

# Symbol Signal-to-Noise Ratio Loss in Square-Wave Subcarrier Downconversion

Y. Fera and J. Statman

Communications Systems Research Section

*This article presents the simulated results of the signal-to-noise ratio (SNR) loss in the process of a square-wave subcarrier downconversion. In a previous article [2], the SNR degradation was evaluated at the output of the downconverter based on the signal and noise power change. Unlike in the previous article, the SNR loss is defined here as the difference between the actual and theoretical symbol SNR's for the same symbol-error rate at the output of the symbol matched filter. The results show that an average SNR loss of 0.3 dB can be achieved with tenth-order infinite impulse response (IIR) filters. This loss is a 0.2-dB increase over the SNR degradation in the previous analysis where neither the signal distortion nor the symbol detector was considered.*

## I. Introduction

A signal with a downconverted square-wave subcarrier suffers distortion caused by the nonideal filtering, the use of a finite number of harmonics, the data bandwidth cut-off, and so on. This distortion results in a loss of signal-to-noise ratio (SNR). In a previous article [2], the SNR degradation at the output of the downconverter was measured under the following definition:

$$\text{SNR degradation, dB} = \text{SNR}_{\text{ideal}} - \text{SNR}_{\text{real}}$$

where  $\text{SNR}_{\text{ideal}}$  and  $\text{SNR}_{\text{real}}$  are the SNR's after the square-wave subcarrier downconversion using ideal and realizable filters, respectively. The analysis based on this definition, however, does not include the qualitative changes, such as distortion, that may have an impact on the symbol-error rate (see Fig. 1), nor does it include the effect of the symbol detector on the symbol SNR loss.

For a complete study, the downconverter output is further decimated and fed into a symbol detector, and the symbol-error rate is then determined and compared with the theoretical symbol-error probability for binary phase-shift keying (BPSK) without downconversion [3]. Unlike the SNR degradation defined previously [2], the SNR loss now is defined as the difference between the required SNR and the theoretical SNR for the same symbol-error rate.

## II. Simulation Procedure

The simulated procedure is illustrated in Fig. 2, where the subcarrier down-mixer has been described in [1,2]. The simulation assumes symbol synchronization, zero subcarrier phase, and known subcarrier frequency; hence, it focuses only on losses due to the nonideal filtering, the imperfect delay compensation, and the data bandwidth cutoff in the down-mixing and symbol-detecting processes. The simulation employs IIR filters with the bandwidths of 2, 4,

error rate is compared with both the theoretical symbol-error probability for BPSK and the simulated results from the direct symbol detection without the downconversion.

The simulation procedure is as follows: a square-wave subcarrier at a frequency of 22.5 kHz is modulated by a pseudo-random sequence with clock time of 1/1000 sec. A white Gaussian noise is added and its variance is adjusted so that the symbol SNR,  $E_s/N_o$ , takes values from -10 to 10 dB. The relationship between the random noise variance,  $\sigma^2$ , and the symbol SNR,  $E_s/N_o$ , is

$$\sigma^2 = \frac{P_s f_s}{2f_d(E_s/N_o)}$$

where  $P_s$  is the subcarrier power,  $f_s$  is the sample frequency, and  $1/f_d$  is the pseudo-noise (PN) sequence clock time. The noise-contaminated signal is then passed through a bank of bandpass filters (BPF's) with center frequencies at the first, third, and fifth harmonics of the square-wave subcarrier. Each of the BPF outputs is multiplied by a proper down-mixing signal, and then the products are added. The sum of the products is then passed through a lowpass filter (LPF). The subcarrier now has a lower frequency. The total signal bandwidth is reduced from 5 times the original subcarrier frequency plus a single-sided bandwidth of the data to 5 times the new subcarrier frequency plus a single-sided bandwidth of the data.

After the down-mixing, the original high sample rate is no longer necessary; hence, the signal is decimated by an integer number equal to  $\lfloor f_s/(6f_B) \rfloor$ , where  $f_s$  is the original sample rate and  $f_B$  is the BPF bandwidth. The decimated samples are then multiplied by the sum of the first three harmonics at the downconverted subcarrier frequency and fed into an integrate-and-dump filter for the symbol detection. The symbol-error rate is obtained by taking the ratio of the number of the incorrect symbol detections and the total number of symbols.

At the downconverter output, a delay due to the non-ideal filtering is compensated for. The compensation is done in terms of sample periods before the decimation for the higher resolution. Note that this compensation, however, may be off by a fraction of a sample period since the delay period may not be an integer multiple of the sample period. The SNR degradation due to the symbol synchronization offset has been previously studied [4], and is thus not discussed in this article. The symbol SNR loss from the downconversion is then determined by taking the difference between the actual and the ideal  $E_s/N_o$  in decibels required to achieve the same symbol-error rate.

Three different BPF bandwidths are used in the simulation. The sample rates before and after the downconversion, the total number of samples, and the number of symbols used in each case are summarized in Table 1.

### III. Results

For the BPF bandwidths of 2, 4, and 6 kHz, the symbol-error rates are obtained through the simulations with the symbol SNR varied from -10 to 10 dB. The comparisons between the simulated results and the theoretical symbol-error probabilities for BPSK are shown in Fig. 3. The theoretical symbol-error probability for BPSK is computed with the complementary error function [3],  $\text{erfc}(\sqrt{E_s/N_o})$ , with an accuracy of  $10^{-5}$ . Therefore, the measured symbol-error rates for a symbol SNR greater than 6 dB may not be very accurate. Another problem is the limitation of the number of symbols used in the simulations. In this case, 250,000 symbols are used. For a symbol-error rate on the order of  $10^{-5}$ , it may be necessary to simulate  $10^6$  symbols, which is beyond the available time and resource constraints.

To obtain the symbol SNR loss for a given symbol-error rate, first the square of the inverse of the complementary error function of two times the symbol-error rate is evaluated with an accuracy of  $10^{-10}$ . The obtained actual SNR is then compared with the theoretical SNR. The difference of the two SNR's,  $\Delta SNR$ , is the SNR loss. The SNR loss is computed for each simulation point, and the results are shown in Fig. 4. The points that are beyond 6 dB are not accurate, as mentioned earlier; therefore, they may not be shown in the figure.

For each BPF bandwidth, the SNR losses are averaged, and the results are given in Table 2, where DC stands for downconversion, and SD stands for symbol detection. The SNR degradations in the downconversion process without symbol detection in Table 2 were obtained in [2]. The SNR loss in the process of symbol detection without downconversion is also obtained and shown in Table 2. Furthermore, the relationship between the SNR loss and the BPF bandwidth is shown in Fig. 5.

For the BPF bandwidth varied from 2 to 6 kHz, the average symbol SNR loss decreased from 0.8745 to 0.2850 dB in the simulations. The loss is higher than the 0.5- to 0.1-dB SNR degradation computed in the previous analysis [2] without the symbol detection. However, the decrease in SNR loss with wider filter bandwidths agrees with the SNR degradation obtained in the previous analysis. The SNR loss includes the effect of the cutoff of the side lobes of the PN data signal spectrum, the effect of the nonideal filtering, and the imperfect delay compensation.

0.1-dB SNR degradation computed in the previous analysis [2] without the symbol detection. However, the decrease in SNR loss with wider filter bandwidths agrees with the SNR degradation obtained in the previous analysis. The SNR loss includes the effect of the cutoff of the side lobes of the PN data signal spectrum, the effect of the nonideal filtering, and the imperfect delay compensation.

For example, when the BPF bandwidth is 6 kHz, the average symbol SNR loss is 0.2850 dB. This loss is higher than the SNR degradation in the previous analysis [2], which is about 0.1 dB. The loss is due to several factors. First, the nonideal filtering causes signal distortion. Second, when the group delay of the output signal at the downconverter is not an integer multiple of the sample period, there is an SNR loss due to symbol timing misalignment. Third, when the decimated signal is multiplied by the sum of the harmonics, the high-frequency terms of the product can cause aliasing if the original signal is sampled at the Nyquist rate. Last, the integrate-and-dump filter acts as a lowpass filter and its performance is not as good as that of an ideal LPF. As the bandpass filter bandwidth gets narrower, the loss becomes larger. This is partly due to the cutoff of the PN data signal bandwidth [2], and also when the filter bandwidth needs to be narrower, the required filter order may be higher for a better performance.

Several attempts were unsuccessful in reducing the SNR loss. First, since the integrate-and-dump filter does not

perform as an ideal LPF, an LPF is put in front of the integrate-and-dump filter. However, the symbol-error rate increases. The reason is that the transition region is enlarged by lowpass filtering.

It is also found that in the symbol detection process, if the incoming signal is multiplied by a square wave instead of by the first three harmonics of the square wave, the symbol SNR loss increases. The reason is that when a sine wave is multiplied by a square wave, assuming perfect timing and frequency alignment, the resulting rectified sine wave contains a constant term plus an infinite series of harmonics in its Fourier series expansion, which is not band limited. After sampling at a finite rate, aliasing occurs. If, on the other hand, only the first three harmonics are multiplied by the incoming signal, aliasing will not occur given a proper sample rate.

A higher sample rate of 324 kHz was also employed at the input of the downconverter for a 6-kHz BPF bandwidth, but the SNR loss did not reduce significantly.

## IV. Conclusion

This article presents the symbol SNR loss due to the process of the square-wave subcarrier downconversion. The symbol SNR loss is measured as the difference between the actual symbol SNR and the theoretical SNR for the same symbol-error rate. An average loss of 0.3 dB was achieved with tenth-order IIR filters.

## Acknowledgments

The authors thank Dr. Edgar Satorius for his many helpful suggestions and contributions to this work. Thanks are also due to David Rogstad for his helpful discussions. The authors are especially grateful to Todd Chauvin and David Watola for their considerable help in the simulation implementation.

## References

- [1] Y. Feraia and J. Statman, "Bandwidth Compression of Noisy Signal With Square-Wave Subcarrier," *The Telecommunications and Data Acquisition Progress Report 42-109*, vol. January-March 1992, Jet Propulsion Laboratory, Pasadena, California, pp. 170-178, May 15, 1992.

- [2] Y. Feraia and J. Statman, "SNR Degradation in Square-Wave Subcarrier Down-conversion," *The Telecommunications and Data Acquisition Progress Report 42-111*, vol. July–September 1992, Jet Propulsion Laboratory, Pasadena, California, pp. 192–201, November 15, 1992.
- [3] J. G. Proakis, *Digital Communications*, New York: McGraw-Hill, 1983.
- [4] R. Sadr and W. Hurd, "Detection of Signals by the Digital Integrate-and-Dump Filter With Offset Sampling," *The Telecommunications and Data Acquisition Progress Report 42-91*, vol. July–September 1987, Jet Propulsion Laboratory, Pasadena, California, pp. 158–173, November 15, 1987.

**Table 1. Simulation conditions.**

BPF bandwidth, kHz	2	4	6
Original sample rate, kHz	264	264	288
Reduced sample rate, kHz	12	24	36
Samples per symbol	12	24	36
Number of samples	$6.6 \times 10^6$	$6.6 \times 10^6$	$7.2 \times 10^6$
Number of symbols	$25 \times 10^3$	$25 \times 10^3$	$25 \times 10^3$

**Table 2. SNR loss and degradation, dB.**

BPF bandwidth, kHz	2	4	6
Loss in DC with SD	0.8745	0.4218	0.2850
Degradation in DC without SD	0.3964	0.1655	0.0606
Loss in SD without DC	0.0288	0.0288	0.0288

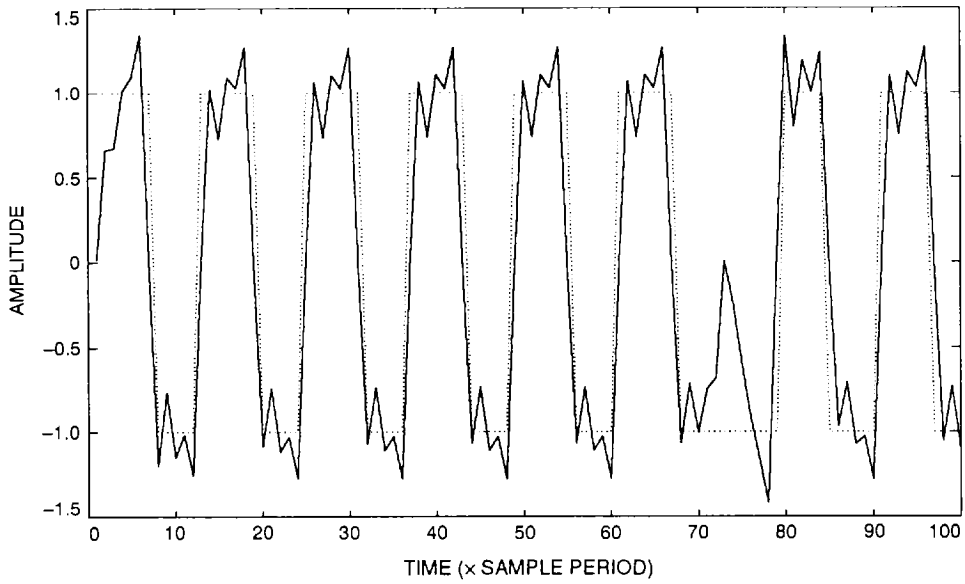


Fig. 1. Signal distortion due to the subcarrier downconversion.

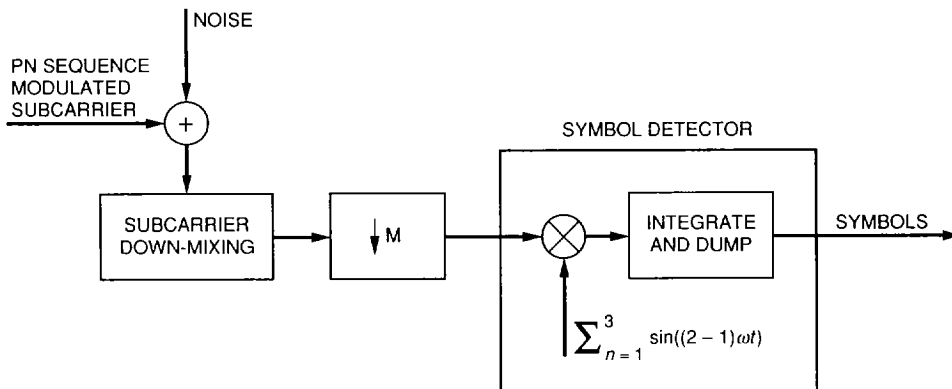
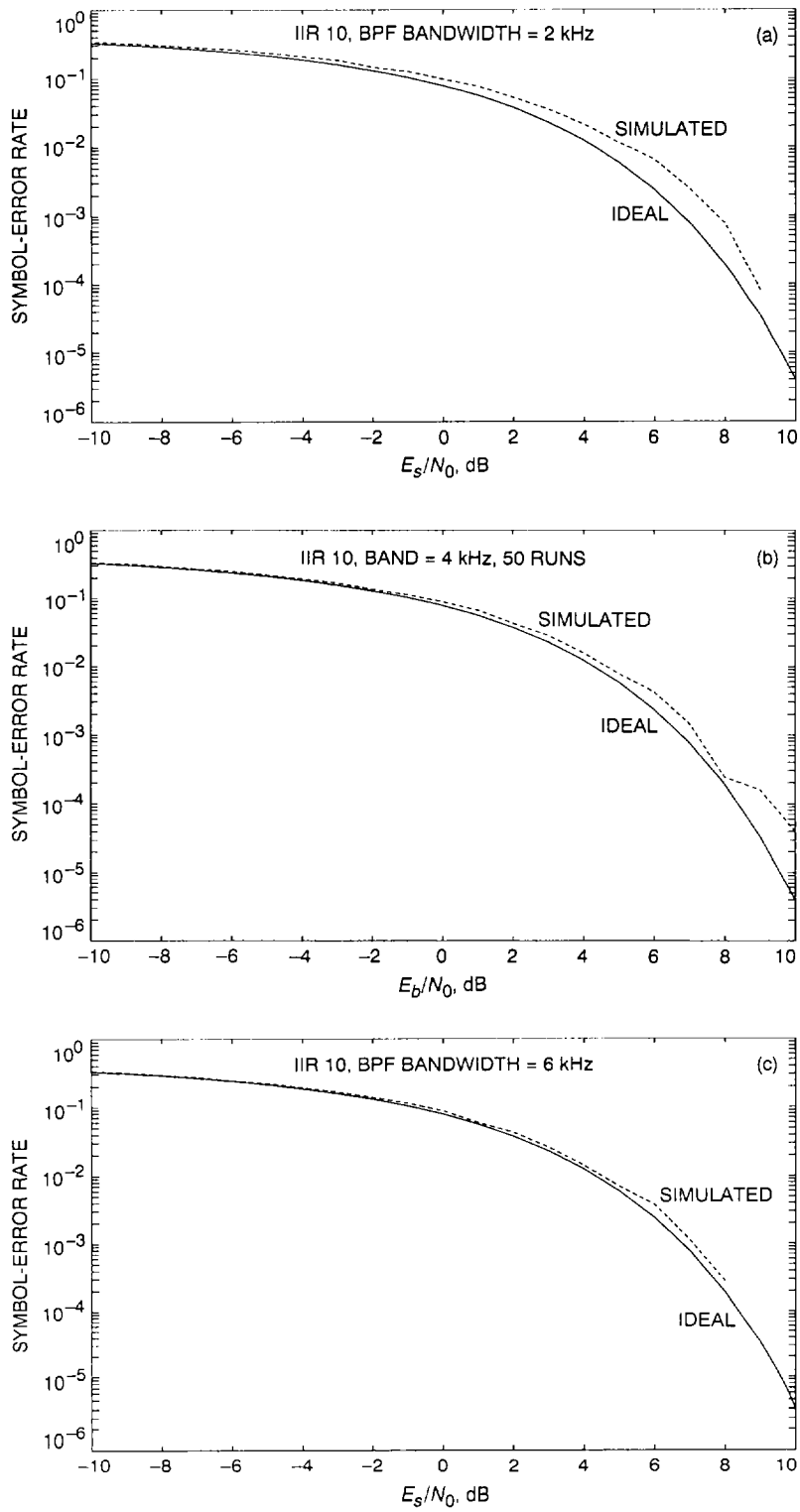
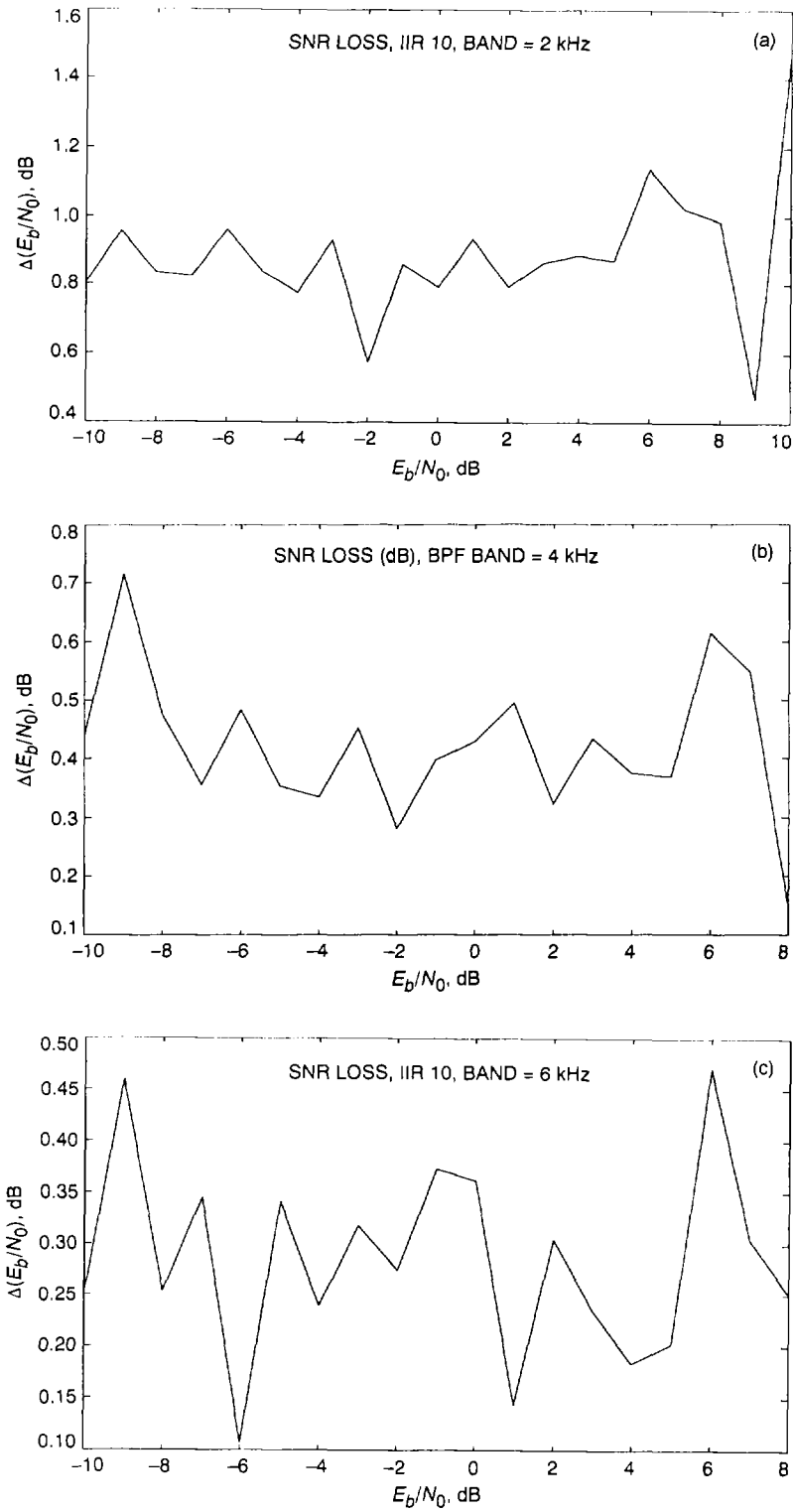


Fig. 2. Subcarrier downconversion and symbol detector.



**Fig. 3. Symbol-error rate versus symbol SNR for BPF bandwidth: (a) 2 kHz, (b) 4 kHz and (c) 6 kHz.**



**Fig. 4.  $\Delta$ (SNR) versus symbol SNR for BPF bandwidth:  
(a) 2 kHz, (b) 4 kHz and (c) 6 kHz.**



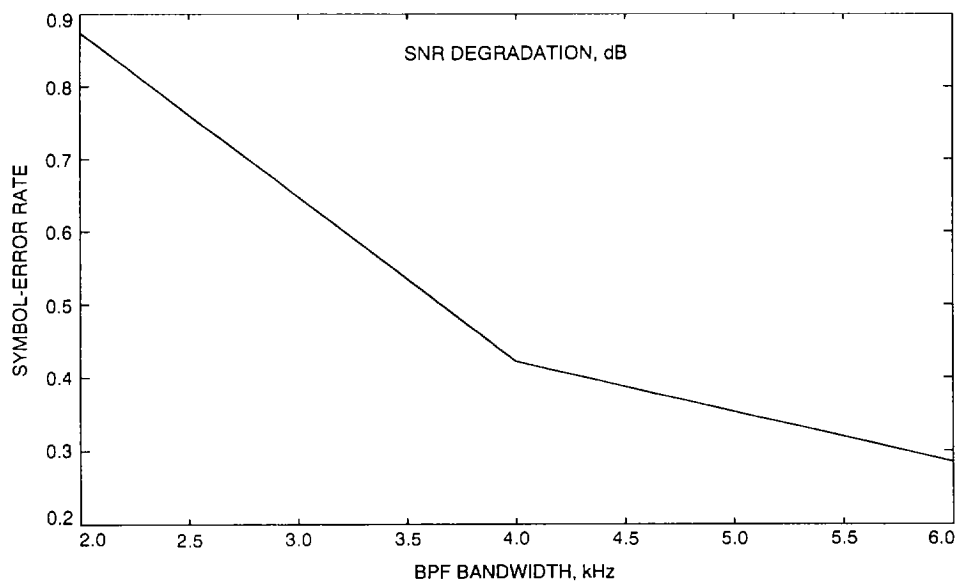


Fig. 5.  $\Delta$ (SNR) versus BPF bandwidth.