Coding Gains and Error Rates From the Big Viterbi Decoder

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A prototype hardware Big Viterbi Decoder (BVD) has been completed for an experiment with the Galileo spacecraft. This decoder resulted from six years of research and development by many members of the Communications Systems Research Section. Searches for new convolutional codes, studies of Viterbi decoder hardware designs and architectures, mathematical formulations and decompositions of the deBruijn graph into identical and hierarchical subgraphs, and VLSI chip design are just a few examples of tasks completed for this project. In this article, BVD bit error rates, measured from hardware and software simulations, are plotted as a function of bit signal-to-noise ratio E_b/N_0 on the additive white Gaussian noise channel. Using the constraint length 15, rate 1/4, experimental convolutional code for the Galileo mission, the BVD gains 1.5 dB over the NASA standard (7,1/2) Maximum-Likelihood Convolutional Decoder (MCD) at a bit error rate (BER) of 0.005. At this BER, the same gain results when the (255,223) NASA standard Reed-Solomon decoder is used, which yields a word error rate of 2.1×10^{-8} and a BER of 1.4×10^{-9}. The (15,1/6) code to be used by the CRAF/Cassini missions yields 1.7 dB of coding gain. These gains are measured with respect to symbols input to the BVD and increase with decreasing BER. Also, 8-bit input symbol quantization makes the BVD resistant to demodulated signal-level variations which may cause 0.1-dB or more loss for the MCD. Since the new rate 1/4 or rate 1/6 codes require higher bandwidth than the NASA (7,1/2) code, these gains are offset by about 0.1 dB of expected additional receiver losses. Coding gains of several decibels are possible by compressing all spacecraft data.

I. Introduction

The Big Viterbi Decoder (BVD) is a programmable, error-correcting machine for convolutional codes with constraint length from 2 to 15 and rate $1/2, 1/3, 1/4, 1/5$, or $1/6$ [7]. Recently, a prototype hardware unit was completed with an $8.\bar{3}$ MHz system clock and tested at 400,000 or more decoded bits/second. Implementation versions of this decoder will be installed at DSN stations to support the Galileo, CRAF/Cassini, and future missions. Searches for new convolutional codes having constraint length 13 to 15 demonstrated that 1.5 to 2.0 dB of coding gain is possible [3,4]. A novel hardware design was completed for a constraint length 15, parallel Viterbi decoder [7]. The arithmetic core contains 8192 simple processing units, interconnected according to a hierarchical decomposition of the deBruijn graph into identical subblocks [8].

The (15,1/4) code described in [4] achieves a 1.5-dB coding gain over the (7,1/2) Maximum-Likelihood Convolutional Decoder (MCD), at a decoded bit error rate (BER) of 0.005, which corresponds to an outer NASA (255,223) Reed-Solomon decoder word error rate of 2.1×10^{-8} and BER of 1.4×10^{-9}. The gain is 1.45 dB at a Reed-Solomon BER of 10^{-6}. The (15,1/6) code described in [3] will be used by the CRAF/Cassini missions for a 1.7-dB gain over the MCD. These gains compare well with the 1.9-dB gain resulting from the upgrade of all three 64-m DSN antennas to 70 m, with surface shaping and improved maser. Also, the coding gains apply for all antennas utilized. On the other hand, compressing all spacecraft data 3:1, perhaps noiselessly, would yield a 4.77-dB gain, less 0.25 dB for increased Reed-Solomon error correction to counteract error propagation.

This article contains bit error rates for the BVD obtained by software simulations of the decoder hardware which were verified by running the BVD hardware on pseudo-random data with added Gaussian noise. Frame error rates for the NASA (255,223) Reed-Solomon decoder with inner BVD are given in another article [9]. All Reed-Solomon error rates given in this article are calculated assuming that symbols from many (i.e., 5 or 8) different codewords are interleaved prior to convolutional encoding. This ensures that symbol errors, which are caused by channel noise, within any particular codeword are statistically independent. For all simulations, decoder "node synchronization" was assumed, meaning that groups of n incoming symbols (where $1/n$ is the code rate) are aligned with the code trellis. This is realistic for the operating E_b/N_0 range and error rates of interest. Algorithms are now being developed to acquire and track node synchronization (i.e., to lock the BVD onto the incoming symbol stream) at BERs up to 0.1. The simulations are described in Section II of this article. Coding gains with respect to the (7,1/2) MCD are discussed in Section III. In Section IV these results are shown to be accurate to within 0.03 dB.

II. Decoder Simulations

Bit error rates as a function of information-bit signal-to-noise ratio E_b/N_0 were obtained from software simulations of the actual BVD hardware. The software program is similar to those used to actually debug and test the BVD. An encoded pseudo-random information sequence with added, precisely controlled, wideband Gaussian noise was decoded. A linear feedback shift register with period $2^{31}-1$ was used to generate 5 million to 60 million data bits for these tests. Each bit yielded n channel symbols output by a convolutional encoder, where $1/n$ is the code rate. Symbols 0 and 1 were mapped to demodulated signal levels of +0.84 V and −0.84 V, respectively. Analog, wideband Gaussian noise was then added to these channel symbols. Exactly as in the baseband assembly (BBA) or symbol synchronizer assembly (SSA), demodulated channel symbols (which include noise) were truncated to ±5 V and then quantized to 8 bits by using a step size of 0.0390625 V. The above procedure of adding noise to constant channel symbols corresponds to optimal SSA/BBA operation. In the BVD hardware simulations, noise samples were quantized before addition to channel symbols, a reversal of operations which makes no significant impact on the Viterbi decoder.

The noise source is based upon a standard multiplicative linear congruential generator for random uniform deviates [6]:

$$x_{n+1} = 16{,}807 x_n \pmod{2^{31} - 1} \quad n \geq 0$$

$$x_0 = 1099 \quad \text{for initial seed}$$

$$u_{n+1} = -1 + 2x_{n+1}/(2^{31} - 1)$$

The sequence of u_n's is uniformly distributed on the open interval (−1, +1). If u_n and u_{n+1}, n odd, both lie within the unit circle, then a Box-Muller transformation yields two independent, normal (Gaussian) random deviates [2], which are multiplied by the desired standard deviation of the additive white Gaussian noise to produce two noise samples for addition with encoded symbols. Although the resulting noise spectrum has nearly flat magnitude, there are significant variations, perhaps because samples output by the linear congruential generator are weakly correlated [5]. Using a 4096-entry shuffling array on samples output by the linear congruential generator flattened the noise spectrum and increased BER, but only in the third significant digit—probably because encoded channel symbols were random and because Viterbi decoders are robust with respect to input signal plus noise. Nonetheless, careful selection of a wideband Gaussian noise source ensured accurate BER measurements. The effect on Viterbi decoding of filtering in the receiver may be analyzed in the future.

III. BVD Coding Gain Over the (7,1/2) MCD

Software BVD bit error rates for several convolutional codes are in Fig. 1, where straight lines connect raw data points. BVD hardware runs, using 5 million bits of repeating pseudo-random data with period 65,535, are also in Fig. 1, and they match the software data to within 0.05 dB. For the hardware tests, E_b/N_0 values were calculated from the 8-bit quantized noise samples added to the encoded symbol stream.

The nominal operating point of a Viterbi decoder in the DSN receiving stations is BER equal to 0.005. At this error rate, coding gains of 1.5 and 1.7 dB are achieved by the BVD with a (15,1/4) or (15,1/6) code, respectively, over the (7,1/2) MCD. These same gains occur when an outer (255,223) Reed-Solomon code is used [9]. The BVD uses all 8 bits of input symbols, instead of only the 3-bit quantization used by the MCD. This feature results in a 0.16-dB gain for the (7,1/2) BVD over the (7,1/2) MCD. It also makes the BVD resistant to variations in the mean absolute symbol level, which is controlled by the BBA/SSA. Such variations may cause a loss of 0.1 dB or more for the MCD. Notice in Fig. 1 that the BVD coding gains increase with decreasing BER. For example, at BER $= 10^{-3}$, the BVD gains 1.7 dB by using the (15,1/4) code and 1.9 dB by using the (15,1/6) code. The corresponding (255,223) Reed-Solomon decoder BER is 10^{-19}.

An (11,1/6) decoder is implemented with one processor board and a 552-wire connector, instead of 16 boards connected by a 28-layer, 4416-wire printed circuit backplane in the full BVD for (15,1/6) codes. The BER curves for an (11,1/6) code show that there is less than 0.09 dB of gain possible for each increase by 1 in encoder constraint length (which doubles the amount of decoder hardware) from 11 to 15 or higher, at which point the gain saturates rapidly. However, all hardware complexity in the BVD may be justified by the economics of upgrading antennas.

The formula[1]

$$\mathrm{BER} \approx \exp(9.807 - 14.064 E_b/N_0)$$

was used for BVD bit error rates for the (15,1/6) code from $E_b/N_0 = -0.1$ to 0.5 dB. (But the formula requires E_b/N_0 to be an arithmetic ratio, not in logarithmic decibel units.) This approximation yields a curve right on top of the one for hardware (15,1/6) BERs in Fig. 1. The formula

$$\mathrm{BER} \approx \exp(10.11 - 13.68 E_b/N_0)$$

was obtained by curve-fitting the BVD (15,1/4) code software simulation data in Fig. 1. Similarly, the formula

$$\mathrm{BER} \approx \exp(4.55 - 6.23 E_b/N_0)$$

was obtained from the BVD (7,1/2) code software simulation data. The last two formulas both match the corresponding software data curves in Fig. 1. All approximations above require that E_b/N_0 be expressed as an arithmetic ratio instead of in logarithmic decibel units.

IV. Accuracy of the Results

Errors from a Viterbi decoder occur in bursts of output bits, during which each decoded bit is correct with a probability of nearly 1/2. The error statistics of the (7,1/2) and (7,1/3) MCD have been analyzed in great detail [1]. That report contains an estimate of the uncertainty in BER measured from simulations. Let σ_{BER} denote the variance in BER caused by different possible noise sequences and by a limited number of error bursts during a simulation. Then from [1],

$$\frac{\sigma_{\mathrm{BER}}}{\mathrm{BER}} \approx \frac{\alpha}{\sqrt{nbursts}}$$

where *nbursts* is the number of error bursts and α is a small constant that depends upon E_b/N_0 and the code. Simulations for the (7,1/2), (15,1/4), and (15,1/6) codes verified that the factor α is a constant less than 1.4, and decreases to 1.0 at high E_b/N_0. Hence, $\alpha = 1.4$ will be used herein and the ratio $\alpha/\sqrt{nbursts}$ will be called the relative BER error. For a fixed number of decoded bits, the largest relative BER error will occur at the highest E_b/N_0, when there will be fewest error bursts.

The formulas given in Section III may be used to convert relative BER error into uncertainty in E_b/N_0 for a given, fixed BER. For the BVD (7,1/2) software simulations, the relative BER error was a maximum of 0.022, which occurred at $E_b/N_0 = 3.0$ dB. By using the approximation formula, this error corresponds to uncertainty in E_b/N_0 of 0.015 dB. At $E_b/N_0 = 1.0$ dB, the relative BER error was 0.01115, which corresponds to an uncertainty in E_b/N_0 of 0.013 dB. For the (15,1/6) and (15,1/4) code software simulations, the E_b/N_0 uncertainty was typically 0.03 dB and at most 0.05 dB.

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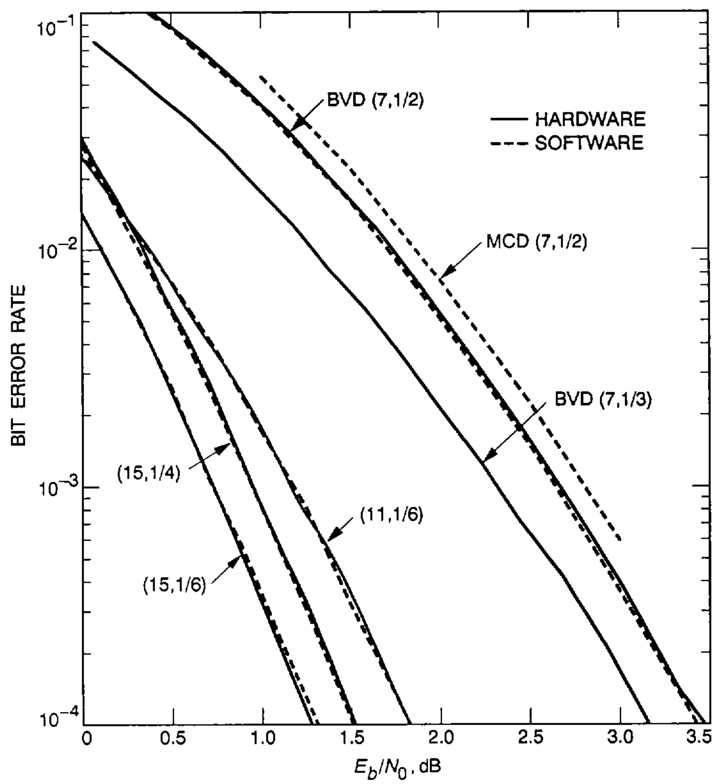

Fig. 1. Viterbi decoder bit error rates for several codes.