A Computational Technique for the Means and Variances of Modulation Losses

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This article presents a technique for computation of the means and variances of modulation losses for phase-modulated residual carrier communication systems. The emphasis is on the coherent turnaround ranging signal that operates simultaneously with the data channels (command data channel on the uplink and telemetry on the downlink). Effects of the automatic gain control (AGC) loop on turnaround simultaneous uplink commanding and ranging are considered in the computation of the downlink modulation losses. Finally, algorithms with flow charts for computing the means and variances of the modulation losses for both uplink and downlink are presented.

I. Introduction

The Consultative Committee for Space Data Systems (CCSDS) has adopted a proposal for a standardized link design control table (DCT) [1]. The DCT allows the analysis of space telecommunication links for five different cases:

(1) uplink telecommand (only),

(2) downlink telemetry (only),

(3) turnaround ranging (only),

(4) simultaneous telecommand and ranging, and

(5) simultaneous telemetry and turnaround ranging.

The computation of the carrier, command, telemetry, and ranging performance margins for these cases requires an evaluation of the modulation losses. The term *modulation loss* as used in this article does not mean the loss

due to modulation; rather, it means a fraction of the total transmitted power allotted to a designated channel. As an example, the command modulation loss means the ratio of the command power-to-total transmitted power.

Since the CCSDS link DCT uses a statistical technique for analyzing the telecommunications link performance, every parameter in the DCT requires the specification of the design value along with its favorable and adverse tolerances. The design value is the most likely value based upon measurement, experience, or computation. Adverse tolerance is the maximum expected unfavorable variation from the design value. Conversely, favorable tolerance is the maximum expected beneficial change from the design value. Once the probability density function (pdf) is assigned to each parameter, the mean and variance for the parameters can be computed, based on the design values and tolerances. However, the specification of the favor-

able and adverse tolerances for those parameters associated with the modulation losses is not apparent to the designers. In practice, the tolerances for the modulation losses are calculated based on the variations of the peak phase deviations (modulation indices) that are specified by the manufacturer. This calculation can be tedious because it requires the designers to evaluate the modulation losses for all possible combinations of the modulation indices. For instance, there are 128 possible combinations for computing the tolerances for downlink telemetry modulation loss, assuming simultaneous range, command, and telemetry operations (Cases 4 and 5 combined).

This article describes a technique for computing the means and variances of the modulation losses for Case 4 (simultaneous command and range operation on the uplink) and Case 5 (simultaneous telemetry and range operation on the downlink). The spacecraft onboard processing (such as the automatic gain control [AGC] operation) of the turnaround ranging signal and the feedthrough telecommand is considered in the computation of the statistical values for the downlink modulation losses.

The communications system analyzed in this article consists of an uplink and a downlink. For the uplink, the carrier is phase modulated by a sine-wave command subcarrier data channel and a ranging channel. The uplink signal is tracked by the phase-locked loop in the spacecraft (S/C) subsystem. The carrier tracking at the S/C demodulates the carrier and downconverts the uplink frequency signal to an intermediate frequency (IF) for command and ranging demodulations. The downlink ranging signal consists of the filtered versions of the uplink ranging, telecommand, and noise. For the downlink, the telemetry data are phase modulated by a telemetry subcarrier data channel and a downlink ranging signal. The block diagram for this two-way communications link is shown in Fig. 1. Algorithms and flow charts based on the proposed technique, along with numerical results demonstrating their applicability, are also presented.

II. Computation of the Means and Variances for Uplink Modulation Losses

The uplink carrier is assumed to be phase modulated by a sine-wave telecommand subcarrier signal [2], along with a ranging signal. The ranging signal will be either a square wave, which is used by the National Aeronautics and Space Administration and the Jet Propulsion Laboratory, or a sine wave, which is used by the European Space Agency's Deep Space Tracking Stations. The modulation is done so that a small amount of power is left in

the carrier component for carrier tracking purposes. The mathematical expression for the uplink signal is given by

$$S_1(t) = (2P_{T1})^{1/2} \sin(\omega_{c1}t + m_c d_1(t)\sin(\omega_{sc}t) + m_{r1}R_1(t))$$ (1)

where P_{T1} is the total uplink received power at the satellite, ω_{c1} is the uplink angular carrier frequency, m_c is the telecommand modulation index, $d_1(t)$ is the telecommand non-return-to-zero (NRZ) data (±1), ω_{sc} is the subcarrier angular frequency, m_{r1} is the uplink ranging modulation index, and $R_1(t)$ is the uplink ranging signal.

From Eq. (1), assuming a square-wave ranging signal, the carrier modulation loss is

$$\frac{P_{C1}}{P_{T1}} = \left(\cos(m_{r1})J_0(m_c)\right)^2$$ (2)

where $J_0(\cdot)$ is the Bessel function of the first kind of order zero.

The telecommand modulation loss in the first-order sideband of the sine-wave subcarrier is

$$\frac{P_{CD}}{P_{T1}} = 2\left(\cos(m_{r1})J_1(m_c)\right)^2$$ (3)

where $J_1(\cdot)$ is the first order of the Bessel function of the first kind.

The uplink ranging modulation loss using the fundamental harmonic of the square-wave ranging signal is

$$\frac{P_{R1}}{P_{T1}} = \frac{8}{\pi^2}\left(\sin(m_{r1})J_0(m_c)\right)^2$$ (4)

As mentioned earlier, the computation of mean and variance requires a knowledge of the tolerances. The favorable tolerance (F_x) and adverse tolerance (A_x) of the modulation loss can be defined as

$$F_x = (P_x/P_T)_{\max} - D_x$$ (5)

$$A_x = (P_x/P_T)_{\min} - D_x$$ (6)

where $(P_x/P_T)_{\max}$ and $(P_x/P_T)_{\min}$ are the maximum and minimum values of the modulation loss, respectively, and $D_x = (P_x/P_T)_n$ is the design or nominal value of the modulation loss. Note that the subscript x in Eqs. (5) and (6) denotes the modulation loss parameter being computed; e.g., for $x = C1$ means the tolerances computed are for the uplink carrier.

For a given F_x and A_x, the mean and variance of the modulation loss in question can be computed using the formulas shown in [3] for a specified pdf. As an example, if the uniform pdf is assumed for the modulation loss in question, the mean and variance are given by, respectively [3]

$$M_x = D_x + \frac{F_x + A_x}{2} \qquad (7)$$

$$V_x = \frac{F_x - A_x^2}{12} \qquad (8)$$

The maximum and minimum values of the modulation losses shown in Eqs. (5) and (6) result from the variations of modulation indices of the subcarrier and the ranging channels. Assume that these modulation indices will vary independently from each other by a certain fraction of their design values (or nominal value). Since the modulator is one component of variation, this is a very conservative assumption.

If Δm_c and Δm_{r1} are defined as the variations of the command and uplink ranging modulation indices, respectively, then the corresponding maximum and minimum values for the command and uplink ranging modulation indices are given by, respectively

$$m_c(\text{max}) = m_{cn} + \Delta m_c \qquad (9)$$

$$m_c(\text{min}) = m_{cn} - \Delta m_c \qquad (10)$$

$$m_{r1}(\text{max}) = m_{r1n} + \Delta m_{r1} \qquad (11)$$

$$m_{r1}(\text{min}) = m_{r1n} - \Delta m_{r1} \qquad (12)$$

where m_{cn} and m_{r1n} denote the nominal or design values for the command and uplink ranging modulation indices, respectively.

Therefore, the maximum and minimum values of the modulation losses for uplink carrier, command, and ranging can be obtained by using Eqs. (2)-(12). Because Bessel and trigonometric functions are involved in the above computations, the maximum and minimum values of the command and uplink ranging modulation losses do not occur when all modulation indices are simultaneously at their maxima or minima, respectively. On the other hand, the extreme values of the uplink carrier modulation losses do occur when all modulation indices are simultaneously at their maxima or minima. The Bessel and trigonometric functions plotted in Fig. 2 depict this situation. Using the algorithm of Fig. 4, and by inspection, from Eqs. (2)-(4),

the maximum and minimum uplink carrier, command, and uplink ranging modulation losses for a square-wave ranging signal are found to be, respectively

$$(P_{C1}/P_{T1})_{\text{max}} = \left(\cos\left(m_{r1}(\text{min})J_0(m_c(\text{min}))\right)\right)^2 \qquad (13)$$

$$(P_{C1}/P_{T1})_{\text{min}} = \left(\cos\left(m_{r1}(\text{max})J_0(m_c(\text{max}))\right)\right)^2 \qquad (14)$$

$$(P_{CD}/P_{T1})_{\text{max}} = 2\left(\cos\left(m_{r1}(\text{max})\right)J_1\left(m_c(\text{min})\right)\right)^2 \qquad (15)$$

$$(P_{CD}/P_{T1})_{\text{min}} = 2\left(\cos\left(m_{r1}(\text{min})\right)J_1\left(m_c(\text{max})\right)\right)^2 \qquad (16)$$

$$(P_{R1}/P_{T1})_{\text{max}} = (8/\pi^2)\left(\sin\left(m_{r1}(\text{max})\right)J_0\left(m_c(\text{min})\right)\right)^2 \qquad (17)$$

$$(P_{R1}/P_{T1})_{\text{min}} = (8/\pi^2)\left(\sin\left(m_{r1}(\text{min})\right)J_0\left(m_c(\text{max})\right)\right)^2 \qquad (18)$$

For a sine-wave ranging signal, the modulation losses can be obtained from the above equations by replacing $(8/\pi^2)\sin(\cdot)$ and $\cos(\cdot)$ for $2J_1(\cdot)$ and $J_0(\cdot)$, respectively. As an example, the carrier modulation loss for a sine-wave ranging signal is $(J_0(m_{r1})J_0(m_c))^2$, and the ranging modulation loss is $2(J_1(m_{r1})J_0(m_c))^2$.

The favorable and adverse tolerances of the uplink carrier, command, and uplink ranging modulation losses can be calculated using Eqs. (5) and (6). If the above modulation losses have uniform probability density functions, then Eqs. (7) and (8) can be used to compute the mean and variance.

Figure 3 illustrates a flow-chart implementation, which represents an algorithm to compute the means and variances of the uplink modulation losses for simultaneous range and command operation. All of the possible cases shown in this flow chart are in accordance with the CCSDS recommendations. The CCSDS recommends that the sine-wave subcarrier be used for the command signal and the square-wave ranging signal be used for deep-space missions. The sine-wave ranging signal is allowed for non-deep-space missions. The use of the algorithm presented in this flow chart results in the optimal computing time.

III. Computation of the Means and Variances for Downlink Modulation Losses

Figure 1 shows a simplified block diagram for ranging demodulation and simultaneous telemetry and range transmission on the downlink for coherent turnaround

ranging operation. From this figure, the downlink signal $S_2(t)$ for simultaneous telemetry and range can be written as

$$S_2(t) = (2P_{T2})^{1/2} \sin(\omega_{C2}t + m_t d_2(t)P(t) + m_{r2}R_2(t) + m_{r2}N\prime_1)$$ (19)

where

P_{T2} = total transmitted power at the spacecraft, W

ω_{C2} = angular carrier frequency, rad/sec

m_t = telemetry modulation index, rad-peak

$d_2(t)$ = bi-phase telemetry data, ± 1

$P(t)$ = unit power square-wave subcarrier

m_{r2} = downlink ranging modulation index, rad

$R_2(t)$ = downlink ranging signal

$N\prime_1$ = white Gaussian noise with zero mean and variance equal to σ^2

The noise $N\prime_1$ is the result of the uplink noise that feeds through the ranging transponder filter. The variance σ^2 of the feedthrough noise $N\prime_1$ is found to be

$$\sigma^2 = N_{01}B_R$$ (20)

where N_{01} is the uplink noise spectral density and B_R is the ranging transponder bandwidth.

It has been shown[1] that the downlink ranging signal (with angular frequency of ω_R) at the output of the power controlled AGC is given by

$$R_2(t) = \underbrace{\tau\prime_1 d_1(t)\sin(\omega_{sc}t)}_{\text{command feedthrough}}$$

$$+ \underbrace{\tau\prime_2 \sin(\omega_R t)}_{\text{ranging}} + \underbrace{\tau\prime_3 N_1}_{\text{noise}}$$ (21)

where

$$\tau\prime_1 = 2\left[\alpha_C/\beta\right]^{1/2}$$ (22)

$$\tau\prime_2 = (4/\pi)\left[\alpha_R/\beta\right]^{1/2}$$ (23)

[1] T. M. Nguyen, *Selection of Modulation Indices for Optimum Power Division for Simultaneous Range/Command/Telemetry Operations*, JPL D-5062 (internal document), Jet Propulsion Laboratory, Pasadena, California, January 12, 1988.

$$\tau\prime_3 = [1/\beta]^{1/2}$$ (24)

$$\beta = [2\alpha_C + (8/\pi^2)\alpha_R + 1]$$ (25)

and N_1 is the white Gaussian noise with zero mean and unity variance.

Here, α_C and α_R are the total telecommand power-to-noise ratio and total ranging-to-noise power ratio at the output of the ranging filter, respectively. They are found to be[2]

$$\alpha_R = \frac{\pi^2}{8}(\text{SNR})_{R_0}$$ (26)

$$\alpha_R = 1/2\left[\frac{P_{CD}}{P_{R1}}\right](\text{SNR})_{R_0}$$ (27)

where $(\text{SNR})_{R_0}$ is the output ranging signal-to-noise ratio (SNR) of the ranging transponder filter not expressed in dB:

$$(\text{SNR})_{R_0} = \frac{P_{R1}}{\sigma^2}$$ (28)

and where P_{R1} and P_{CD} are found from the uplink computation, Eqs. (4) and (3), respectively. Note that the value of $(\text{SNR})_{R_0}$ can be obtained from the link budget computation for the uplink. This parameter carries the impact of the uplink on the downlink computations.

Equation (21) has been derived for the case recommended by the CCSDS, where the highest ranging clock frequency f_R is at 1 MHz and the ranging transponder bandwidth is 3 MHz. This implies that the transponder will pass only the fundamental harmonic of the ranging signal along with the command signal.

The downlink carrier modulation loss is given[3] as

$$(P_{C2}/P_{T2}) = [J_0(\tau_1)J_0(\tau_2)\cos(m_t)]^2\exp(-\tau_3^2)$$ (29)

The telemetry modulation loss is

$$(P_{TLM}/P_{T2}) = [J_0(\tau_1)J_0(\tau_2)\sin(m_t)]^2\exp(-\tau_3^2)$$ (30)

The downlink ranging modulation loss is

$$(P_{R2}/P_T) = 2[J_0(\tau_1)J_1(\tau_2)\cos(m_t)]^2\exp(-\tau_3^2)$$ (31)

[2] Ibid.

[3] Ibid.

where

$$\left.\begin{array}{l} \tau_1 = m_{r2}\tau\prime_1 \\ \\ \tau_2 = m_{r2}\tau\prime_2 \\ \\ \tau_3 = m_{r2}\tau\prime_3 \end{array}\right\} \qquad (32)$$

are the effective downlink modulation indices for the feed-through command, downlink ranging, and feedthrough noise, respectively.

In calculating the means and variances of the downlink modulation losses and using the technique presented above, it is necessary to compute the maximum and minimum values for the modulation losses described in Eqs. (29), (30), and (31).

Again, if Δm_t and Δm_{r2} are defined as the variations of the telemetry and downlink ranging modulation indices, respectively, then the corresponding maximum and minimum values for the telemetry and downlink ranging modulation indices are given by, respectively

$$m_t(\text{max}) = m_{tn} + \Delta m_t \qquad (33)$$

$$m_t(\text{min}) = m_{tn} - \Delta m_t \qquad (34)$$

$$m_{r2}(\text{max}) = m_{r2n} + \Delta m_{r2} \qquad (35)$$

$$m_{r2}(\text{min}) = m_{r2n} - \Delta m_{r2} \qquad (36)$$

where m_{tn} and m_{r2n} denote the nominal or design values for the telemetry and downlink ranging modulation indices, respectively.

From Eq. (32), the maximum and minimum values of the effective modulation indices for the feedthrough command, downlink ranging, and feedthrough noise are given by, respectively

$$\tau_1(\text{max}) = m_{r2}(\text{max})\tau\prime_1(\text{max}) \qquad (37)$$

$$\tau_1(\text{min}) = m_{r2}(\text{min})\tau\prime_1(\text{min}) \qquad (38)$$

$$\tau_2(\text{max}) = m_{r2}(\text{max})\tau\prime_2(\text{max}) \qquad (39)$$

$$\tau_2(\text{min}) = m_{r2}(\text{min})\tau\prime_2(\text{min}) \qquad (40)$$

$$\tau_3(\text{max}) = m_{r2}(\text{max})\tau\prime_3(\text{max}) \qquad (41)$$

$$\tau_3(\text{min}) = m_{r2}(\text{min})\tau\prime_3(\text{min}) \qquad (42)$$

where $m_{r2}(\text{max})$ and $m_{r2}(\text{min})$ are given by Eqs. (35) and (36), respectively. The parameters $\tau\prime_1(\text{max})$, $\tau\prime_1(\text{min})$, $\tau\prime_2(\text{max})$, $\tau\prime_2(\text{min})$, $\tau\prime_3(\text{max})$, and $\tau\prime_3(\text{min})$ can be shown to have the following forms, from Eqs. (22), (23), and (24)

$$\tau\prime_1(\text{max}) = 2[\alpha_C(\text{max})/\beta(\text{min})]^{1/2} \qquad (43)$$

$$\tau\prime_1(\text{min}) = 2[\alpha_C(\text{min})/\beta(\text{max})]^{1/2} \qquad (44)$$

$$\tau\prime_2(\text{max}) = (4/\pi)[\alpha_R(\text{max})/\beta(\text{min})]^{1/2} \qquad (45)$$

$$\tau\prime_2(\text{min}) = (4/\pi)[\alpha_R(\text{min})/\beta(\text{max})]^{1/2} \qquad (46)$$

$$\tau\prime_3(\text{max}) = [1/\beta(\text{min})]^{1/2} \qquad (47)$$

$$\tau\prime_3(\text{min}) = [1/\beta(\text{max})]^{1/2} \qquad (48)$$

From Eqs. (25), (26), and (27), $\alpha_C(\text{max})$, $\alpha_C(\text{min})$, $\alpha_R(\text{max})$, $\alpha_R(\text{min})$, $\beta(\text{max})$ and $\beta(\text{min})$ can easily be shown to have the forms

$$\alpha_R(\text{max}) = \frac{\pi^2}{8}(\text{SNR})_{R_0}(\text{max}) \qquad (49)$$

$$\alpha_R(\text{min}) = \frac{\pi^2}{8}(\text{SNR})_{R_0}(\text{min}) \qquad (50)$$

$$\alpha_C(\text{max}) = 1/2\left[\frac{(P_{CD}/P_{T1})_{\text{max}}}{(P_{R1}/P_{T1})_{\text{min}}}\right](\text{SNR})_{R_0}(\text{max}) \qquad (51)$$

$$\alpha_C(\text{min}) = 1/2\left[\frac{(P_{CD}/P_{T1})_{\text{min}}}{(P_{R1}/P_{T1})_{\text{max}}}\right](\text{SNR})_{R_0}(\text{min}) \qquad (52)$$

$$\beta(\text{max}) = [2\alpha_C(\text{max}) + (8/\pi^2)\alpha_R(\text{max}) + 1] \qquad (53)$$

$$\beta(\text{min}) = [2\alpha_C(\text{min}) + (8/\pi^2)\alpha_R(\text{min}) + 1] \qquad (54)$$

Here, $(P_{CD}/P_{T1})_{\text{max}}$, $(P_{CD}/P_{T1})_{\text{min}}$, $(P_{R1}/P_{T1})_{\text{max}}$, and $(P_{R1}/P_{T1})_{\text{min}}$ are given by Eqs. (15), (16), (17), and (18), respectively.

Suppose that the brute-force technique is used in the computation of the maximum value of the downlink carrier modulation loss; an evaluation of Eq. (29) is then required for all possible combinations of the modulation indices shown in Eqs. (33)–(42). Specifically, there are 128 possible combinations of modulation indices for computing this particular modulation loss. To avoid this tedious task, the procedure described above is now applied to the downlink case.

Since the downlink modulation losses are related to the modulation indices given by Bessel, trigonometric, and exponential functions, the maximum and minimum values of these losses can be obtained by inspecting Fig. 2. From Fig. 2 and Eqs. (29), (30), and (31), the modulation losses for the downlink carrier, telemetry, and downlink ranging are easily shown to have the forms

$$(P_{C2}/P_{T2})_{\text{max}} = \Big[J_0\big(\tau_1(\text{min})\big) J_0\big(\tau_2(\text{min})\big)$$

$$\times \cos\big(m_t(\text{min})\big)\Big]^2 \exp\big(-\tau_3{}^2(\text{min})\big)$$

(55)

$$(P_{C2}/P_{T2})_{\text{min}} = \Big[J_0\big(\tau_1(\text{max})\big) J_0\big(\tau_2(\text{max})\big)$$

$$\times \cos\big(m_t(\text{max})\big)\Big]^2 \exp\big(-\tau_3{}^2(\text{max})\big)$$

(56)

$$(P_{TLM}/P_{T2})_{\text{max}} = \Big[J_0\big(\tau_1(\text{min})\big) J_0\big(\tau_2(\text{min})\big)$$

$$\times \sin\big(m_t(\text{max})\big)\Big]^2 \exp\big(-\tau_3{}^2(\text{min})\big)$$

(57)

$$(P_{TLM}/P_{T2})_{\text{min}} = \Big[J_0\big(\tau_1(\text{max})\big) J_0\big(\tau_2(\text{max})\big)$$

$$\times \sin\big(m_t(\text{min})\big)\Big]^2 \exp\big(-\tau_3{}^2(\text{max})\big)$$

(58)

$$(P_{R2}/P_T)_{\text{max}} = 2\Big[J_0\big(\tau_1(\text{min})\big) J_1\big(\tau_2(\text{max})\big)$$

$$\times \cos\big(m_t(\text{min})\big)\Big]^2 \exp\big(-\tau_3{}^2(\text{min})\big)$$

(59)

$$(P_{R2}/P_T)_{\text{min}} = 2\Big[J_0\big(\tau_1(\text{max})\big) J_1\big(\tau_2(\text{min})\big)$$

$$\times \cos\big(m_t(\text{max})\big)\Big]^2 \exp\big(-\tau_3{}^2(\text{max})\big)$$

(60)

where the parameters $m_t(\text{max})$, $m_t(\text{min})$, $\tau_1(\text{max})$, $\tau_1(\text{min})$, $\tau_2(\text{max})$, $\tau_2(\text{min})$, $\tau_3(\text{max})$, and $\tau_3(\text{min})$ are given by Eqs. (33), (34), (37), (38), (39), (40), (41), and (42), respectively.

For a sine-wave telemetry subcarrier, the modulation losses can be obtained from the above equations by replacing $\sin(\cdot)$ and $\cos(\cdot)$ for $2J_1(\cdot)$ and $J_0(\cdot)$, respectively. Furthermore, for bi-phase telemetry data that are directly phase modulated on the radio-frequency (rf) carrier, the downlink modulation losses are the same as the case for a square-wave telemetry subcarrier.

Having determined the maximum and minimum values of the downlink modulation losses, the favorable and adverse tolerances can be calculated using Eqs. (5) and (6). Hence, the means and variances of the downlink modulation losses can be determined after their probability density functions are assigned.

An algorithm and flow chart for calculating the downlink means and variances of the modulation losses is presented in Fig. 4. This figure shows the logic required in the computation of downlink modulation losses for simultaneous telemetry and range operation, including the power-controlled AGC action on the coherent turnaround ranging signal. The use of this algorithm greatly reduces the number of computations.

IV. Application and Numerical Results

The algorithm illustrated in Fig. 3 is applied to a simultaneous range and command operation for a hypothetical communications system. This communications system consists of an uplink that transmits a signal in which the carrier is phase modulated by a sine-wave command subcarrier data channel and a square-wave ranging channel. Assume that the nominal values for the command and uplink ranging modulation indices are m_{cn} = 1.1 rads and m_{r1n} = 0.774 rads, respectively. Further, assume that the modulation indices of the command subcarrier and uplink ranging channels vary independently of each other by 10 percent of their nominal values, i.e., Δm_c = 0.11 rads and Δm_{r1} = 0.0774 rads. From Eqs. (9), (10), (11), and (12), the maximum and minimum values for the command and uplink ranging modulation indices are found to be, respectively, $m_c(\text{max})$ = 1.210 rads, $m_c(\text{min})$ = 0.990 rads, $m_{r1}(\text{max})$ = 0.851 rads, and $m_{r1}(\text{min})$ = 0.697 rads.

Using Eqs. (2), (3), and (4), the nominal values for the uplink carrier, command, and uplink ranging modulations are found to be $(P_{C1}/P_{T1})_n$ = 0.265, $(P_{CD}/P_{T1})_n$ = 0.227, and $(P_{R1}/P_{T1})_n$ = 0.205. Similarly, the maximum and minimum values for the modulation losses can be obtained using Eqs. (13), (14), (15), (16), (17), and (18). They are given by $(P_{C1}/P_{T1})_{\text{max}}$ = 0.348,

$(P_{C1}/P_{T1})_{min} = 0.193$, $(P_{CD}/P_{T1})_{max} = 0.295$, $(P_{CD}/P_{T1})_{min} = 0.166$, $(P_{R1}/P_{T1})_{max} = 0.271$, and $(P_{R1}/P_{T1})_{min} = 0.148$.

Hence, the favorable and adverse tolerances for the uplink carrier, command, and uplink ranging modulation losses can be obtained from Eqs. (5) and (6). The results are $F_{C1} = 0.083$, $A_{C1} = -0.072$, $F_{CD} = 0.068$, $A_{CD} = -0.061$, $F_{R1} = 0.066$, and $A_{R1} = -0.057$.

Suppose that uniform probability density functions are assigned to the above modulation losses. Substituting the above results into Eqs. (7) and (8) obtains the means and variances for the uplink carrier, command, and uplink ranging modulation losses, respectively, as $M_{C1} = 0.271$, $V_{C1} = 0.002$, $M_{CD} = 0.23$, $V_{CD} = 0.001$, $M_{C1} = 0.21$, and $V_{C1} = 0.001$.

V. Conclusions

A detailed description of the technique for computing the means and variances of modulation losses was given. It was shown that the technique provides an optimum method, in terms of computing time, for determining the statistical values of modulation losses for both uplink and downlink with the turnaround ranging signal. Simple algorithms to compute the means and variances of the modulation losses were presented. These algorithms conform with the CCSDS recommendations concerning simultaneous range, command, and telemetry operations. The presented algorithms are straightforward and have been successfully implemented in the CCSDS link design control table. It was found that, in practice, the link design control table implementing these algorithms performs its calculations about three times faster than the table using the brute-force technique.

Acknowledgments

References

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UPLINK

TELECOMMAND
DATA → SUBCARRIER GENERATION AND MODULATION → (+) → PHASE MODULATOR

RANGING SIGNAL

CARRIER TRACKING AND DEMODULATION → COMMAND DEMODULATION → DISTRIBUTION TO SPACECRAFT SUBSYSTEM

FREQUENCY MULTIPLIER

RANGING DEMODULATOR

FILTER

⊗ ← IF REF   POWER-CONTROLLED AGC

RANGING FILTER

TELEMETRY FORMATTING AND GROUND TRANSMISSION ← TELEMETRY DEMODULATION ← CARRIER TRACKING AND DEMODULATION

A/D CONVERTER IN DIGITAL SUBSYSTEM ← RANGING DEMODULATOR ASSEMBLY

(+) ← SUBCARRIER GENERATION AND MODULATION ← SPACECRAFT TELEMETRY DATA SOURCES

PHASE MODULATOR

GROUND SYSTEM

SPACECRAFT SYSTEM

Fig. 1. A simplified diagram for two-way simultaneous telecommand–range and telemetry–range operation.
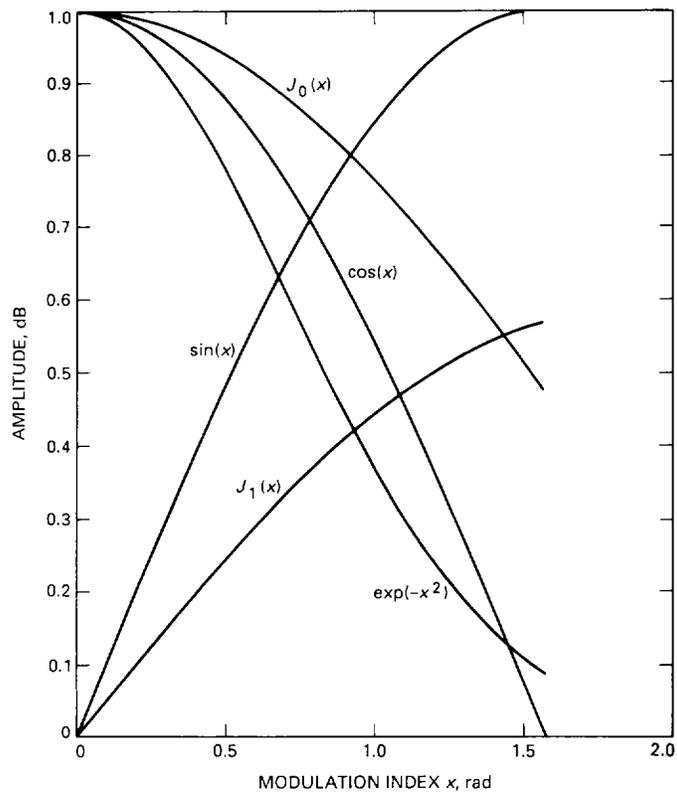
Fig. 2. Bessel, trigonometric, and exponential curves.

START

READ THE INPUT DATA:
$m_{cn}, m_{r1n}, \Delta m_c, \Delta m_{r1}$

CALCULATE $m_c(\text{max}), m_c(\text{min}), m_{r1}(\text{max}), m_{r1}(\text{min})$

SINE-WAVE COMMAND SUBCARRIER?

NO → END

YES

SPECIFY THE PROBABILITY DENSITY FUNCTIONS FOR THE MODULATION LOSSES

SPECIFY THE RANGING WAVEFORM (ONLY SQUARE WAVE OR SINE WAVE IS ALLOWED)

SQUARE-WAVE RANGING?

NO

YES

CALCULATE $J_0(m_{cn}), J_1(m_{cn}),$ $J_0(m_{r1n}), J_1(m_{r1n}), J_0(m_{r1}(\text{max})),$ $J_1(m_{r1}(\text{min})), J_0(m_c(\text{max})), J_0(m_c(\text{min}))$

CALCULATE $J_0(m_{cn}), J_1(m_{cn}),$ $\cos(m_{r1n}), \sin(m_{r1n}), \cos(m_{r1}(\text{max})),$ $\sin(m_{r1}(\text{min})), J_c(m_c(\text{max})), J_c(m_c(\text{min}))$

FORM THE RATIOS $(P_{CD}/P_{T1}), (P_{R1}/P_{T1}),$ (P_{C1}/P_{T1})

FORM THE RATIOS $(P_{CD}/P_{T1}), (P_{R1}/P_{T1}),$ (P_{C1}/P_{T1})

CALCULATE $(P_{CD}/P_{T1})_n, (P_{R1}/P_{T1})_n, (P_{C1}/P_{T1})_n$ $(P_{C1}/P_{T1})_{\text{max}}, (P_{CD}/P_{T1})_{\text{max}}, (P_{R1}/P_{T1})_{\text{max}},$ $(P_{CD}/P_{T1})_{\text{min}}, (P_{R1}/P_{T1})_{\text{min}}, (P_{C1}/P_{T1})_{\text{min}}$

CALCULATE THE FAVORABLE AND ADVERSE TOLERANCES

A_1

Fig. 3. An algorithm to compute the mean and variance of uplink modulation losses for simultaneous range and telecommand operations.

(A_1)

CALCULATE THE MEAN AND VARIANCE
(P_{R1}/P_{T1}), (P_{C1}/P_{T1}), (P_{C1}/P_{T1})

WRITE THE MEAN AND VARIANCE OF
(P_{CD}/P_{T1}), (P_{R1}/P_{T1}), (P_{C1}/P_{T1})

END

(P_{CD}/P_{T1})	COMMAND MODULATION (CM) LOSS, dB
(P_{R1}/P_{T1})	UPLINK RANGING MODULATION (URM) LOSS, dB
(P_{C1}/P_{T1})	UPLINK CARRIER MODULATION (UCM) LOSS, dB
m_{cn}	NOMINAL VALUE FOR TELECOMMAND MODULATION INDEX, radian
m_{r1n}	NOMINAL VALUE FOR URM INDEX, radian
Δm_c	VARIATION OF CM INDEX, radian
Δm_{r1}	VARIATION OF URM INDEX, radian
$m_c(max)$	MAXIMUM VALUE FOR CM AND DRM INDEX, radian
$m_{r1}(max)$	MINIMUM VALUE FOR URM INDEX, radian
$m_c(min)$	MINIMUM VALUE FOR CM INDEX, radian
$m_{r1}(min)$	MINIMUM VALUE FOR URM INDEX, radian
$(P_{CD}/P_{T1})_n$	NOMINAL VALUE FOR CM LOSS, dB
$(P_{R1}/P_{T1})_n$	NOMINAL VALUE FOR URM LOSS, dB
$(P_{C1}/P_{T1})_n$	NOMINAL VALUE FOR UCM LOSS, dB
$(P_{CD}/P_{T1})_{max}$	MAXIMUM VALUE FOR CM LOSS, dB
$(P_{R1}/P_{T1})_{max}$	MAXIMUM VALUE FOR URM LOSS, dB
$(P_{C1}/P_{T1})_{max}$	MAXIMUM VALUE FOR UCM LOSS, dB

Fig. 3 (contd)

START

READ THE INPUT DATA: m_{tn}, Δm_t, m_{r2n}, Δm_{r2}, $(P_{R1}/P_{T1})_{max}$, $(P_{R1}/P_{T1})_{min}$, $(P_{CD}/P_{T1})_{max}$, $(P_{CD}/P_{T1})_{min}$, $(SNR)_{RG}$, $(SNR)_{RG}$ (max), $(SNR)_{RG}$ (min)

SPECIFY THE PROBABILITY DENSITY FUNCTIONS FOR THE MODULATION LOSSES

A_2

CALCULATE m_t (max), m_t (min), Δ_S m_{r1}(max), m_{r2}(min), Δ_S(max), Δ_S(min), α_R(max), α_R(min), α_R

CALCULATE α_C, α_C(max), α_C(min)

CALCULATE β, β(max), β(min)

CALCULATE γ_1, γ_2, γ_3, γ_1(max), γ_2(max), γ_3(max), γ_1(min), γ_2(min), γ_3(min)

SINE-WAVE RANGING SIGNAL?

NO — END

YES

USE SUBCARRIER MODULATION?

NO

USE BI-ϕ MODULATED DIRECTLY ON THE CARRIER?

NO — END

YES

A_4

YES

SPECIFY SUBCARRIER WAVEFORM (SQUARE WAVE FOR DEEP-SPACE MISSIONS AND SINE WAVE FOR NEAR-EARTH MISSION)

A_3

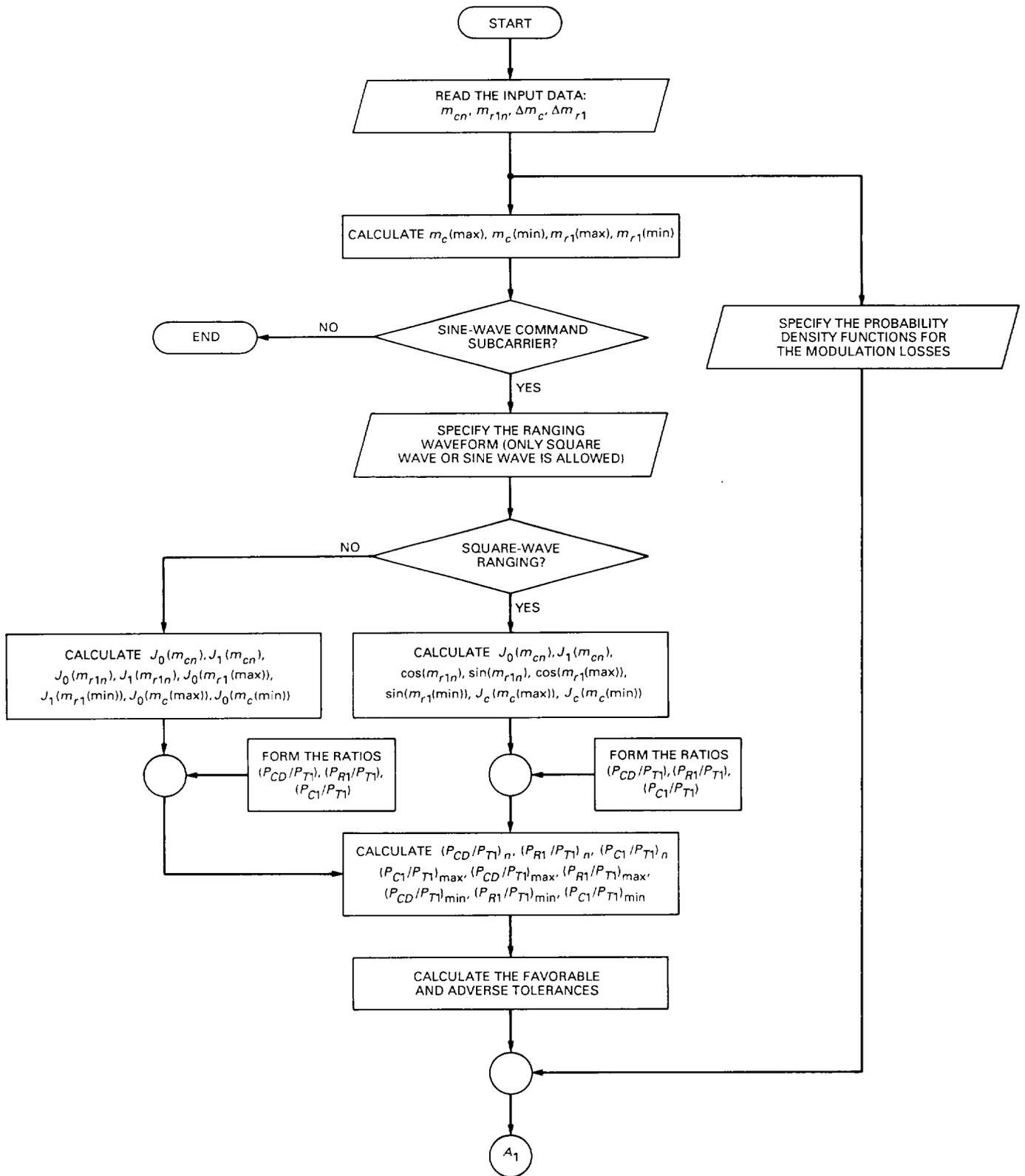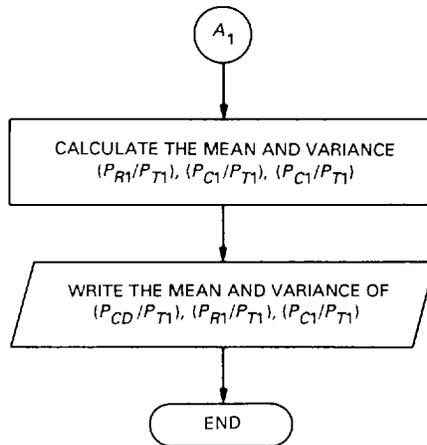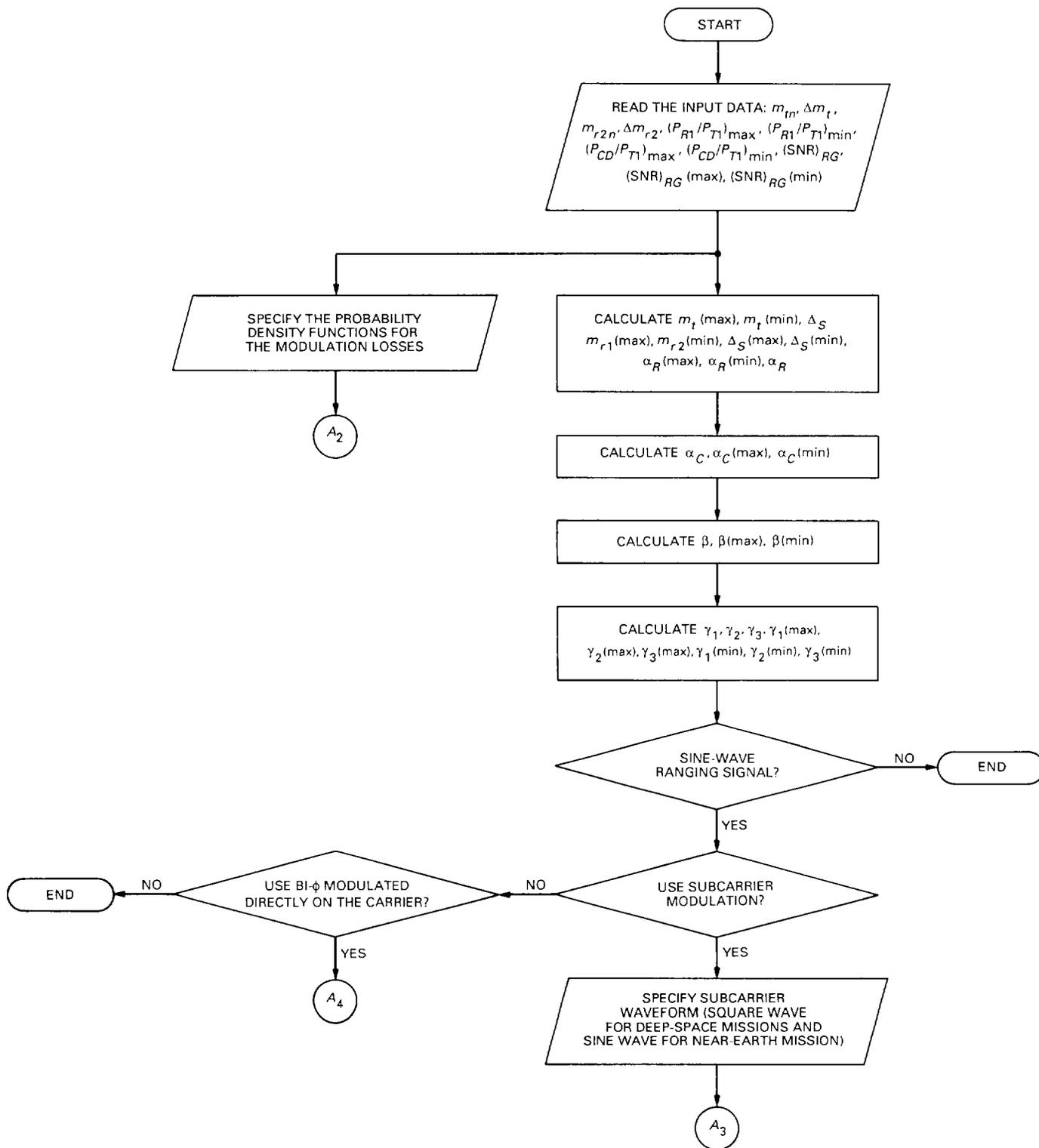Fig. 4. An algorithm to compute the mean and variance of downlink modulation losses for simultaneous range and telemetry operations with power-controlled AGC on the turnaround ranging channel.
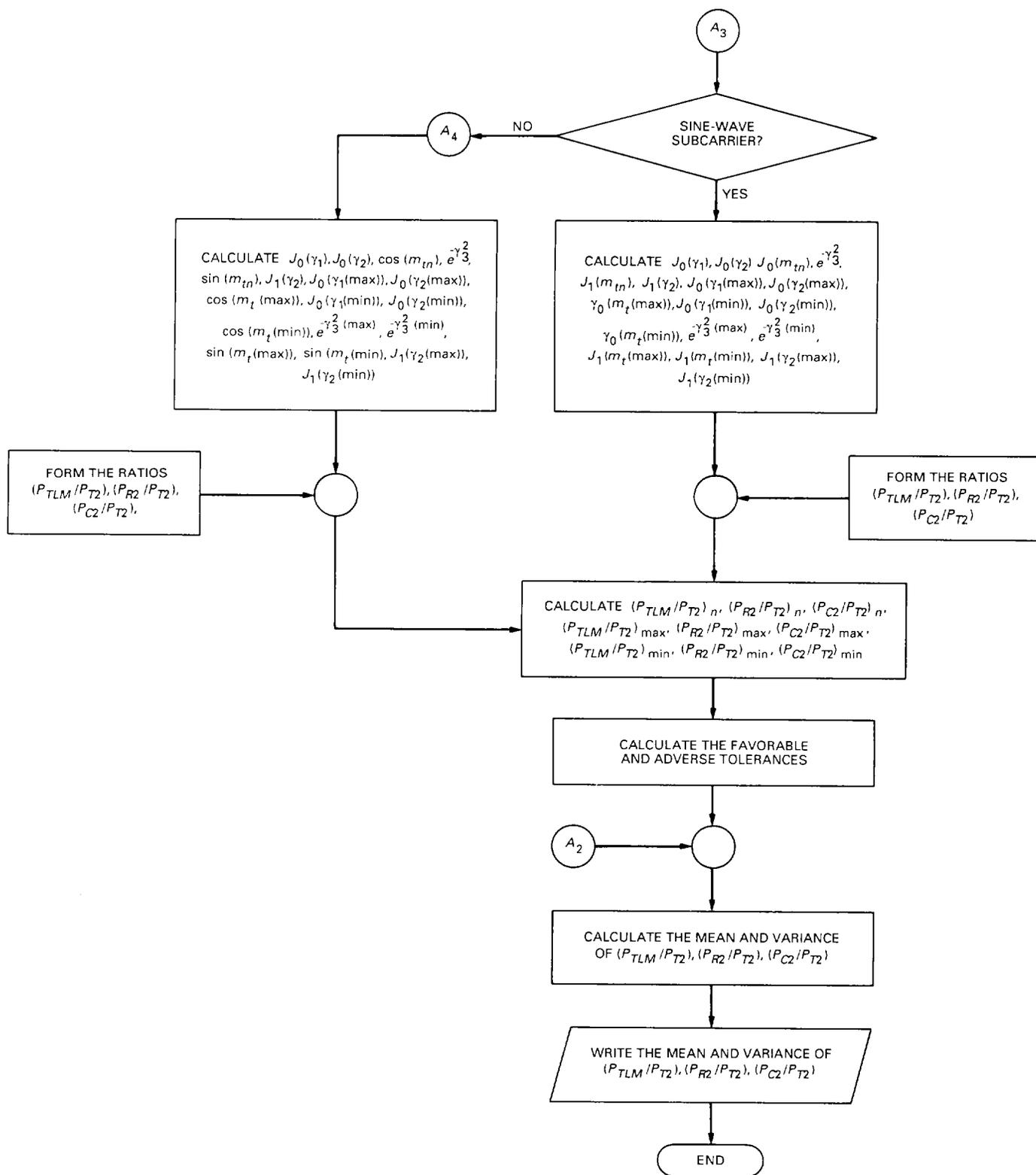
A_3

SINE-WAVE SUBCARRIER?

NO → A_4

YES

CALCULATE $J_0(\gamma_1), J_0(\gamma_2), \cos(m_{tn}), e^{-\gamma_3^2},$
$\sin(m_{tn}), J_1(\gamma_2), J_0(\gamma_1(\max)), J_0(\gamma_2(\max)),$
$\cos(m_t(\max)), J_0(\gamma_1(\min)), J_0(\gamma_2(\min)),$
$\cos(m_t(\min)), e^{-\gamma_3^2(\max)}, e^{-\gamma_3^2(\min)},$
$\sin(m_t(\max)), \sin(m_t(\min)), J_1(\gamma_2(\max)),$
$J_1(\gamma_2(\min))$

CALCULATE $J_0(\gamma_1), J_0(\gamma_2) J_0(m_{tn}), e^{-\gamma_3^2},$
$J_1(m_{tn}), J_1(\gamma_2), J_0(\gamma_1(\max)), J_0(\gamma_2(\max)),$
$\gamma_0(m_t(\max)), J_0(\gamma_1(\min)), J_0(\gamma_2(\min)),$
$\gamma_0(m_t(\min)), e^{-\gamma_3^2(\max)}, e^{-\gamma_3^2(\min)},$
$J_1(m_t(\max)), J_1(m_t(\min)), J_1(\gamma_2(\max)),$
$J_1(\gamma_2(\min))$

FORM THE RATIOS
$(P_{TLM}/P_{T2}), (P_{R2}/P_{T2}),$
$(P_{C2}/P_{T2}),$

FORM THE RATIOS
$(P_{TLM}/P_{T2}), (P_{R2}/P_{T2}),$
(P_{C2}/P_{T2})

CALCULATE $(P_{TLM}/P_{T2})_n, (P_{R2}/P_{T2})_n, (P_{C2}/P_{T2})_n,$
$(P_{TLM}/P_{T2})_{\max}, (P_{R2}/P_{T2})_{\max}, (P_{C2}/P_{T2})_{\max},$
$(P_{TLM}/P_{T2})_{\min}, (P_{R2}/P_{T2})_{\min}, (P_{C2}/P_{T2})_{\min}$

CALCULATE THE FAVORABLE
AND ADVERSE TOLERANCES

A_2

CALCULATE THE MEAN AND VARIANCE
OF $(P_{TLM}/P_{T2}), (P_{R2}/P_{T2}), (P_{C2}/P_{T2})$

WRITE THE MEAN AND VARIANCE OF
$(P_{TLM}/P_{T2}), (P_{R2}/P_{T2}), (P_{C2}/P_{T2})$

END

Fig. 4 (contd)

(P_{TLM}/P_{T2})	TELEMETRY MODULATION (TM) LOSS, dB
(P_{R2}/P_{T2})	DOWNLINK RANGING MODULATION (DRM) LOSS, dB
(P_{C2}/P_{T2})	DOWNLINK CARRIER MODULATION (DCM) LOSS, dB
m_{tn}	NOMINAL VALUE FOR TM INDEX, radian
m_{c2n}	NOMINAL VALUE FOR DRM INDEX, radian
Δm_t	VARIATION OF TM INDEX, radian
Δm_{r2}	VARIATION OF DRM INDEX, radian
$m_t(max)$, $m_{r2}(max)$	MAXIMUM VALUES FOR TM AND DRM INDICES
$m_t(min)$, $m_{r2}(min)$	MINIMUM VALUES FOR TM AND DRM INDICES, radian
$(SNR)_{RG}$	RANGING SIGNAL-TO-NOISE POWER RATIO (SNR) AT THE OUTPUT OF THE RANGING TRANSPONDER
$(SNR)_{RG}(max)$, $(SNR)_{RG}(min)$	MAXIMUM AND MINIMUM RANGING SNR AT THE OUTPUT OF THE RANGING TRANSPONDER
$\Delta_S = (P_{CD}/P_{T1}) - (P_{r1}/P_{T1})$	DIFFERENCE BETWEEN CM AND URM LOSSES, dB
$\Delta_S(max)$, $\Delta_S(min)$	MAXIMUM AND MINIMUM VALUES OF THE DIFFERENCE BETWEEN THE CM AND URM LOSSES, dB
α_R	TOTAL RANGING SNR AT THE INPUT TO THE RANGING TRANSPONDER, dB
$\alpha_R(max)$, $\alpha_R(min)$	MAXIMUM AND MINIMUM VALUES OF α_R, dB
α_C	TOTAL COMMAND-TO-NOISE POWER RATIO
$\alpha_C(max)$, $\alpha_C(min)$	MAXIMUM AND MINIMUM VALUES OF α_C
β	GAIN COEFFICIENT DUE TO AUTOMATIC GAIN CONTROL
$\beta(max)$, $\beta(min)$	MAXIMUM AND MINIMUM VALUES OF β
γ_1	EFFECTIVE MODULATION INDEX FOR TURNAROUND COMMAND, radian
γ_2	EFFECTIVE MODULATION INDEX FOR TURNAROUND RANGING SIGNAL, OR THE EFFECTIVE MODULATION INDEX FOR DOWNLINK RANGING SIGNAL, radian
γ_3	EFFECTIVE MODULATION INDEX FOR TURNAROUND NOISE, radian
$\gamma_1(max)$, $\gamma_2(max)$, $\gamma_3(max)$	MAXIMUM VALUES FOR γ_1, γ_2, γ_3, radian
$\gamma_1(min)$, $\gamma_2(min)$, $\gamma_3(min)$	MINIMUM VALUES FOR γ_1, γ_2, γ_3, radian
$(P_{TLM}/P_{T2})_{max}$	MAXIMUM VALUE FOR (P_{TLM}/P_{T2}), dB
$(P_{R2}/P_{T2})_{max}$	MAXIMUM VALUE FOR (P_{R2}/P_{T2}), dB
$(P_{C2}/P_{T2})_{max}$	MAXIMUM VALUE FOR (P_{C2}/P_{T2}), dB
$(P_{TLM}/P_{T2})_{min}$	MINIMUM VALUE FOR (P_{TLM}/P_{T2}), dB
$(P_{R2}/P_{T2})_{min}$	MINIMUM VALUE FOR (P_{R2}/P_{T2}), dB
$(P_{C2}/P_{T2})_{min}$	MINIMUM VALUE FOR (P_{C2}/P_{T2}), dB

Fig. 4 (contd)