

Correlator Data Analysis for the Array Feed Compensation System

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The real-time array feed compensation system is currently being evaluated at DSS 13. This system recovers signal-to-noise ratio (SNR) loss due to mechanical antenna deformations by using an array of seven Ka-band (33.7-GHz) horns to collect the defocused signal fields. The received signals are downconverted and digitized, in-phase and quadrature samples are generated, and combining weights are applied before the samples are recombined. It is shown that when optimum combining weights are employed, the SNR of the combined signal approaches the sum of the channel SNRs. The optimum combining weights are estimated directly from the signals in each channel by the Real-Time Block II (RTB2) correlator; since it was designed for very-long-baseline interferometer (VLBI) applications, it can process broadband signals as well as tones to extract the required weight estimates. The estimation algorithms for the optimum combining weights are described for tones and broadband sources. Data recorded in correlator output files can also be used off-line to estimate combiner performance by estimating the SNR in each channel, which was done for data taken during a Jupiter track at DSS 13.

I. Introduction

The advantages of array feed combining for recovering signal-to-noise ratio (SNR) lost to mechanical deformations of large receiving antennas have been described in previous articles [1,2]. Typically, SNR losses become significant when carrier wavelengths smaller than the design tolerance of the reflector are employed. This is the case in the DSN, where Ka-band (33.7-GHz) reception is contem-

plated with the large 34- and 70-m antennas, whose surfaces are subject to considerable deformation from gravity and wind.

The array feed compensation system currently being evaluated at DSS 13 has been designed to recover SNR losses at both low and high elevations, where losses are most severe. The idea is to collect some of the deflected

signal power using an array of feeds in the focal plane and to optimally recombine it for maximum SNR. In this implementation, combining is carried out on complex baseband samples using complex combining weights estimated in real time from the received signals. The estimates are obtained from 1-bit quantized samples processed by the Real-Time Block II correlator (RTB2) [3] over a specified time interval. The estimated weight vectors are supplied to a combining subsystem via an RS232 interface and used for combining over the following interval, during which time the next weight vector is estimated.

Independent confirmation of experimental results is always desirable, particularly when new concepts or techniques are analyzed. In the context of the array feed compensation system, the correlator can be used to predict combiner performance as well as to obtain real-time weight estimates from the 1-bit quantized bit stream. Direct comparison of the predicted and measured SNRs of the combined signal can be used to verify system performance. In the following sections, a simple expression will be derived for the SNR of the combined signal when accurate combining weights are available, and its implementation with recorded data will be described.

II. Optimum Combining Weights for Independent Noise

Suppose that the antenna points to a distant natural radio source (such as a planet or a quasar) that generates a broadband signal, and that an independent noise waveform is added to each channel of the array. The received RF signal in the k th channel may then be represented by

$$r_k(t) = s_k(t) + n_k(t) \quad k = 1, 2, \dots, K \quad (1a)$$

with source signal and background noise components

$$s_k(t) = \sqrt{2}S_k [a_c(t) \cos(\omega t + \theta_k) + a_s(t) \sin(\omega t + \theta_k)] \quad (1b)$$

$$n_k(t) = \sqrt{2}[n_{ck}(t) \cos(\omega t) + n_{sk}(t) \sin(\omega t)] \quad (1c)$$

where $a_c(t)$ and $a_s(t)$ are uncorrelated random processes representing the source signal, as are $n_{ck}(t)$ and $n_{sk}(t)$, which represent the background noise. The bandwidths

of these random processes are assumed to be narrow compared to the center frequency ω ; however, when downconverted to baseband, these processes are considered to be "broadband" signals. Since the source signals in the various channels differ from each other only in amplitude and phase (hence $a_c(t)$ and $a_s(t)$ are independent of k) and the time-varying envelopes are identical, the random processes $s_k(t)$ are correlated. The background noise processes are assumed to be uncorrelated in this analysis, as these consist of noise generated within the receivers plus background radiation arriving from different directions in space. (Actually, correlation coefficients on the order of 0.01 are typical between the feeds, probably due to near-field atmospheric noise; this issue is currently being investigated.) Following baseband downconversion and sampling, the in-phase and quadrature samples may be represented by the complex process

$$\tilde{r}_k(i) = \tilde{s}_k(i) + \tilde{n}_k(i) \quad (2)$$

where $\tilde{s}_k(i) = \tilde{S}_k \tilde{a}(i)$, and $\tilde{S}_k = S_k e^{j\theta_k}$. Each component of the complex noise is independent with variance σ_k^2 , but we shall assume that the real and imaginary components of $\tilde{a}(i)$ have variance 1/2 to avoid introducing additional scaling. Thus, the SNR of $\tilde{r}_k(i)$ becomes $S_k^2/2\sigma_k^2$.

Let $\{\tilde{w}_k\}$ be a set of complex weights and form the sum

$$\tilde{z}(i) = \sum_{k=1}^K \tilde{r}_k(i) \tilde{w}_k \quad (3)$$

where $\tilde{z}(i)$ is the weighted sum. With

$$\tilde{s}_c(i) = \sum_{k=1}^K \tilde{s}_k(i) \tilde{w}_k$$

denoting the combined source signal component, the SNR of the combined sequence can be defined as

$$SNR_C = \frac{\langle |\tilde{s}_c(i)|^2 \rangle}{\text{var} \left\{ \sum_{k=1}^K \tilde{w}_k \tilde{n}_k(i) \right\}} \quad (4)$$

where the numerator is the expected value of the magnitude squared of the combined signal, and the denominator is just the variance of the weighted noise sum. As shown in [1,2], the SNR of $\tilde{z}(i)$ is maximized when the complex weights $\{\tilde{w}_k\}$ are selected according to the formula

$$\tilde{w}_k = \frac{\tilde{S}_k^*}{2\sigma_k^2} \quad (5)$$

(The symbol * denotes a complex conjugate.) Substituting these complex combining weights into Eq. (4) yields

$$SNR_C = \frac{\left| \sum_{k=1}^K \tilde{S}_k \tilde{w}_k \right|^2}{\sum_{k=1}^K 2 |\tilde{w}_k|^2 \sigma_k^2} = \sum_{k=1}^K \frac{S_k^2}{2\sigma_k^2} \quad (6)$$

Thus, the SNR of the combined sequence is equal to the sum of the individual channel SNRs when optimum combining weights are used. The problem is that the optimum weights are not known a priori, so they must be estimated and, in the presence of noise, are always subject to error.

The expression for optimum combining weights for a continuous wave source is very similar to the natural radio source case. The continuous wave signal in each channel can be written as

$$s_k(t) = \sqrt{2}S_k \cos[\phi_{cw}(t) + \theta_k]$$

where $\phi_{cw}(t)$ is the RF signal phase. Following baseband downconversion and sampling, the in-phase and quadrature samples of the continuous wave signal may be represented by the complex process $\tilde{s}_k(i) = \tilde{S}_k e^{j\phi_{BB}}$, where $\tilde{S}_k = S_k e^{j\theta_k}$ and $\phi_{BB}(t)$ is the baseband signal phase. With these definitions, Eqs. (2) through (6) apply in the continuous wave case as well as in the natural radio source case.

III. Array Feed Combining Demonstration Setup

A schematic of the array feed combining demonstration is shown in Fig. 1. The signals in the seven feeds are amplified using high electron mobility transistor low-noise amplifiers (HEMT LNAs) and are sent to the DSS-13 control room after downconversion to IF. (Typically, the IF frequency is in the 250–300 MHz range.) Each of the seven IFs is split in two signals, with one signal fed directly into the Mark III VLBI data-acquisition terminal (Mark III DAT), where it is downconverted to baseband. The second signal goes into a “trombone” before being downconverted to baseband. The trombone is a waveguide whose length

can be adjusted so that, at IF, a narrowband signal passes through it 90 deg out of phase from the direct signal, giving rise to in-phase (I) and quadrature (Q) baseband signals for each of the seven feeds. After downconversion, the baseband signals are sent to the digital signal processor (DSP) subsystem for combining and to RTB2, where the combining weights are produced. The baseband signals are sampled at the Nyquist rate and single-bit quantized by the formatter of the Mark III DAT before being sent to RTB2. In the broadband signal case, the IF signal from the central feed is divided again and downconverted using a heterodyne frequency 10 kHz less than that used for the other 14 signals; this frequency-offset signal is also sent to RTB2. The 10-kHz offset ensures that both correlated signal amplitude and phase can be extracted from the correlation product, as in Eqs. (9a) through (9c). Finally, RTB2 produces complex combining weights, which are sent to the DSP where the 14 baseband signals are combined.

The real-time correlator is being used to compute (1) combining weights in real-time and (2) the SNR for the optimally combined array feed signal (performed off-line). The computation of the combining weights is described in Section IV. The computation of the expected optimally combined SNR is described in Section V.

IV. Computing the Combining Weights

The computation of the combining weights is discussed for two signal types: a continuous wave signal, such as a spacecraft carrier wave, and a broadband noise source.

RTB2 has two primary functions: (1) cross-correlating pairs of signal bit streams and (2) extracting tones (approximately sinusoidal signals) from individual signal bit streams. In the cross-correlation process, pairs of signal bit streams are multiplied together with a complex sinusoid and integrated. In the tone-extraction process, individual signal bit streams are multiplied by complex sinusoids and integrated. When weights are computed for a spacecraft carrier wave, tone extraction is used. When weights are computed for a broadband noise source, cross-correlation is used.

A. Computing Combining Weights for a Continuous Wave Source

Combining weights for a continuous wave source are computed as follows: RTB2 multiplies the in-phase signal

bit streams by the complex sinusoid $\exp(-j\phi_m(t))$ and integrates them, where $\phi_m(t)$ is a model for the signal phase, thus producing complex correlation sums for each feed. These have the expectation values [4]

$$\rho_k(t) = \sqrt{\frac{1}{\pi}} \sqrt{\frac{P_{sk}}{P_{nk}}} e^{j\theta_k} e^{j\phi_r(t)} \quad k = 1, \dots, 7 \quad (7)$$

where P_{sk} is the signal power for feed k ; P_{nk} is the noise power for feed k ; θ_k is a phase shift for feed k due to spatial propagation and electronic phase shifts [see Eq. (1b)]; and $\theta_k + \phi_r(t)$ is the residual of the true signal phase relative to the model signal phase, $\phi_m(t)$. The factor of π is a vestige of the single-bit quantized nature of the data going into RTB2. A low SNR assumption has been used in deriving Eq. (7). Three parameters are needed to compute the combining weights: each feed's SNR, phase shift, and average noise power. If the noise power P_{nk} is measured separately, the complex combining weights can be estimated as follows:

$$\tilde{w}_k = \frac{\rho_k^* \rho_1}{|\rho_1|^2} \sqrt{\frac{P_{n1}}{P_{nk}}} \quad k = 1, \dots, 7 \quad (8)$$

The overall phase and magnitude of the combining weights for the seven feeds are arbitrary and have been chosen so that the central feed (feed 1) has weight 1; this renders the weights dimensionless. In practice, the feeds have nearly identical noise power, so the last factor typically is ignored.

B. Computing Combining Weights for a Broadband Noise Source

Computation of combining weights for an unresolved far-field broadband noise source is somewhat more complicated than for continuous waves. Unlike the continuous-wave case, the single-bit signals from the feeds must be correlated with each other to measure the common noise signal from the source, leading to a requirement for additional information to get the SNR of each channel. Although not modeled in the derivation of the optimal weights above, an additional complication arises from the existence of low-level background signals common to the feeds (primarily near-field atmospheric noise) that must be accounted for when computing the weights. This is particularly important when observing weak sources, where the weight estimates could be corrupted by background noise correlation between the feeds.

RTB2 cross-correlates the signal from the frequency-offset central feed signal (described in Section III) with

each of the seven in-phase signal bit streams. That is, each of the seven in-phase signal bit streams is multiplied by the frequency-offset bit stream and integrated. The product stream will contain a 10-kHz signal from the source (introduced by the frequency offset described earlier). Before integration, the product stream is multiplied by a 10-kHz complex sinusoid so that the source signal will integrate coherently. The results of the cross-correlation are correlation sums for each of the seven feeds with expectation values [5]

$$\rho_1 = A_1 e^{j\theta_1} e^{-j\theta_{freq\ offset}} \quad (9a)$$

$$\rho_k = A_k e^{j\theta_k} e^{-j\theta_{freq\ offset}} + \rho_{bk} \quad k = 2, \dots, 7 \quad (9b)$$

where

$$A_k = \frac{a}{\pi} \sqrt{\frac{P_{sk} P_{s1}}{P_{Tk} P_{T1}}} \quad k = 2, \dots, 7 \quad (9c)$$

and $k = 1$ is the central feed. In feed k , P_{sk} is the source signal power and P_{Tk} is the total signal power (source power plus background noise power). The constant a equals 1.176 and is a vestige of the three-level sinusoid used by RTB2 in the cross-correlation process; $\theta_{freq\ offset}$ is the phase shift for the frequency-offset central feed signal; and ρ_{bk} is the correlation of the background noise in the central feed with the background noise in feed k . A low SNR assumption has been used in the derivation of Eq. (9c). The term ρ_1 is the correlation of the center feed signal with itself offset by 10 kHz; A_1 contains no SNR information since it results from the product of a single-bit quantized bit stream with itself.

In order to compute the combining weights, the background noise correlation ρ_{bk} must be removed from the correlation sums. Even after doing so, phase offset information is complete, but SNR information is not. These two problems are resolved by performing correlation and central-feed temperature measurements both on source and 84 mdeg off source in cross-elevation. The background noise correlation is nearly identical at angularly close points, so by performing the cross-correlation off source, ρ_{bk} can be measured and subsequently removed when back on source. Also, by measuring the central feed temperature on source and off source, the central feed SNR can be measured. With this information, it is then possible to compute the SNR in each feed.

The array feed combining experiments thus far have been performed during boresight tracks, in which the elevation boresights alternate with cross-elevation boresights.

Thus, on-source and slightly off-source pointing alternate regularly. The correlation ρ_{bk} is measured while off source and is used to calibrate the subsequent on-source correlation. This yields the calibrated on-source correlation sums

$$\rho_k^{cal} = A_k e^{j\theta_k} e^{-j\theta_{freq\ off\ set}} \quad k = 1, \dots, 7 \quad (10)$$

Combining this with measurements of the central feed SNR and noise voltage measurements, one can compute the combining weights

$$\tilde{w}_k = \frac{\rho_k^{cal*} \rho_1}{|\rho_k^{cal} \rho_1|} \sqrt{\frac{P_{sk}/P_{nk}}{P_{s1}/P_{n1}}} \sqrt{\frac{P_{n1}}{P_{nk}}} \quad k = 1, \dots, 7 \quad (11)$$

where the SNR in outer feed k is computed from

$$\frac{P_{sk}}{P_{nk}} = \frac{(P_{T1}/P_{s1}) (\pi/a)^2 (|\rho_k^{cal}|^2 - 2\sigma_c^2)}{1 - (P_{T1}/P_{s1}) (\pi/a)^2 (|\rho_k^{cal}|^2 - 2\sigma_c^2)} \quad k = 2, \dots, 7 \quad (12)$$

and where σ_c^2 is a correlation sum variance, negligible for a strong source, which is subtracted to remove a positive bias on the square magnitude $|\rho_k^{cal}|^2$ (see Appendix). The overall magnitude and phase of the weights are arbitrary and have been chosen to set \tilde{w}_1 equal to 1. In practice, the feeds have nearly identical noise power, so that the last factor in Eq. (11) is typically ignored.

V. Computation of Optimal Combined SNR

In Eq. (6) it is shown that, when the optimal weights are used, the SNR for the combined signal is the sum of the individual feed SNRs. By combining the direct radiometric measurement of the central feed SNR and the calibrated correlation sums ρ_k^{cal} , one can compute the combined SNR expected for optimal weighting. Correlation sums were written to a postcorrelation record file and processed off-line to produce this combined SNR.

The above computations have been carried out for a Jupiter boresight track on 1993 day-of-year (DOY) 335 using the 34-m antenna at DSS 13. Plotted in Fig. 2 are the central feed SNR and the optimal combined SNR, the sum of the central feed SNR plus the six ring-feed SNRs as computed from Eq. (12). Also plotted are modifications to the SNR plot, eliminating the effects of elevation-dependent noise power and signal attenuation.

The elevation dependence in the noise power was obtained directly from the off-source power measurements. The elevation-dependent signal loss was estimated using station weather data (temperature, pressure, and humidity) during tracking, computing a zenith attenuation coefficient, and applying this value to an atmospheric loss model. Note that even at a 42.5-deg elevation the SNR of the combined channels (SNR_C) exceeds that of the central channel (SNR_1) by about 0.5 dB. (SNR_1 has been effectively “smoothed” on a greater time scale than the SNR_C curve.) The corrected combined signal-to-noise ratio SNR_C^{**} decreases by only 0.54 dB at low elevations, while the corrected central channel loses about 1.12 dB of SNR at 9 deg. (This 34-m antenna is actually much better at Ka-band than the 70-m antennas used by the DSN, which suffer as much as a 5-dB SNR loss over the same elevation range [6]; hence, much greater SNR recovery can be expected for a 70-m antenna.) Thus, combining tends to provide uniform SNR over a range of elevations.

The rms scatter of the estimates of the combined SNR is roughly 0.035 dB. This is largely due to the inherent limit on accuracy imposed by the signal strength, the noise power in each channel, and the integration time. The variance of the estimates of the combined SNR (derived in the Appendix) is roughly

$$(\sigma_{SNR_C})^2 = \frac{3\pi^2}{2a^2 B T_{integration}} \frac{SNR_C - SNR_1}{SNR_1} \quad (13)$$

where B is the recorded bandwidth and $T_{integration}$ is the integration time, which is comparable with the results shown in Fig. 2.

VI. Summary and Conclusions

A technique has been described for estimating the optimum combining weights for the array feed compensation system, both for received tones and broadband signals, observed in the presence of additive noise. In the derivation of the optimum combining weights, the noise components in the various channels were assumed to be independent, even though correlations on the order of 1 percent are routinely observed. These low-level correlations are believed to be primarily due to near-field atmospheric radiation; their presence could introduce a large error into the weight estimates, particularly when weak broadband sources are observed, i.e., when ρ_{bk} is comparable to A_k in Eq. (9b). Therefore, the noise correlations are measured slightly off source and subtracted from ρ_k before computing the combining weights. The combining weights thus

obtained can then be used for combining the signals from all of the feeds. Modifications to the combining weights to account for noise correlations are currently being examined.

The ability to compute channel SNRs based on the correlation coefficients is useful for estimating the SNR of the combined signal, which is simply the sum of the channel SNRs when accurate combining weights are available. (If the combining weights are in error, a loss in combined SNR

will occur.) The above procedure has been carried out for a Jupiter track, from which we conclude that on the 34-m antenna at DSS 13 the combined channel loses only 0.54 dB in efficiency as the elevation varies from 43 deg down to 9 deg, after correction for elevation-dependent losses. This result confirms the ability of the array feed combining system to improve the efficiency response of large DSN antennas. The technique will be applied to subsequent tracks in order to accumulate data on combiner performance with a variety of sources under various conditions.

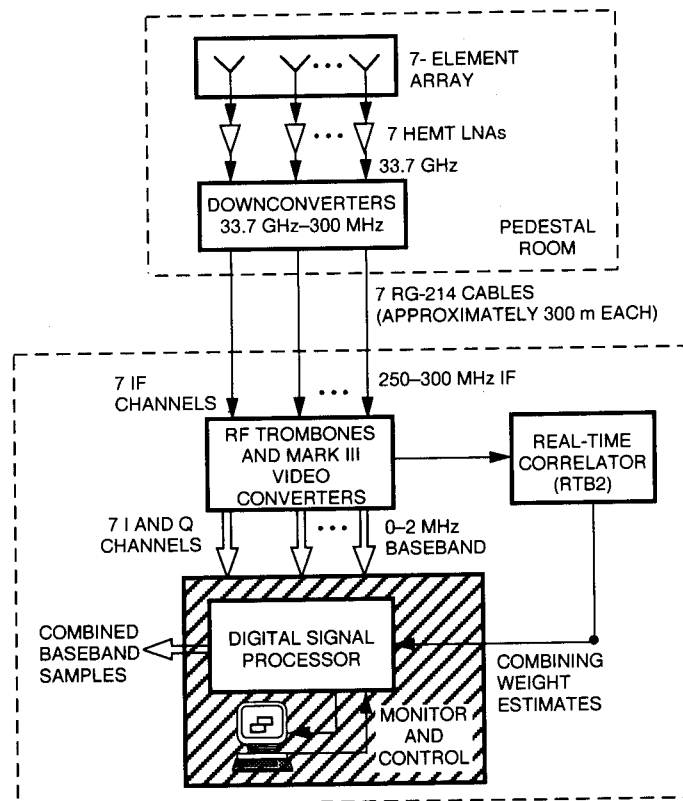


Fig. 1. Array feed combining system.

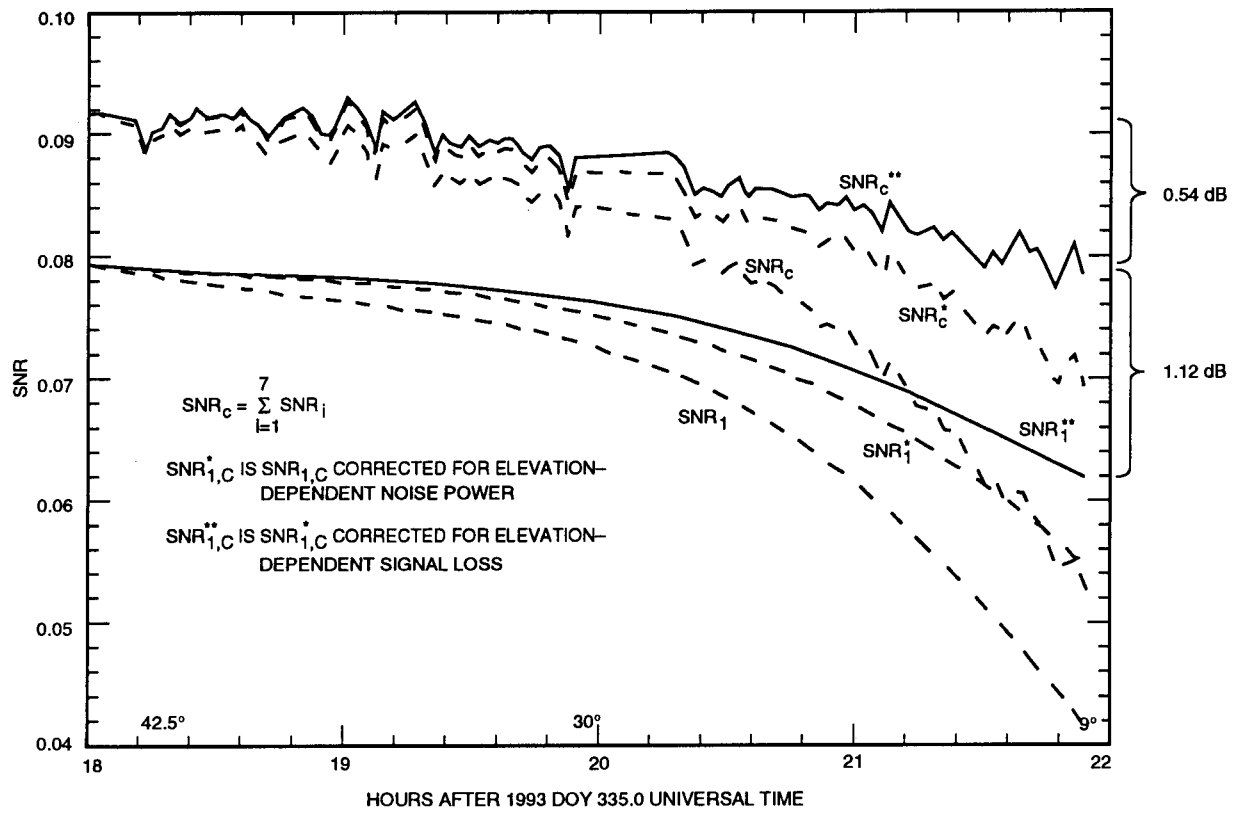


Fig. 2. SNR estimates for central (SNR_1) and combined (SNR_C) channels.

Appendix

Correlation Sum, Amplitude, and SNR Variances

The complex correlation sum ρ can be written as a two-component vector, $\rho = (c_0 + \delta c, s_0 + \delta s)$, where (c_0, s_0) is the expectation value of ρ , and δc and δs are deviations with variances $\langle \delta c^2 \rangle = \langle \delta s^2 \rangle = \sigma_c^2$ where $\langle \rangle$ indicates ensemble average. The correlation sum variance then can be written [7]

$$\sigma_c^2 = \frac{3}{8BT_{integration}} \quad (\text{A-1})$$

where B is the recorded bandwidth and $T_{integration}$ is the integration time. The expected value of $|\rho|^2$ is

$$\langle |\rho|^2 \rangle = c_0^2 + s_0^2 + 2\sigma_c^2 = A^2 + 2\sigma_c^2 \quad (\text{A-2})$$

where A is the amplitude in Eq. (9c) when ρ is the calibrated correlation sum in Eq. (10), so an unbiased estimator for A^2 is $|\rho|^2 - 2\sigma_c^2$. This is used in deriving Eq. (12). The variance of $|\rho|^2$ is

$$(\sigma_{|\rho|^2})^2 = 4A^2\sigma_c^2 \quad (\text{A-3})$$

This is used in the derivation of combined SNR uncertainty below.

The estimate of the total SNR in the outer feeds is approximately

$$SNR_O = \sum_{k=2}^7 \left(\frac{\pi}{a}\right)^2 A_i^2 \frac{1}{SNR_1} \quad (\text{A-4})$$

and its variance is

$$(\sigma_{SNR_O})^2 = \sum_{k=2}^7 \left[\left(\frac{\pi}{a}\right)^2 \frac{1}{SNR_1} \right]^2 (\sigma_{|\rho_i|^2})^2 \quad (\text{A-5})$$

Combining Eqs. (A-1), (A-3), and (A-5) gives

$$(\sigma_{SNR_O})^2 = \frac{3\pi^2}{2a^2 BT_{integration}} \frac{SNR_O}{SNR_1}$$

which, when rewritten, is the optimal combined SNR variance given in Eq. (13).

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