Enhanced Orbit Determination Filter: Inclusion of Ground System Errors as Filter Parameters

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The theoretical aspects of an orbit determination filter that incorporates ground-system error sources as model parameters for use in interplanetary navigation are presented in this article. This filter, which is derived from sequential filtering theory, allows a systematic treatment of errors in calibrations of transmission media, station locations, and Earth orientation models associated with ground-based radio metric data, in addition to the modelling of the spacecraft dynamics. The discussion includes a mathematical description of the filter and an analytical comparison of its characteristics with more traditional filtering techniques used in this application. The analysis in this article shows that this filter has the potential to generate navigation products of substantially greater accuracy than more traditional filtering procedures.

I. Introduction

In JPL's interplanetary orbit determination process, ground error sources associated with radio metric data, such as station location, Earth orientation, and transmission media calibration errors, are usually treated as unmodelled "consider" parameters in a scheme known as the consider option. This method does not utilize any information pertinent to the consider parameters in computing estimates of the trajectory parameters. Rather, the effects of the consider parameters are accounted for by modifying the computed estimation error covariance with preassigned uncertainties of these parameters. In many cases, this method gives satisfactory results and allows reasonable navigational accuracy. However, the consider option sometimes yields unstable and unpredictable results when used for interplanetary navigation.

Recently, an enhanced orbit determination scheme (which will be referred to throughout this article as the "enhanced" filter) has been developed. The enhanced filter explicitly models ground error sources as random processes simultaneously with the trajectory-related parameters. This method exploits the full information content of the data pertaining to these ground errors in the filtering process and may thereby improve the knowledge of these parameters and the overall accuracy of the estimated flight path.

The enhanced filter has been successfully applied to selected navigation problems in some interplanetary orbit-determination case studies at JPL [1,2]. These studies have shown an increase in orbit-determination accuracy using the enhanced filter method of factors of two to four over the use of more conventional techniques. Interestingly, in the field of Earth satellite orbit determination, similar filtering techniques have been in use for some time. Numerous articles exist in the literature. For example, in the article by Lichten and Border [3], stochastic models for the tropospheric delays are used for problems in Global Positioning System orbit determination.

This article describes the framework of the enhanced filter model and gives an analytical comparison of the enhanced filter with the traditional consider option. Since there are many concepts involved, a quick review of the basic filter model from which the enhanced filter evolved will be given first. A detailed discussion of the consider option will also be presented, so that its characteristics can be compared with those of the enhanced filter.

II. Basic Filter Model

All of the filters discussed herein employ a linear representation of the process dynamics and the measurements. In this formulation, the state space and measurements are described as follows:

$$\mathbf{x}_{j+1} = \Phi(j+1,j)\,\mathbf{x}_j + \mathbf{w}_j \tag{1}$$

$$\mathbf{z}_j = \mathbf{H}_j\,\mathbf{x}_j + \mathbf{v}_j \tag{2}$$

where $\mathbf{x}$ denotes the extended state, which includes the spacecraft state and the dynamic process perturbation parameters. The dynamic process perturbations may be treated as random or deterministic parameters. The transition matrix from time t_j to t_{j+1} is $\Phi(j+1,j)$, and $\mathbf{w}$ represents the state-space modelling errors. The jth data point is $\mathbf{z}_j$, and $\mathbf{v}_j$ is the corresponding data noise. The term $\mathbf{H}_j$ is a matrix that contains the partial derivatives of the jth data point with respect to the extended state.

The computations for the estimate and covariance matrix follow the basic square-root information filtering procedure discussed by Bierman [4]. This procedure can be described as follows: The a priori values and covariance matrix for the estimated parameters are first transformed into the a priori residual and the a priori information matrix. To ensure numerical stability, the Cholesky decomposition is applied to the a priori information matrix to obtain an upper triangular factorization, hence the term "square root" of the matrix. This a priori square-root information matrix is augmented by the a priori residual. Measurements are then included into this augmented matrix using the Householder orthogonal transformations, resulting in the following a posteriori square-root information and residual matrix:

$$\begin{pmatrix} \mathbf{R} & \mathbf{z} \\ & \mathbf{e} \end{pmatrix} \tag{3}$$

where $\mathbf{R}$ denotes the information matrix, $\mathbf{z}$ denotes the residual, and $\mathbf{e}$ denotes the sum of squares of the residual errors as defined in the classical least-squares problem.

The extended state estimate $\hat{\mathbf{x}}$ and the error covariance matrix $\mathbf{P}_x$ for the extended state are computed using the values in the above matrix.

$$\hat{\mathbf{x}} = \mathbf{R}^{-1}\mathbf{z} \tag{4}$$

$$\mathbf{P}_x = \mathbf{R}^{-1}\mathbf{R}^{-T} \tag{5}$$

III. Consider Option

The consider option is based on the assumption that there is a set of parameters that affects the performance of the filter and that it is unnecessary or impractical to model these parameters accurately. These parameters are referred to as consider parameters. Since consider parameters are not included in the model, they are not estimated. Moreover, the estimation and covariance computation process for the extended state is not aware of the presence of these parameters and any errors in their values. Instead, once the state estimation is performed and the error covariance computed, the consider filter modifies the computed error covariance to account for a constant uncertainty in the consider parameters. This new covariance is called the "consider covariance." If $\mathbf{P}_x$ denotes the computed error covariance matrix and $\mathbf{P}_{y_0}$ denotes the a priori covariance of the consider parameters, then the consider covariance, $\mathbf{P}_c$, is computed as follows:

$$\mathbf{P}_c = \mathbf{P}_x + \mathbf{S}\mathbf{P}_{y_0}\mathbf{S}^T \tag{6}$$

The matrix $\mathbf{S}$ above is commonly called the sensitivity matrix and contains the partial derivatives of the estimated state with respect to the consider parameters. In the consider filter, it is a function of the measurements only. More details of the sensitivity matrix can be found in [4].

The consider option is often used to treat ground error sources that affect the measurements. The rationalization is that since these error sources do not affect the spacecraft dynamics, estimating them will not improve the knowledge of the spacecraft state, and therefore it is adequate to just characterize them by their uncertainties. In this approach, the presence of the ground errors is acknowledged in a conservative fashion, which overlooks the important fact that these error sources, since they affect the measurements, do affect the spacecraft state estimates. Note also that this method assumes that the ground error parameters are constants with no dynamics. This simplified modelling does not allow an improvement in knowledge of these parameters to be obtained.

The most obvious disadvantage of the consider option is that when the sensitivity of the state with respect to the ground error parameters increases, the consider covariance increases, as can be seen clearly from Eq. (6). This implies a greater uncertainty in the spacecraft state. When this happens, the only remedy available within the context of the consider option is to decrease the "weight" of the data, i.e., assume that each measurement contains less information than it does in actuality. This and other characteristics of the consider option are discussed by Scheeres [5].

There is, however, a deeper significance here. If the sensitivity of the state with respect to the ground error parameters grows large, then this implies that there is significant information contained in the measurements concerning these parameters. The logical recourse would be to exploit this information in order to learn something about the characteristics of the ground error sources. This implies that the ground error parameters should be incorporated into the filter model.

IV. Enhanced Filter

The enhanced filter provides for the inclusion of the ground error parameters in the filter model so that the estimation process incorporates the information pertinent to the behavior of these parameters. In this treatment, the ground error sources are modelled as dynamic entities. By doing this, an automatic feedback mechanism is created so that no artificial data weighting is needed to keep the covariance of the state estimate from diverging.

A. Enhanced Filter Model

The enhanced filter uses the extended state model for the basic batch sequential model given by Eq. (1). In addition, there are models for ground error parameters and a modified measurement model. The ground error parameters are modelled as a discrete first-order Markov process in a vector form given by

$$\mathbf{y}_{j+1} = \Psi(j+1, j)\,\mathbf{y}_j + \mathbf{u}_j \tag{7}$$

where $\mathbf{y}$ denotes the vector consisting of the ground error parameters, $\Psi(j+1, j)$ accounts for the time dependency of $\mathbf{y}$ from time t_j to t_{j+1}, and $\mathbf{u}$ represents the random driving term.

The measurement model given by Eq. (2) is modified to include information pertaining to the ground error parameters.

$$\mathbf{z}_j = \mathbf{H}_j\,\mathbf{x}_j + \mathbf{G}_j\,\mathbf{y}_j + \mathbf{v}_j \tag{8}$$

where the meanings of $\mathbf{H}_j$ and $\mathbf{v}_j$ are the same as before and $\mathbf{G}_j$ contains the partial derivatives of the jth data point with respect to the ground error parameters.

The enhanced filter model thus consists of Eqs. (1), (7), and (8). The characteristics of this model can be described as follows. Ground error sources are treated as system parameters that can be estimated. The extended spacecraft state space evolves independently from the ground system parameters. The measurements have explicit dependency on the ground system parameters.

B. Estimate and Covariance

In the enhanced filter, estimated parameters include both the extended spacecraft state and the ground system parameters. The computations for the estimate and covariance matrix follow the basic square-root information filtering procedure described in Section II. After processing measurements, the a posteriori information and the residual matrix have the following form:

$$\begin{pmatrix} \mathbf{R}_x & \mathbf{R}_{xy} & \mathbf{z}_x \\ & \mathbf{R}_y & \mathbf{z}_y \\ & & \mathbf{e} \end{pmatrix} \tag{9}$$

In Eq. (9), subscripts are given to show the relationship of the quantities in the matrix with respect to the parameters. Here, $\mathbf{R}_x$ represents the information with respect to the extended state and is the same as $\mathbf{R}$ in Eq. (3); $\mathbf{R}_y$ represents the information with respect to the ground system parameters; $\mathbf{R}_{xy}$ denotes the information concerning the extended state affected by the ground system parameters; and $\mathbf{z}_x$ and $\mathbf{z}_y$ are residual components corresponding to the extended state and ground system parameters, respectively. From this matrix, the extended state and ground system parameter estimates are computed as

$$\hat{\mathbf{x}} = \mathbf{R}_x^{-1}[\mathbf{z}_x - \mathbf{R}_{xy}\hat{\mathbf{y}}] \tag{10}$$

$$\hat{\mathbf{y}} = \mathbf{R}_y^{-1}\mathbf{z}_y \tag{11}$$

Equations (10) and (11) show that the information content pertinent to the ground system parameters, $\mathbf{R}_y$ and $\mathbf{z}_y$, is used to estimate these parameters, yielding a new estimate, $\hat{\mathbf{y}}$. This estimate is then used to obtain a new state estimate, $\hat{\mathbf{x}}$, together with the information pertinent to the state, $\mathbf{R}_x$, the state with respect to the ground system parameters, $\mathbf{R}_{xy}$, and the residual, $\mathbf{z}_x$. In the consider option, the state estimate would have used only $\mathbf{R}_x$ and $\mathbf{z}_x$, and no estimate for the ground system parameters would have been computed.

Since all parameters are estimated, the covariance matrix for the extended state is then

$$\hat{\mathbf{P}} = \mathbf{R}_x^{-1}\mathbf{R}_x^{-T} + [-\mathbf{R}_x^{-1}\mathbf{R}_{xy}\mathbf{R}_y^{-1}]\,[-\mathbf{R}_x^{-1}\mathbf{R}_{xy}\mathbf{R}_y^{-1}]^T \tag{12}$$

Recognizing that $-\mathbf{R}_x^{-1}\mathbf{R}_{xy}$ is the definition of the sensitivity matrix, Eq. (12) may be represented as

$$\hat{\mathbf{P}} = \hat{\mathbf{P}}_x + \mathbf{S}\hat{\mathbf{P}}_y\mathbf{S}^T \tag{13}$$

In this equation, $\hat{\mathbf{P}}_x = \mathbf{R}_x^{-1}\mathbf{R}_x^{-T}$ and $\hat{\mathbf{P}}_y = \mathbf{R}_y^{-1}\mathbf{R}_y^{-T}$, where $\hat{\mathbf{P}}_x$ and $\hat{\mathbf{P}}_y$ represent the blocks of the computed covariance matrix corresponding to the extended state and the ground system parameters, respectively.

Using the same notations, the full covariance matrix corresponding to the estimate for both the extended state and the ground system parameters is

$$\begin{pmatrix} \hat{\mathbf{P}}_x + \mathbf{S}\hat{\mathbf{P}}_y\mathbf{S}^T & \mathbf{S}\hat{\mathbf{P}}_y \\ \hat{\mathbf{P}}_y\mathbf{S}^T & \hat{\mathbf{P}}_y \end{pmatrix} \tag{14}$$

Note that the $\hat{\mathbf{P}}_x$ in both Eqs. (13) and (14) is exactly the computed covariance in the consider option. Equation (13) has the same form as Eq. (6), which gives the computation for the consider covariance. But the important difference here is that the covariance for the ground system parameters, $\hat{\mathbf{P}}_y$ in Eqs. (13) and (14), is updated using the measurements and their modelled behavior, while in the consider option only the constant a priori covariance is used.

Further, in the enhanced filter, the full covariance matrix is used to automatically weight the measurement data, thus using the modelled behavior of ground system parameters to control the relative weight given to the measurements. In the consider option, only the computed covariance $\hat{\mathbf{P}}_x$ is used to weight the measurements; therefore, the ground system parameters have no effects on the data weights. In particular, using the enhanced filter formulation, the computed covariance stabilizes as the information contained in $\mathbf{R}_y$ increases. This is transparent since $\hat{\mathbf{P}}_y = \mathbf{R}_y^{-1}\mathbf{R}_y^{-T}$. Thus, there is no need to artificially deweight the data when the information content of the ground system parameters is large. On the other hand, when there is little or no information pertinent to the ground system parameters, the enhanced filter then behaves as in the consider option. In this case, the covariance matrix with respect to the ground system parameters, $\hat{\mathbf{P}}_y$, will consist primarily of the a priori constant covariance for these parameters.

C. Features of the Enhanced Filter

The advantage of modelling measurement error sources as system parameters is that this makes it possible to estimate these parameters. Thus, the knowledge of them may be improved. This approach distinguishes the enhanced filter strategy from other commonly used schemes that treat ground system errors as measurement noise. Many problems arise with such schemes. For example, because of the very assumption that the ground system errors are noise-like, no information regarding these parameters can be extracted from the measurements. Very often, this leads to degenerate covariance matrices; see [6,7] for more discussions on such treatments.

The advantage of modelling the evolution of the ground system parameters independently from the state dynamics is that, while not affecting the state evolution, the ground errors are allowed to evolve according to their own dynamics. This is necessary since some of the error sources, such as Earth orientation and transmission media, are dynamic. This model demonstrates that system parameters do not have to be included in the state dynamics in order to improve the state estimation. Finally, the advantage of modelling the dependency of the measurements on the ground system parameters explicitly is that the best possible weighting of the data can be utilized in the filtering process to generate estimates of the spacecraft trajectory.

The net combination of the above features provides a procedure that fully exploits the information content of the measurements pertaining to the ground system parameters and systematically assigns the proper weight for each measurement according to the modelled behavior. This is achieved in the enhanced filter by using user-input parameters that describe the stochastic nature of the ground system parameters. By choosing the statistics of these parameters properly, accurate estimates of both the ground system parameters and the spacecraft state may be obtained. In addition, the covariance of the ground system parameters may be effectively controlled, ensuring that the estimates for these parameters will be within the accuracy with which they can be modelled.

V. Conclusions

To summarize, the enhanced filter has been shown to offer some advantages over the traditional consider option in the following ways: First, the enhanced filter allows the behavior of the ground system parameters to affect the spacecraft state estimates. This helps to ensure that the spacecraft state is being estimated with a more complete representation of the physical world. Second, the enhanced filter can exploit the information contained in the data pertaining to the ground system parameters, possibly helping to improve the knowledge of these parameters. Moreover, the improved knowledge is fed back into the filtering process automatically, which effectively adjusts the weighting of the data systematically, helping to stabilize the state covariance. Third, the performance of the enhanced filter is no worse than that of the consider option in the case when the information content of the data with respect to the ground system parameters is small.

In reviewing the history and evolution of the sequential filtering techniques, a major motivation for using the consider option in the past was that it was cost effective, in terms of computation time, to deal with a lower dimensional model, even if the accuracy of the filtering product was compromised. With modern computers, this motivation is no longer a valid concern. The enhanced filter model can use this computational power to achieve an unprecedented degree of accuracy and robustness in many orbit determination problems.

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