Frame Synchronization for the Galileo Code

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This article reports results on the performance of the Deep Space Network's frame synchronizer for the (15,1/4) convolutional code after Viterbi decoding.

I. Introduction

Most deep-space mission communication systems use a 32-bit unique word (synchronization or sync word) to identify the beginning of a telemetry frame. The sync word is incorporated into the spacecraft's telemetry data stream after it has been divided into frames. The data stream is then encoded, transmitted, and received at a Deep Space Network (DSN) tracking station. The ability of the tracking station to achieve frame synchronization requires the appearance of the sync word within the decoded bit stream. Difficulty arises when the sync word has been corrupted by bit errors, which the Viterbi decoder tends to create in bursts. As a result, the DSN has incorporated frame synchronizers into its communication systems based on an algorithm that attempts to find the sync word even if it is corrupted.

The performance of the DSN frame synchronization algorithm has been analyzed for various frame lengths at various signal-to-noise ratios (SNRs) and thresholds using the NASA (7,1/2) convolutional code [1]. The (15,1/4) experimental convolutional code developed for the Galileo mission to Jupiter will use the same frame synchronizers. Although the present DSN frame synchronization scheme is adequate for the (7,1/2) code, its performance for the (15,1/4) code is unknown. Given the size of the Galileo

code, the average error bursts generated from the output of the (15,1/4) Viterbi decoder are twice as long as the bursts from the (7,1/2) Viterbi decoder. See [2,3] for further discussion of the burst statistics for the NASA code and the Galileo code. This article determines the performance of the frame synchronizer for the (15,1/4) convolutional code after Viterbi decoding and finds the threshold that optimizes the probability of acquiring true sync within four frames using a strategy that requires next-frame verification.

II. Method for Finding Sync

Many different frame synchronization methods have been studied, but the DSN's current method is to compare the true sync word (noiseless version) to a 32-bit segment of decoded bits. Those bits found to be in disagreement are counted. This count is then compared to a predetermined threshold, T, optimized for a given bit error rate (BER). If the number of disagreements is greater than T, those 32 bits are rejected as the sync word. Otherwise, the 32 bits are recorded as a sync word candidate. Successive one-bit shifts of 32-bit received signal segments (sliding window) are compared to the true sync word until the threshold test is passed at the same location in two consecutive frames. Once sync has been declared, testing for the sync word

continues through all succeeding frames. If sync is lost, the sliding window process is started again.

III. Analysis

Decoded data bits were generated using the 1-kbps Viterbi decoder [4], also known as the Little Viterbi Decoder (LVD). The LVD is a hardware decoder developed for use in testing long constraint-length convolutional codes. The received symbols fed into the decoder represent encoded symbols generated from the all-zero information bit sequence with pseudo-random noise added (i.e., noise only). The LVD generated enough data to ensure that 100 error bursts were produced for each SNR of interest. For each SNR tested, the decoded bits were subjected to the threshold test for possible threshold values T, where $0 \leq T \leq 10$. From a random 32-bit window of decoded bits, this test determines whether the number of decoded bit errors in the observed window exceeds the given threshold. A count is maintained of the number of 32-bit windows for which the number of errors exceeds T. The 32-bit window is then shifted to the right one bit until all possible 32-bit segments have been tested.

The two error components that influence the overall performance of the frame synchronization scheme are the probability of miss P_m and the probability of false alarm P_f. P_m is the likelihood that the sync word is not detected in the decoded bit stream. P_f is the likelihood that the sync word is falsely detected in an incorrect position in the decoded bit stream. For this article, P_m was estimated from the LVD error data as

$$P_m = \frac{X}{Y} \tag{1}$$

where

X = number of 32-bit windows where the number of errors exceeds T

Y = number of 32-bit windows tested within a given file

Assuming random data, P_f is given by [5]

$$P_f = \sum_{k=0}^{T} \binom{32}{k} 2^{-32} \tag{2}$$

Note that P_m depends on the code and the SNR, but P_f does not.

Failure to acquire sync in one frame will occur if either the location of the true sync word fails the threshold test or if the sync word is falsely detected before the location of the true sync word. The probability of acquiring sync correctly within one frame P_C can be approximated for small values of P_m and P_f by [6]

$$P_C \approx 1 - P_m - \frac{B-1}{2}P_f \tag{3}$$

where

$$B = \text{length of data frame}$$

Next-frame verification requires that the 32-bit sync word candidate found in the current frame be verified in the next succeeding frame. To acquire sync correctly within four frames, the threshold test must be passed correctly in one of three ways: within the first and second frames; in the second and third frames after failing in the first frame; or in the third and fourth frames after failing in the second frame. Equation (3) can be extended to the probability of acquiring sync correctly within four frames with next-frame verification P_{C4} for small values of P_m and P_f by [6]

$$P_{C4} \approx 1 - 3P_m^2 - \frac{B-1}{2}P_f^2 \tag{4}$$

assuming that the decoded bit errors in the four frames tested are independent.

IV. Results

If the SNR over the DSN channel were sufficiently strong to ensure no bit errors, the received (decoded) sync word would be identical to the true sync word. However, no matter what the SNR and threshold are, there is always a nonzero probability that the sync word will not be found correctly due to the possibility of random data mimicking the sync word within T or fewer disagreements. Given this possibility, the choice of threshold T requires a trade-off between P_m and P_f. As T increases, P_m improves very little while P_f increases substantially. This effect can be seen in Fig. 4. Figure 5 shows how individual error components P_m and P_f work to influence the overall performance of the frame synchronization scheme.

Several figures have been drawn to quantitatively describe sync-acquisition probability using the (15,1/4) convolutional code. In all cases, P_m is determined from the output data of the LVD; P_f is calculated from Eq. (2); and

the overall probability of acquiring sync correctly within four frames is calculated from Eq. (4). The sync marker length is 32 bits. Figures 1 and 2 plot the probability of not finding sync correctly within four frames with next-frame verification for a frame length of 5120 bits with increasing SNR and T, respectively. Figures 3 and 4 plot the same data for a frame length of 8960 bits. Figure 4 also shows a limiting case, SNR $= \infty$ ($P_m = 0$), for which the probability of not correctly acquiring sync is entirely due to the possibility of false alarms. If threshold T is set too high, the result is unacceptable performance no matter how high the SNR. Figure 5 replots one of the curves from Fig. 4 with additional curves showing two individual components ($3P_m{}^2$ and $\frac{B-1}{2}P_f{}^2$) contributing to the probability of not correctly acquiring sync. In this figure, P_f is shown to have little or no influence on the performance until the critical point where P_m and P_f intersect ($T \approx 6$). At this point P_f begins to overwhelm P_m. Note that the P_m component is dominant for smaller thresholds ($T \le 5$) while the P_f component is dominant for higher thresholds ($T \ge 6$). Figure 6 plots P_m versus P_f curves for various SNRs. The plot symbols in this figure represent threshold values T when $0 \le T \le 7$. Observe that the

corresponding thresholds of each curve have the same P_f value, providing evidence as shown in Eq. (2) that P_f is independent of SNR.

V. Conclusion

In general, choosing a threshold for the DSN communication system should be based on the system's operating point. However, other important factors to be considered include the frame-to-frame verification strategy, the length of a frame, and the size of the sync word. Therefore, the following recommendations are based only on the frame synchronization scheme described above. In order to maximize the probability of acquiring sync correctly within four frames with next-frame verification in the area of interest, SNR $= 0.5$ dB (Viterbi decoder BER $= 5 \times 10^{-3}$), a threshold value of five would be optimal for each of the frame lengths tested (5120 and 8960 bits). However, using a threshold of five for SNRs above those tested would result in a sync acquisition rate that is less than optimal. If slightly higher SNRs are anticipated, a threshold of four would be more appropriate.

References

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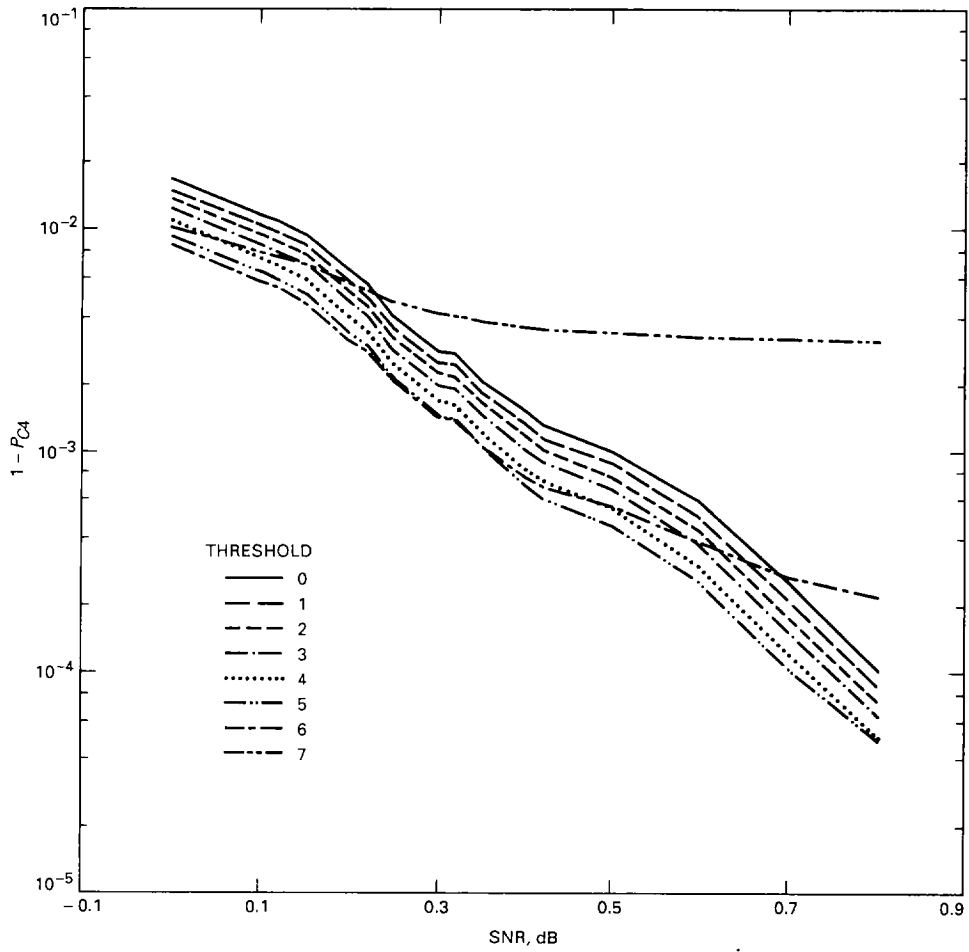
Fig. 1. The effect of an increasing SNR on the probability of not correctly acquiring sync within four frames with next-frame verification, assuming a frame length of 5120 bits.
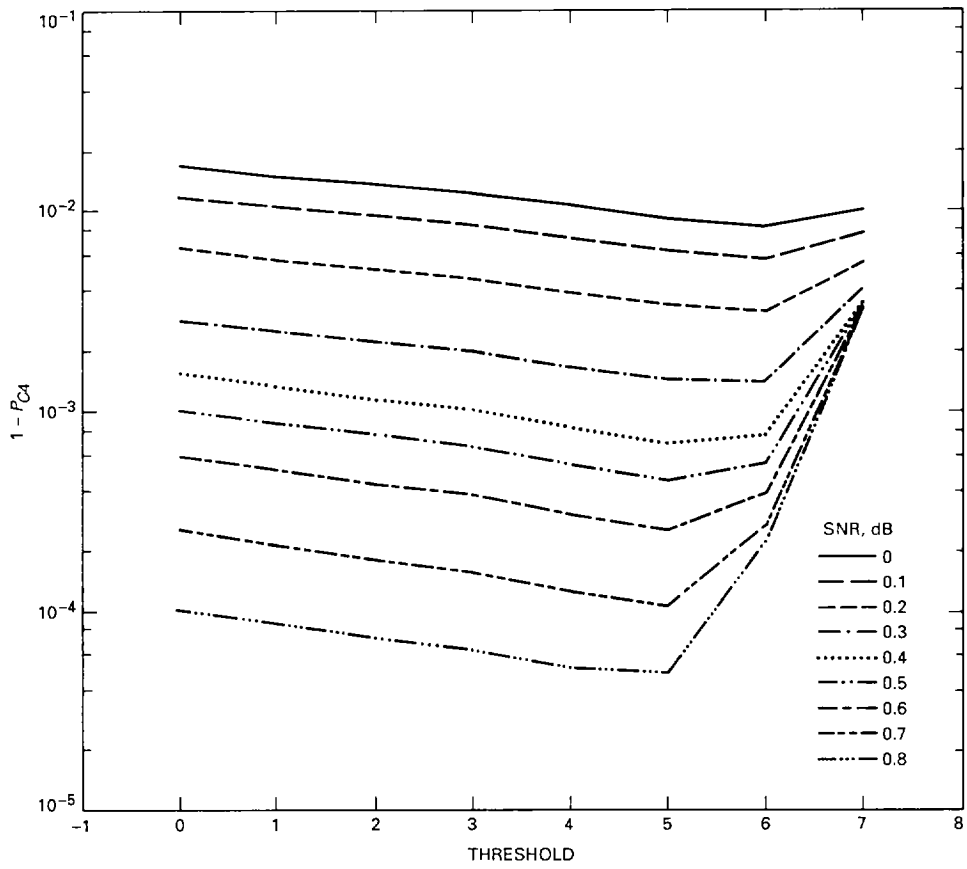
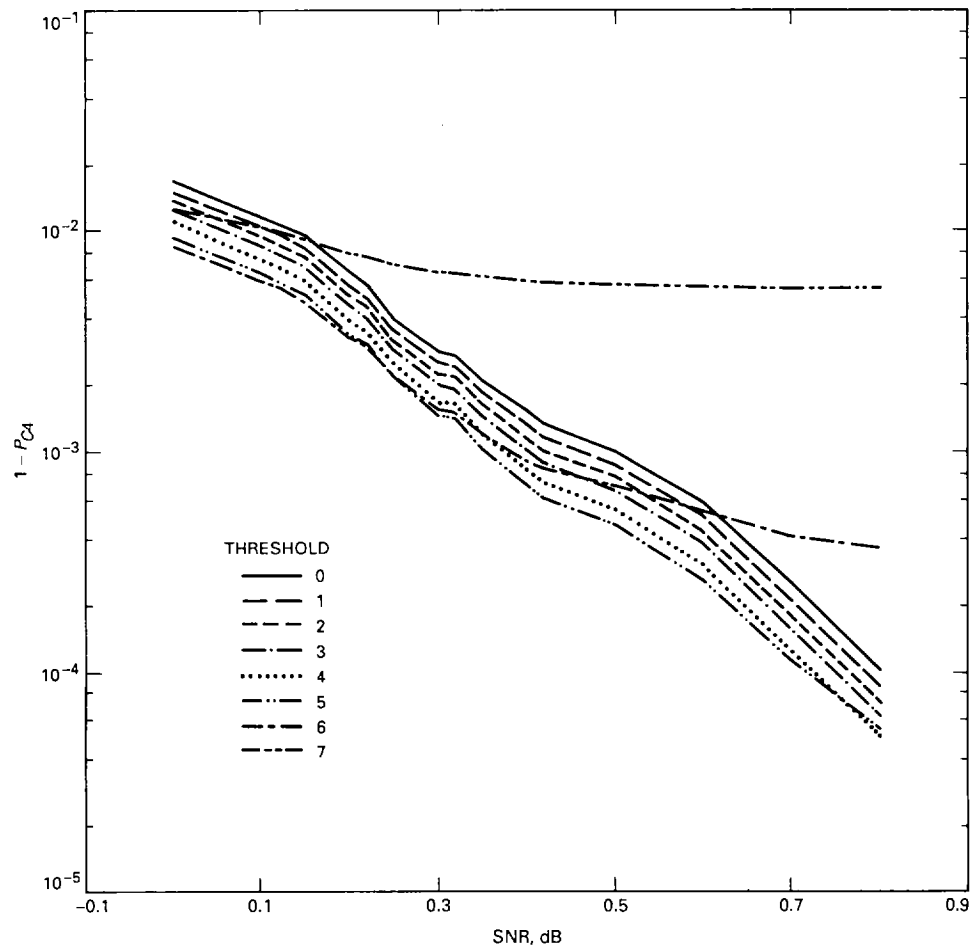Fig. 2. The effect of threshold on the same probability as described in Fig. 1.

Fig. 3. The effect of SNR on the same probability as Figs. 1 and 2, assuming a frame length of 8960 bits.
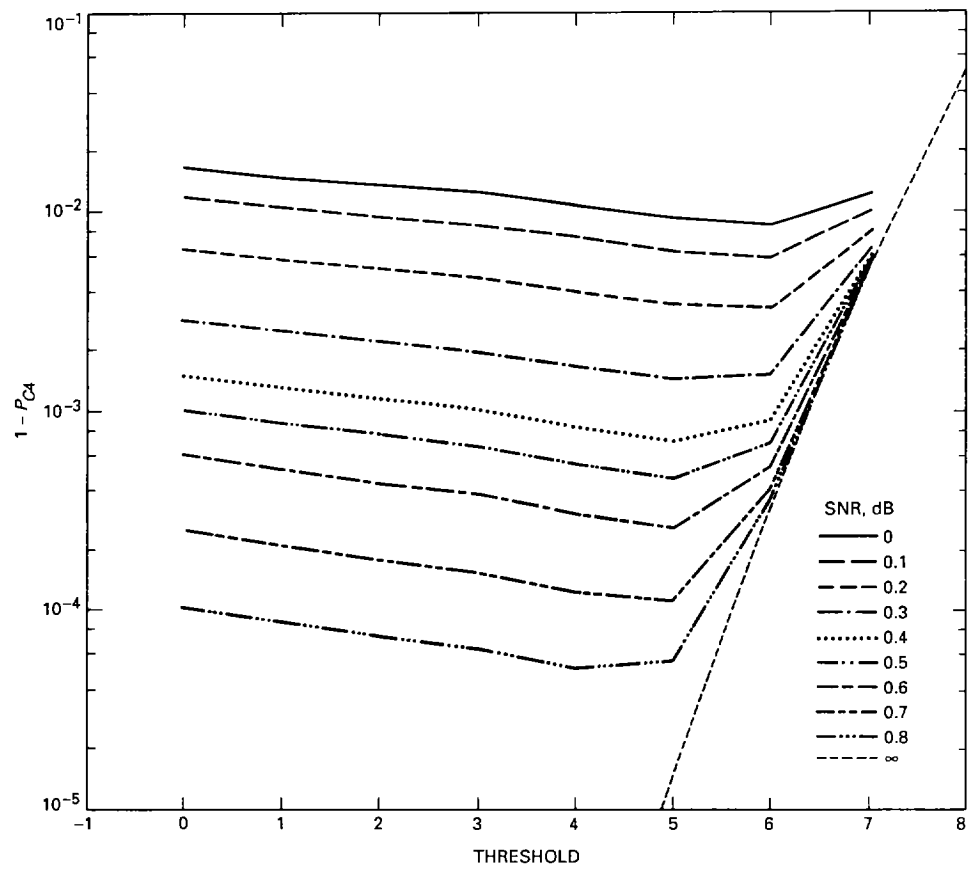
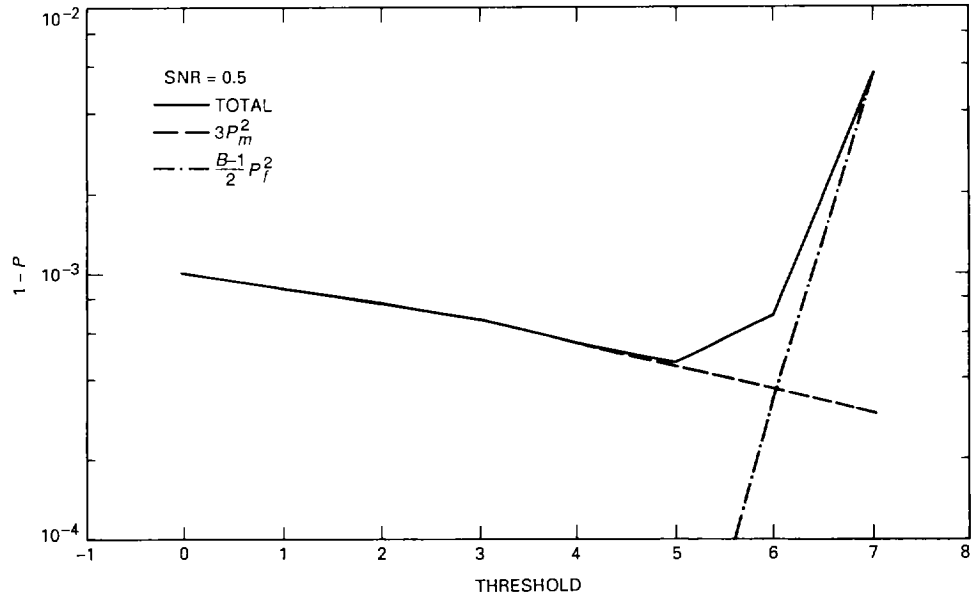Fig. 4. The effect of threshold on the same probability as Fig. 3.

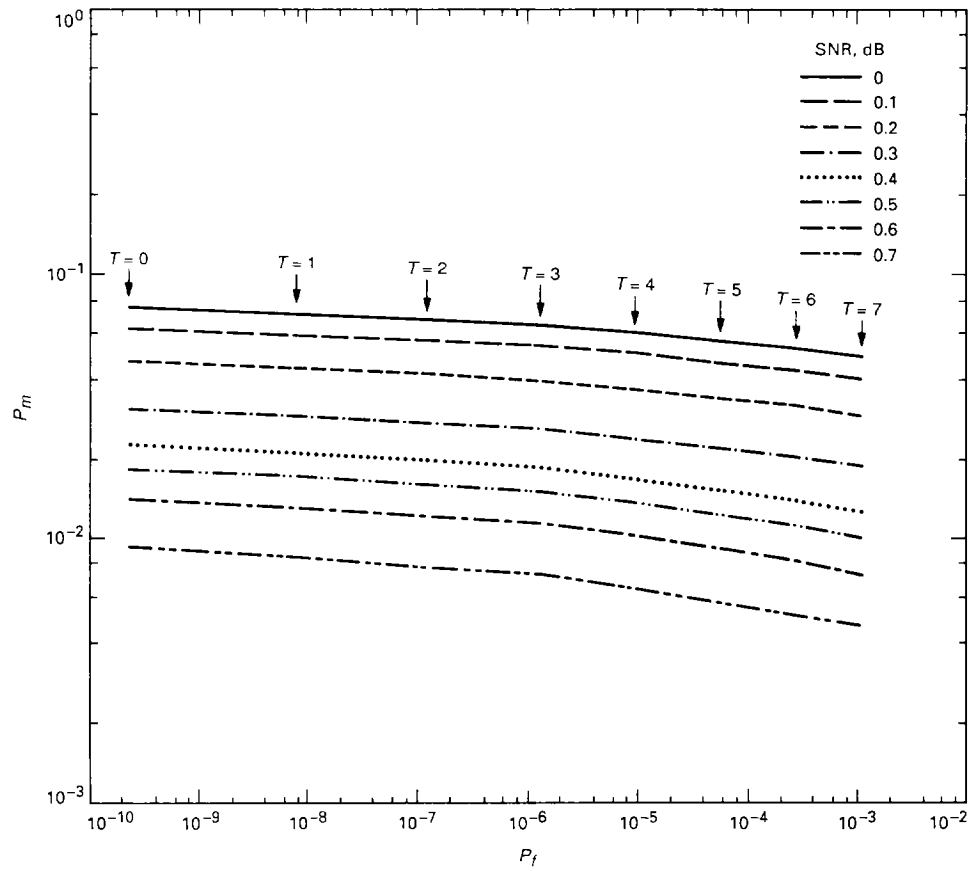Fig. 5. The effect of two additional error factors to the probability described in Fig. 4.

Fig. 6. The probability of miss P_m versus the probability of false alarm P_f for various SNRs.