBI measurements of spacecraft angular coordinates; these measurements are accurate to about 20–50 nrad, with good prospects for future significant measurement improvements [5]. These measurements, combined with conventional DSN range and Doppler data, can

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[2] C. M. Yeates, Galileo Science and Mission Design Manager, Jet Propulsion Laboratory, Pasadena, California, private communication, May 21, 1990.

[3] T. V. Johnson, Galileo Project Scientist, Jet Propulsion Laboratory, Pasadena, California, private communication, May 21, 1990.

be used to accurately predict the three-dimensional geocentric coordinates of the spacecraft.

C. Determining the Asteroid Position

However, approach-phase navigation also requires accurate target positions, since the ultimate navigation product is target-relative spacecraft coordinates. As the spacecraft approaches the target, onboard optical angular tracking gradually determines the target coordinates in the spacecraft skyplane, but the target coordinate in the spacecraft time-of-flight direction (S) remains poorly determined [2].

D. Consequences for Instrument Sequencing

Time-of-flight accuracy has important consequences for asteroid encounters, since poor knowledge of S results in correspondingly degraded near-encounter instrument-pointing predictions. Since the Galileo spacecraft does not have any autonomous "smart-platform" pointing capabilities, a mosaic of onboard images must be obtained to ensure adequate imaging of the asteroid. As a result, Galileo mission analysts[4] currently expect to take roughly 70 images of blank sky for each good image of the asteroid target. Thus, better a priori knowledge of the target position would reduce the number of wasted images, which would improve the science return for these encounters.

E. Observation Overview

Asteroid Ida provides several suitable crossing-point observation opportunities, but, unfortunately, no suitable opportunities are available for asteroid Gaspra. Since crossing-point observation techniques are the primary subject of this article, the subsequent discussion focuses on Ida ephemeris accuracy. However, observational techniques for Gaspra are also briefly discussed.

IV. Crossing-Point Observation Methods

This section reviews the rationale for crossing points (including a description of positional errors found in available star catalogs), defines the crossing-point technique, and provides a statistical description of expected stellar proper motions over a typical six-month crossing-point time interval.

A. Rationale

A good illustration of the limitations of available star-catalogs is provided by current preparations for the Galileo encounter with asteroid 951 Gaspra. The catalog error for Gaspra is being minimized by construction of field catalogs based on wide-field astrographic observations at Lick Observatory [6][5]; Owen[6] has investigated the potential accuracy of the catalogs, and concluded that 200-nrad relative accuracy and 500-nrad absolute accuracy may be possible.

Similar catalogs will be constructed for asteroid Ida. However, HST crossing-point measurements for asteroid Ida can potentially provide 100-nrad accuracy, which is not limited by the 500-nrad star-catalog absolute errors. This has motivated the present effort to improve Ida ephemeris accuracy. The crossing-point techniques developed for Ida may also be useful for later missions to other asteroids. The first such missions are late 1990s asteroid flybys by the Comet Rendezvous Asteroid Flyby (CRAF) and Cassini spacecraft.

B. Crossing-Point Definitions

Two types of crossing points are defined in this article: "self-crossings," in which the asteroid is observed relative to the same background stars at two different times (for Ida, typically separated by about six months), and "two-body" crossings, in which the asteroid and another body (with an a priori accurately known position) are observed relative to the same background stars, but usually at different observation epochs. For both types of crossings, the star position errors essentially cancel out of the reduction, so that relative positions of the bodies can be obtained. As will be discussed, self-crossings can provide important orbit improvement information for Ida, but two-body crossings are subject to significant additional errors that occur during the transformation from the planetary ephemeris reference system to the catalog star-reference system. Therefore, two-body crossings will not be used in the present analysis.

C. Relationship to Previous Work

Previous crossing-point investigations [1,7] have a long-term (15-year) objective, namely, improvement of the fundamental optical reference system with asteroid–asteroid crossings. The present analysis differs from the previous work, since it is concerned only with the Ida ephemeris (Ida was not part of the previous long-term observing programs).

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[4] See note 2 above.

[5] A. R. Klemola and W. M. Owen, Jr., *Galileo–Gaspra Reference Star Catalog* (data on magnetic tape), Lick Observatory, University of California, Santa Cruz (to be completed).

[6] W. M. Owen, Jr., JPL Interoffice Memorandum 314.8-760 (internal document), Jet Propulsion Laboratory, Pasadena, California, May 1, 1990.

D. Star Motion Over the Crossing-Point Interval

Although crossing-point measurements remove the constant portion of the star-position error, stellar parallax and proper motion over the interval between the two individual measurements still must be considered. Since mean annual parallax is about four times smaller than mean proper motion [8], the main concern is the proper motion. An appropriate star-motion-error budget allocation may be about 50 nrad, significantly less than the total 100-nrad error budget for self-crossings with HST data.

About 75 percent of 11th-magnitude stars have suitably small proper motions (less than 50 nrad over a typical six-month interval between crossing observations), and this percentage increases by about three percentage points per magnitude. Since asteroid Ida is much fainter (13.7–15.7 magnitude), the observed background stars (perhaps 13th magnitude for ground-based and 16th magnitude for HST observations) should have even more favorable percentages of acceptable proper motion. These results were obtained from statistical analysis with stars up to 11th magnitude from [9].

Since crossing-point measurements always include measurement of the same star field at the two epochs, solutions for stellar proper motion could identify stars with unusually large motions.

V. Crossing-Point Opportunities

A. Observational Constraints

One important observational constraint is the avoidance of near-solar observing, thus limiting observations to suitably large Sun–Earth–planetoid (SEP) angles. For HST data, the initial solar constraint is SEP $\geq$ 50 deg, which may be relaxed somewhat after the first year of observation [10]. Ground-based observations have a similar constraint, that the Sun be well below the horizon, and zenith distances sufficiently small. Both Ida crossing-point observations observed from the HST will be made at SEP $\geq$ 66 deg and meet the constraint.

B. Graphical Representation

Figure 1 contains the skyplane tracks of Mars and Ida for the two-body crossing region. The Mars track is marked with a solid line and the Ida track is marked with a broken line. The four-digit numbers appearing at intervals along the tracks are the date (year and month) when the objects reach those positions. Two-body crossings are marked with an "X," and Ida self-crossings are marked with an "S." As can be seen, Fig. 1 contains two Ida/Mars crossings and one Ida self-crossing. Two other good opportunities for Ida self-crossing measurements (not shown) involve roughly six-month time spans in late 1990 and in early 1993, respectively.

It is not necessary that an actual crossing be observed; measurements can also be made if the two adjacent tracks are sufficiently close to each other and to the same set of background stars. Of course, this depends on the detector field of view; larger fields create more opportunities to observe adjacent (but not actually crossing) tracks. For Ida, only the 1990 self-crossing is not an actual crossing.

C. Summary of Observational Opportunities

The three available Ida self-crossing points are shown in chronological order in Table 2. The "Epochs" column contains the crossing-point epochs for the two parts of the crossing, and the next column contains the SEP at the two epochs. The three right-most columns of Table 2 contain the approximate J2000 right ascension (α), declination (δ), and galactic latitude (b) of the crossing point.

Geocentric angular rates for Ida (not shown in Table 1) range from about 0.07 to 0.36 deg per day for the observations made. Ida visual magnitudes are between 14.3 and 15.7, significantly dimmer than the Ida opposition magnitude of 13.7.

D. Number of Stars in the Instrument Fields

The galactic latitudes (b) from Table 2 can be used with star population tables [8] to establish the approximate (statistical) number of stars in the instrument field for each crossing-point region; stars tend to be more numerous nearer the galactic equator. These tables indicate many (> 50) relatively bright ($m_v \leq 13$) stars for ground-based 1 deg × 1 deg photographic fields. There are also sufficient (> 20) stars brighter than 17th magnitude in the HST Fine Guidance Sensor (FGS) high-accuracy subfield (about 4 by 5 arcmin) for the 1991–92 self-crossing, and about five stars each for the 1990 and 1993 crossing points.

Since the 1990 self-crossing will be observed only from the ground (because HST astrometric calibrations will not be completed in time for these measurements), and since HST Wide-Field (WF) Instrument charge-coupled devices (CCDs) are potentially available for the 1993 crossing point, there should be enough stars for each crossing point. If the HST WF Instrument CCDs are used, the smaller instrument field (about 2.7 × 2.7 arcmin for the four combined WF quadrants) is more than compensated for by the ability to observe fainter (more numerous) 20th-magnitude stars.

In conclusion, there appear to be enough stars to provide many different skyplane position angles between observed objects (asteroid and stars) in the instrument field of view. Thus, even though HST accurately measures only angular distances between celestial objects [3], the ensemble of asteroid–star and star–star angular distances should usually provide accurate two-dimensional angular measurements (i.e., both right ascension and declination), and should also provide good averaging for the previously discussed stellar proper-motion solutions.

VI. Crossing-Point Information Content

In this article, discussion is limited to the time-of-flight coordinate (and related parameters), since this is the critical information for Galileo/Ida navigation. Topics include least-squares analysis methods, perturbations by other asteroids, the baseline data set, star-catalog reference-system errors, astrometric data assumptions, and local orbit improvement.

A. Least-Squares Analysis Methods

The analysis was performed with the Comet Error Analysis program (COMEA). The Ida trajectory was obtained by heliocentric integration of the asteroid equations of motion, which included gravitational perturbations from the Earth, Moon, and planets. Least-squares covariance results were obtained from combinations of existing real Earth-based optical astrometric data with simulated spacecraft data and future crossing-point data; these results were mapped into σ_S at the encounter time.

The COMEA program does not model star-catalog reference-system errors. However, later in this article, a "back-of-the-envelope" analysis is presented; this analysis shows that the effect of reference-system errors has an insignificant effect on the Ida S coordinate.

Self-crossing observations were modeled as measures of the difference in right ascension ($\Delta\alpha$) and declination ($\Delta\delta$) between the object positions at two times:

$$\Delta\alpha = (\alpha_2 - \alpha_1)\cos\delta_a$$

$$\Delta\delta = \delta_2 - \delta_1$$

where δ_a is the average of the two declinations. As discussed, the differencing in these equations cancels the constant portion of the star-catalog positional errors. Implementation of these partial derivatives in the COMEA program was checked using finite-difference techniques.

B. Perturbations by Other Asteroids

The present COMEA version does not include gravitational perturbations of the integrated asteroid by other asteroids, leading to obvious questions about the possible effects of perturbing asteroids on asteroid Ida. Duma and Fedij [11] have investigated the effects of the three largest asteroids (Ceres, Vesta, and Pallas) and the 16th-largest asteroid (Juno) on each other and on 16 other asteroids. They found that the maximum systematic residual remaining after a 52-year orbit fit is about 4000 nrad for Vesta, but is much smaller ($<$ 1000 nrad) for most of the asteroids.

Representing these asteroids (and other large asteroids) as perturbing bodies should reduce the error for the target asteroid, so that it is possible, but far from certain, that systematic perturbations (after the fit) for asteroid Ida will be smaller than the assumed 85-nrad HST observational error. Thus, the present analysis is inadequate for the Ida ephemeris improvement purposes discussed here. Future analyses will include the effect of a much larger number of perturbing asteroids. Since all but the three largest asteroids have mass uncertainties of roughly a factor of two, there will be fundamental limitations on Ida ephemeris improvement.

C. Baseline Data Set

The baseline data set is defined as the actual (and planned) data set for Ida ephemeris support, excluding all crossing-point observations. These baseline observations consist of 129 actual ground-based photographic observations over the interval 1905 to 1989, and three simulated observations at each of the remaining oppositions before the 1993 Galileo encounter. Also included were DSN radio metric and ΔVLBI measurements of the Galileo spacecraft, and Galileo onboard imaging measurements of Ida. As discussed, the baseline data is assumed to end with an onboard optical image of Ida, taken one day before encounter. These baseline observations were included in all the covariance analyses discussed here.

The data pattern and accuracies for the DSN and onboard data are the same as those discussed in [2], except that the work described in this article assumes a slightly less conservative accuracy for conventional ground-based data (6800 nrad versus 7700 nrad). The actual root-mean-square residual noise for this data is about 6000 nrad.

D. FK4 Reference-System Errors

The background-star positions were reduced by using star catalogs based on the *Fourth Fundamental Catalogue*

(*FK4*), which is discussed in [12]. For mission navigation, it is necessary to transform the Ida coordinates from the *FK4* system to the JPL planetary-ephemeris reference system (dynamical equinox), and eventually to the system of the JPL quasar catalog. The transformation from *FK4* to ephemeris dynamical equinox is especially uncertain, due mainly to the fact that the origin of the *FK4* frame is not well defined. A 500-nrad uncertainty in this transformation is indicated in [13], and [14] subsequently identified a 5000-nrad-per-century linear drift in the right ascension of the optical system relative to the dynamical equinox.

FK4 transformation errors affect the use of Ida/planet crossing-point data, since these data provide accurate positional information relative to the planetary ephemeris dynamical equinox. These positions must be converted to the *FK4* system before combination with the baseline data set. The relatively large transformation errors and the small number of data (only one available Ida/Mars crossing with HST data) suggest that these data will be seriously degraded, since a calibration cannot be confidently performed by using the current poorly understood drift solution. Therefore, the Ida/Mars data are not included in the present analysis.

Transformation of the *FK4*-relative solution to the planetary ephemeris system introduces a negligible error in the critical S position component of Ida. This was verified with a simplified "back-of-the-envelope" error analysis for the effect of the linear drift on the solution for S, using Kepler's third law. This analysis assumes that onboard data accurately determine the position components normal to the S direction, and that Earth-based data determine the heliocentric radial component (R). The analysis predicts an error in S of about 12 km; this insignificant error is ignored in the Table 1 results. Other errors, such as the effect of orbital longitude changes on the eccentric ($e = 0.046$) Ida orbit, appear to be much smaller.

E. Astrometric Data Assumptions

The assumed error budget for self-crossings is relatively simple, consisting (for HST data) of 50 nrad for stellar proper motion and 85 nrad for HST observational error. The total (root-sum-square) error is about 100 nrad. The HST portion of the budget assumes two observations at each of the two crossing-point epochs, so the error budget value is the same as the error for a single HST observation.

Since the 1990 self-crossing point must be observed from the ground, its total error budget must be increased. The error level has been set to 300 nrad, a difficult but potentially achievable goal. The acquisition and reduction of these data are being conducted as a joint effort with the Allegheny Observatory of the University of Pittsburgh.

In summary, out of three Ida self-crossing opportunities, it is assumed that the 1990 self-crossing will be observed from the ground, and the 1991/92 and 1993 self-crossings will be observed from the HST.

F. Orbit Knowledge Improvement

Other results (not shown in the tables) indicate that an individual crossing-point observation contributes to knowledge of the orbit in the neighborhood of the crossing point. The subsequent heliocentric radial error improvement oscillates with a period of about half the orbital period. Thus, the improvement provided for the Ida encounter by an individual crossing-point observation depends strongly on the times of the two parts of the crossing. Further discussion is beyond the scope of the present article.

VII. Summary

Self-crossing observations potentially provide significant (factor of 2.5) accuracy improvements for time-of-flight accuracy for Galileo's encounter with asteroid Ida, but this accuracy is critically dependent on acquisition of accurate HST data, starting in August 1991. The improvement in the time-of-flight component of the Galileo spacecraft at Ida encounter will enable correspondingly accurate Galileo instrument pointing, which may yield additional onboard optical pictures of Ida.

Although this scenario seems conceptually sound, there are many uncertainties about its actual implementation. First, the Galileo Project may decide not to target the Ida encounter (roughly a 50-percent probability). Second, although the HST instrument teams still expect good astrometric performance, this remains to be demonstrated. Finally, inter-asteroid gravitational perturbations may cause significant systematic trends in the fitted-orbit data residuals; these effects will be investigated prior to any request for HST data.

In any case, the present analysis indicates the potential benefit of self-crossing data, provided that sufficiently accurate angular data can be processed with adequate dynamical models. Crossing-point techniques may also be useful for late 1990s asteroid encounters by the CRAF and Cassini missions.

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Table 1. Reductions in time-of-flight position component error σ_S. The baseline case is $\sigma_S = 139$: No self-crossing data were used.

Aug. '90/Dec. '90	Aug. '91/Feb. '92	Mar. '93/Jul. '93	σ_S, km
—	—	—	139
√	—	—	122
—	√	—	64
—	—	√	56
√	√	—	45
√	—	√	46
—	√	√	52
√	√	√	38

"—" means that the crossing-point observation is not included.
"√" means that the crossing-point observation is included.

Table 2. Self-crossing observation opportunities for asteroid 243 Ida

Opportunity	Epochs	SEP, deg	α, deg	δ, deg	b, deg
1	Aug. '90/Dec. '90	135/66	350.0	−4.0	−57.5
2	Aug. '91/Feb. '92	69/113	77.0	23.9	−10.5
3	Mar. '93/Jul. '93	154/70	184.5	−2.9	58.5

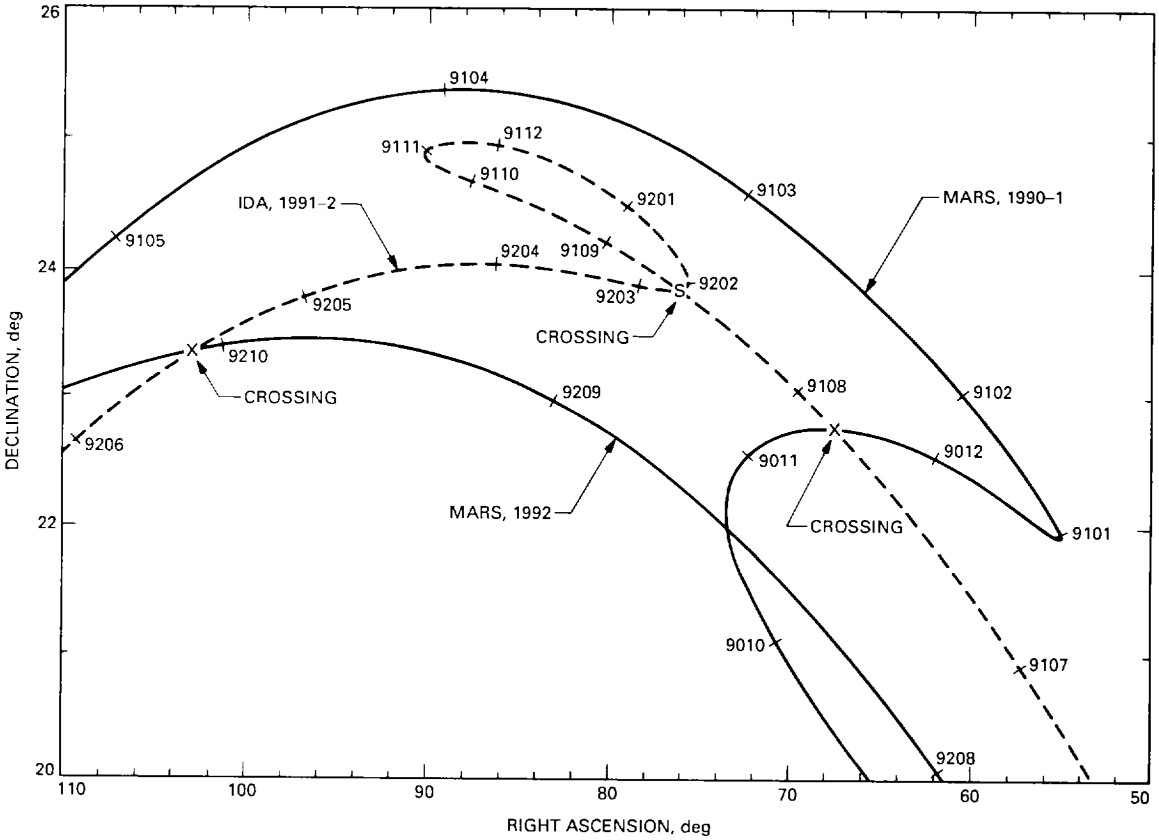

Fig. 1. Apparent paths of Ida and Mars as viewed from the Earth.