Frame Error Rate of the NASA Concatenated Coding System

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For a concatenated coding system, the effect of interleaving depth on interleaved frame-error rate is compiled by simulation. Comparisons are made for three different inner convolutional codes; all three systems have a (255,223) 8-bit Reed-Solomon code as an outer code. For all the inner convolutional codes tested, an interleaving depth smaller than 4 causes substantial loss.

I. Introduction

Communication between spacecraft and the DSN has, for many years, included error-correcting codes, also known as channel codes. A JPL, NASA, and Consultative Committee for Space Data Standards (CCSDS) standard for channel coding is the concatenated code shown in Fig. 1. On the spacecraft, data are first encoded with a Reed-Solomon code, which adds 32 parity bytes to each block of 223 (8-bit) information bytes, and then the output of that block is encoded by a constraint length 7, rate 1/2 convolutional code. On the ground, the process is reversed: a convolutional decoder using the Viterbi algorithm is followed by a Reed-Solomon decoder. Presently, the DSN performs the convolutional decoding and passes the convolutionally decoded data to the project for Reed-Solomon decoding; in the future (beginning with Mars Observer) the DSN will do Reed-Solomon decoding. This system was first used by NASA during Voyager 2's encounters with Uranus and Neptune to protect compressed imaging data, and is typically used by a project to protect compressed data, because it is much more efficient than a convolutional code alone at obtaining low (10^{-6}) bit error rates.

There are several reasons for this concatenated system. The convolutional code with a Viterbi decoder is very good at dealing with extremely noisy data, while the Reed-Solomon code makes a medium channel extremely good, and identifies almost all of its own errors. The channel errors which are not corrected by the convolutional code tend to occur in bursts. The Reed-Solomon code is good at correcting short bursts of errors because it considers any errors in one byte as one error.

Because the error bursts in Viterbi-decoded data are often longer than one byte, the Reed-Solomon codewords are block interleaved to disperse error bursts over several codewords. Interleaving is best described by Fig. 2. The data stream is read into the array horizontally, and then block encoded vertically. Then the data stream is read out horizontally, as it was originally read in, and goes to the convolutional encoder. This means that a byte from one Reed-Solomon codeword is convolutionally encoded next to a byte from a different Reed-Solomon codeword. Because of this, bursts of Viterbi decoder errors cause a small number of errors in each of several Reed-Solomon words instead of causing a large number of errors in one Reed-

Solomon word. The horizontal dimension of the array in Fig. 2 is the interleaving depth, and the vertical dimension is the Reed-Solomon codeword size (255 for these uses). The whole array constitutes one interleaved frame. The interleaving depth for the CCSDS standards is five.

Galileo is using an experimental constraint length 15 convolutional code as the inner code in its concatenated system, and its Reed-Solomon code is interleaved to depth two. The constraint length 15 code was chosen to increase performance substantially compared to the standard constraint length 7 code. But the error bursts from a constraint length 15 code are typically about twice as long as those from a constraint length 7 code. On the other hand, the fact that (for historical reasons involving the length of data blocks) Galileo's Reed-Solomon code is interleaved to depth two means that it cannot handle long bursts of errors as well as it would with a larger interleaving depth.

An earlier article [1] quantified the loss in coding gain from the use of a small interleaving depth for outer Reed-Solomon codes concatenated with long constraint-length convolutional inner codes. That article measured performance in terms of average bit error rates. For some applications, bit error rate may not be the right measure of performance. Because bytes from one Reed-Solomon codeword are interleaved in the data with bytes from the other codewords in the same interleaved frame, it could be necessary to declare an entire interleaved frame in error whenever a decoded word in the frame has a detected error. The effect of the interleaving depth on frame errors is not immediately clear. Once codewords are interleaved "enough," an additional increase in interleaving depth will not cause much decrease in Reed-Solomon word error rate, but frame error rates will rise because the frames are longer. One would expect the frame error rate, as a function of interleaving depth, to fall for a while, and then to rise towards 1 as the interleaving depth goes to infinity; it is desirable to know where this turnaround takes place. Thus, the frame error rate statistics have been compiled in this article. A frame is declared in error if any Reed-Solomon word in the frame has more than the 16 byte errors which the code can correct. (In practice, a frame is not declared in error unless the Reed-Solomon decoder detects at least one word error in the frame, but almost every Reed-Solomon decoder error is detected, see [2].)

II. Simulation Results and Analysis

A study was conducted by simulation of a Gaussian channel at various signal-to-noise ratios using the Little Viterbi Decoder, a 100-bit-per-second decoder developed as a research tool by the Coding and Modulation Work Unit for the DSN Advanced Systems Program. The plots for various interleaving depth of frame error rates as a function of bit signal-to-noise ratio, E_b/N_0, are shown in Fig. 3 for the constraint length 7, rate 1/2 code. Figure 4 shows the same thing with Galileo's constraint length 15, rate 1/4 code used as the inner code. Figure 5 uses the constraint length 15, rate 1/6 code described in [3], which will be used by the Comet Rendezvous/Asteroid Flyby (CRAF) and Cassini missions. As can be seen in the graphs, interleaving depths between 4 and 8 give the best performance. The graphs do not have as much data, and especially as much low error-rate data, as one might like, because of the great length of simulation time required to tell anything about 10^{-5} error rates.

Acknowledgment

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References

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[2] R. J. McEliece and L. Swanson, "On the Decoder Error Probability for Reed-Solomon Codes," *IEEE Transactions on Information Theory*, vol. IT-32, no. 5, pp. 701–703, September 1986.

[3] J. H. Yuen and Q. D. Vo, "In Search of a 2-dB Coding Gain," *TDA Progress Report 42-83*, vol. July–September 1985, Jet Propulsion Laboratory, Pasadena, California, November 15, 1985.

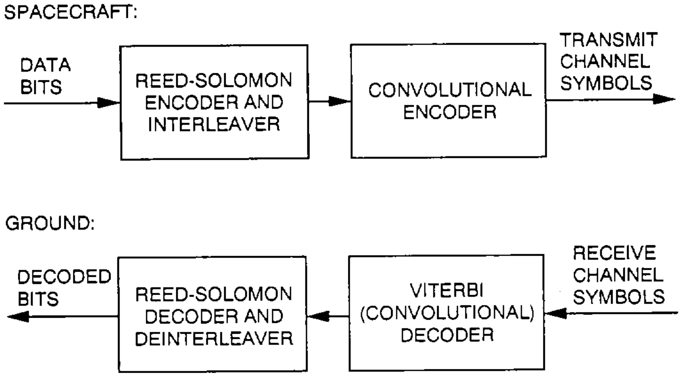

Fig. 1. Concatenated coding system.

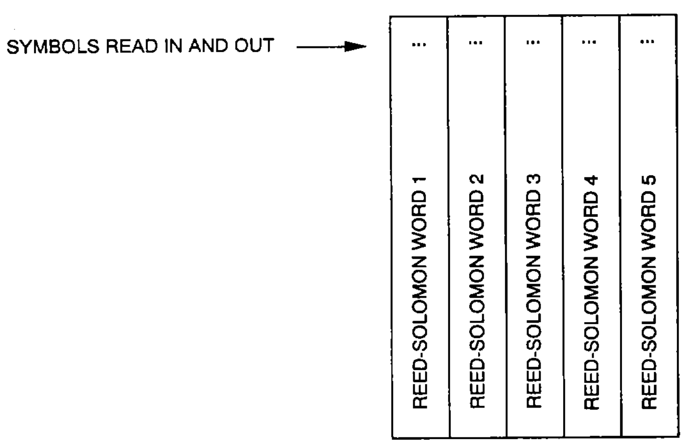

Fig. 2. Block interleaving (depth 5).

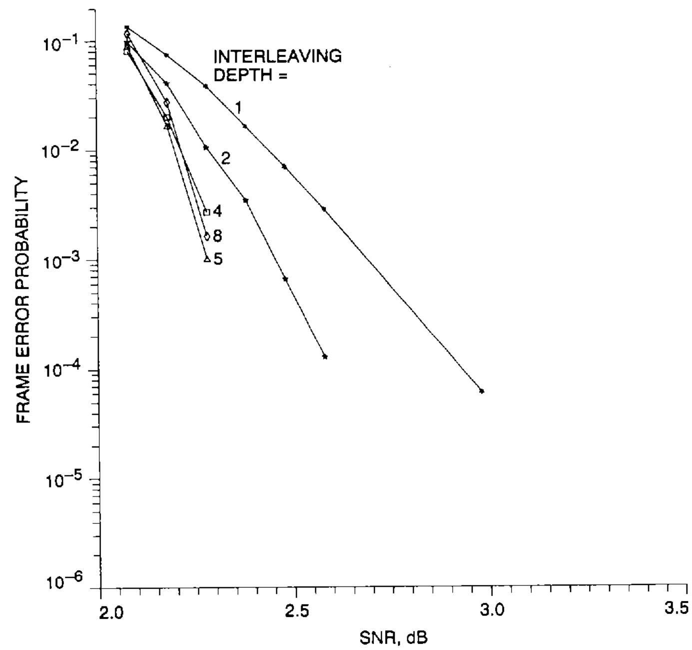

Fig. 3. Frame error probability of the concatenated coding scheme with (7,1/2) convolutional code with (255,223) Reed-Solomon code.

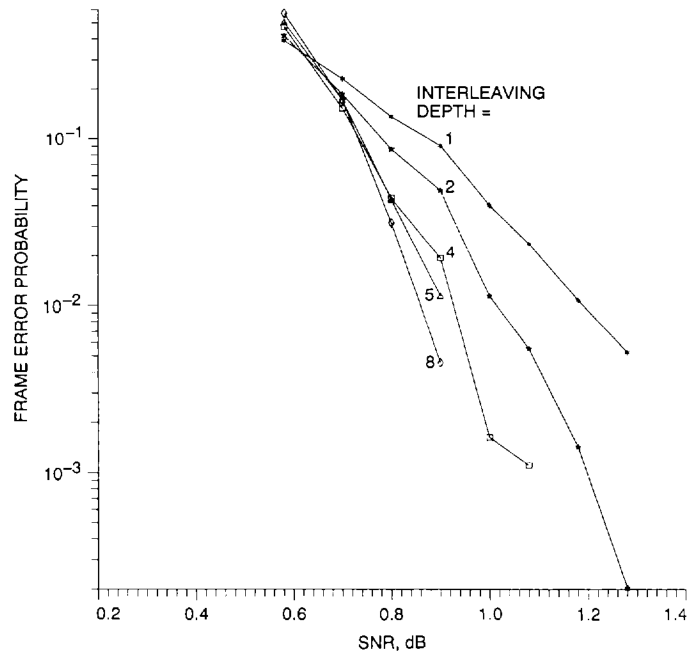

Fig. 4. Frame error probability of the concatenated coding scheme with (15,1/4) convolutional code with (255,223) Reed-Solomon code.

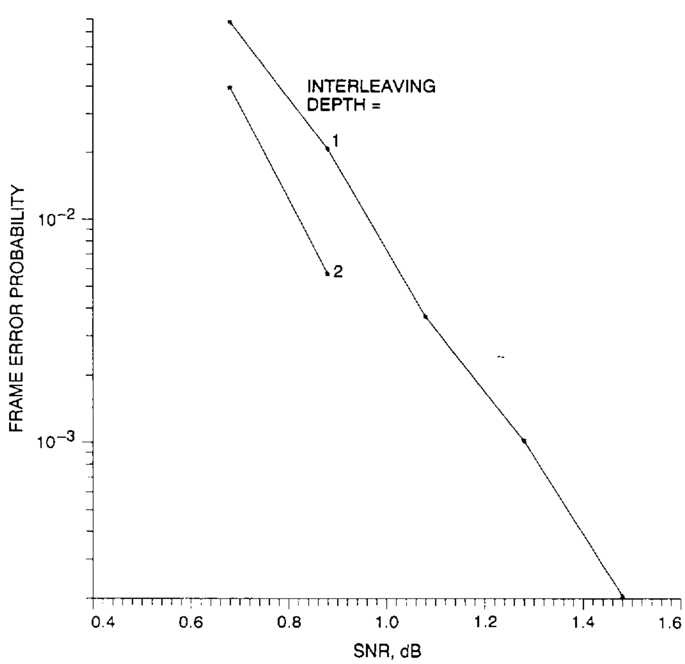

Fig. 5. Frame error probability of the concatenated coding scheme with (15,1/6) convolutional code with (255,223) Reed-Solomon code.