# CDMA With Interference Cancellation for Multiprobe Missions

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*Code division multiple-access spread spectrum has been proposed for use in future multiprobe/multispacecraft missions. This article considers a general parallel interference-cancellation scheme that significantly reduces the degradation effect of probe (user) interference but with a lesser implementation complexity than the maximum-likelihood technique. The scheme operates on the fact that parallel processing simultaneously removes from each probe (user) the total interference produced by the remaining most reliably received probes (users) accessing the channel. The parallel processing can be done in multiple stages. The proposed scheme uses tentative decision devices with different optimum thresholds at the multiple stages to produce the most reliably received data for generation and cancellation of probe/spacecraft interference. The one-stage interference cancellation was analyzed for two types of tentative decision devices, namely, hard and null zone decisions. Simulation results are given for one- and two-stage interference cancellation for equal as well as unequal received power probes.*

## I. Introduction

Historically, the Pioneer Venus Mission employed frequency division multiple-access (FDMA) multiplexing to prevent any possible interference between the four probe signals and the two redundant flyby spacecraft bus signals. A 2-MHz open-loop recording bandwidth was sufficient to capture all six S-band (2.3-GHz) signals. Future missions have been proposed with significantly more probes and use of X-band for improved radio metric tracking. The addition of the higher X-band (8.4-GHz) frequency with higher Doppler shifts, and the much larger number of probes,[1] implies that a much larger bandwidth would be required if an FDMA scheme were used. As a result, Charles D. Edwards proposed that code division multiple-access (CDMA) multiplexing be used for future multiprobe missions.[2] This approach has the advantage of greatly reducing the required open-loop recording bandwidth, reducing the cost of the

[1] For example, the Venus Multiprobe Mission (VMPM) is a proposed Discovery Mission concept that delivers 18 small probes into the Venus atmosphere and two redundant flyby spacecraft bus signals. The science goal of the mission is to understand the superrotation of the Venusian atmosphere, which causes the clouds of Venus to rotate 60 times faster than the surface.

[2] C. Edwards, "VMPM Wind Experiment," viewgraph presentation (internal document), Jet Propulsion Laboratory, Pasadena, California, February 24, 1994.

recording systems and the complexity of the postencounter data reduction, and simplifying the probe design by allowing all probes to use the same ultrastable oscillator frequency and identical transmitter structure, with the exception that different seeds should be used for the pseudorandom noise (PN) codes.

In the following, the term "user" will be used for "probe," "spacecraft," or any direct sequence spread-spectrum transmitter that communicates with a central receiving Earth station, usually called a "base-station receiver."

Multiuser communications systems that employ CDMA exhibit a user capacity limit in the sense that there exists a maximum number of users that can simultaneously communicate over the channel for a specified level of performance per user. This limitation is brought about by the ultimate domination of the other user interference over the additive thermal noise. Over the years, researchers have sought ways to extend the user capacity of CDMA systems either by employing optimum (maximum-likelihood) detection [1] or interference-cancellation methods [2–4]. In this article, we discuss a general parallel interference-cancellation scheme that significantly reduces the degradation effect of user interference but with a lesser implementation complexity than the maximum-likelihood technique. The proposed scheme operates on the fact that parallel processing simultaneously removes from each user the total interference produced by the remaining reliably received users accessing the channel. In this way, each user in the system receives equal treatment in so far as the attempt is made to completely cancel his or her multiple user interference.

When compared with classical CDMA, which has no interference cancellation, and also with the successive (serial) interference-cancellation technique previously proposed by Viterbi [3], in which user interference is sequentially removed one user at a time (the first user sees all of the interference and the last user sees none), the parallel cancellation scheme discussed here achieves a significant improvement in performance. Aside from increasing the user capacity, the parallel cancellation scheme has a further advantage over the serial cancellation scheme with regard to the delay necessary to fully accomplish the interference cancellation for all users in the system. Since in the latter the interference cancellation proceeds serially, a delay on the order of $M$ bit times ($M$ denotes the number of simultaneous users in the CDMA system) is required, whereas in the former, since the interference cancellation is performed in parallel for all users, the delay required is only 1 bit time (for a single-stage scheme).

## II. Single-Stage Interference Cancellation

### A. Tentative Hard Decisions—Equal Power, Synchronous Users

We consider first the performance of the single-stage parallel interference-cancellation scheme illustrated in Fig. 1, where the tentative decision devices associated with each user are 1-bit quantizers (hard decisions). This particular case corresponds to the scheme proposed in [2] and [4]. We assume that all users have the same power; thus, it is sufficient to characterize only the performance of any one user, say the first, which will be typical of all the others. Furthermore, we assume that all users have synchronous data streams and purely random PN codes.[3] While the assumption of synchronous users is perhaps unrealistic from a practical standpoint, it can be shown that the synchronous user case results in worst-case performance and thus serves as a lower bound on the user capacity achievable with this scheme. Alternately stated, any degree of data asynchronism among the users will yield a better performance, e.g., more users capable of being supported for a given amount of signal-to-noise ratio (SNR) degradation, than that arrived at in this section.

In general, the received signal in Fig. 1 is the sum of $M$ direct-sequence binary phase shift key (BPSK) signals, each with power $S_i$, bit time $T_b$, PN chip time $T_c$, and additive white Gaussian noise with

[3] For very long linear feedback shift registers, PN codes can be assumed to be purely random.

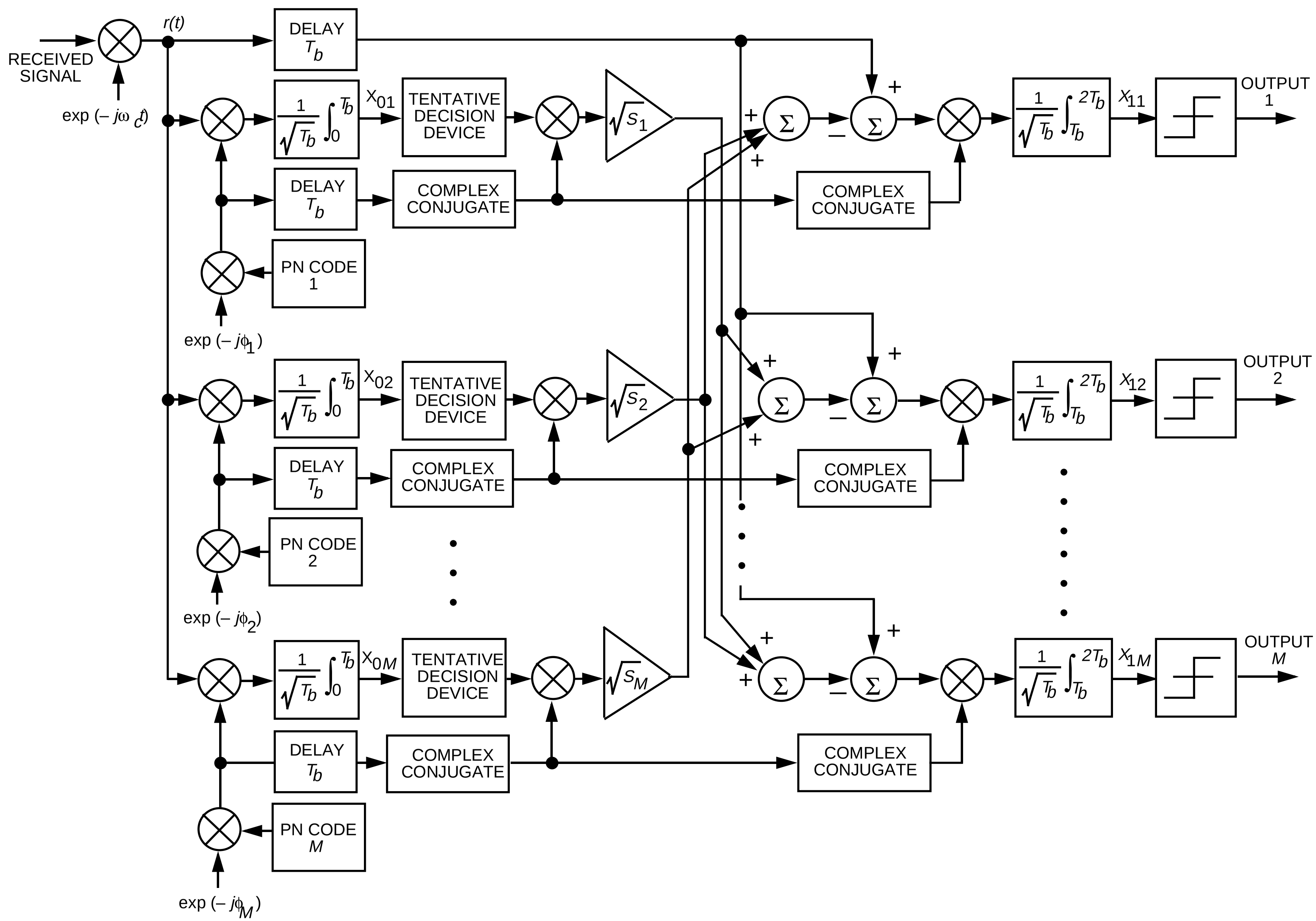


**Fig. 1. A single-stage interference cancellation scheme with parallel processing for CDMA (complex baseband model).**

single-sided power spectral density (PSD) $N_0$ W/Hz, which at baseband can be written in the complex form[4]

$$r(t) = \sum_{i=1}^{M} \sqrt{S_i} m_i(t) PN_i(t) e^{j\phi_i} + n(t) \tag{1}$$

where, for the $i$th user, $PN_i(t)$ is the PN code; $m_i(t) = \sum_{k=-\infty}^{\infty} a_{ki} p(t - kT_b)$ is the modulation with the $k$th bit $a_{ki}$ taking on equiprobable values $\pm 1$ and unit power rectangular pulse shape $p(t)$ of duration $T_b$; and $\phi_i$ is the carrier phase. For our case of interest here, $S_i = S; i = 1, 2, \cdots, M$. After despreading and demodulating[5] r(t) with user 1's PN code and carrier reference signal (both of these operations are assumed to be ideal), the normalized output of the integrate-and-dump (I&D) circuit is given by

$$x_{01} = a_{01}\sqrt{E_b} + n_1 e^{-j\phi_1} + \sum_{i=2}^{M} a_{01} n_{1i} e^{j(\phi_i - \phi_1)} \triangleq a_{01}\sqrt{E_b} + n_1 e^{-j\phi_1} + \sqrt{E_b}\sum_{i=2}^{M} a_{0i}\gamma_{1i} e^{j(\phi_i - \phi_1)} \tag{2}$$

where $E_b = ST_b$ denotes the bit energy; $a_{0i}$ is the polarity of user $i$'s bit in the interval $0 \leq t \leq T_b; n_1 = 1/(\sqrt{T_b}) \int_0^{T_b} n(t) PN_1(t) dt$ is a zero-mean complex Gaussian random variable with variance $E\{|n_1|^2\} = N_0$ representing the thermal noise; and $n_{1i} = \sqrt{S/T_b} \int_0^{T_b} PN_1(t) PN_i(t) dt \triangleq \sqrt{E_b}\gamma_{1i}; i = 2, 3, \cdots, M$ are the interference noises contributed by the other $M - 1$ users, which are modeled as independent zero-mean Gaussian random variables, each with variance $ST_c$.[6] Also, the first subscript on $x$ denotes the stage at which we are observing the I&D output, while the second subscript denotes the particular user. This notation will be useful later on in our discussion of multiple-stage cancellation schemes. The foregoing modeling of user interference as additive Gaussian noise follows from the assumptions made in a similar analysis of a CDMA system [5], namely, a large spreading ratio, $\eta = T_b/T_c$, and purely random PN codes.

*Tentative* hard decisions are made on the signals, $x_{0i}; i - 1, 2, \cdots, M$, and are used in an attempt to cancel the other user interference. If a correct tentative decision is made on a particular other user's bit, then the interference from that user can be completely cancelled. On the other hand, if an incorrect tentative decision is made, then the interference from that user will be enhanced rather than cancelled. A quantitative description of this will be given when we model the signal upon which final decisions are made. As we shall see, the performance analysis associated with this model is complicated by the fact that *the tentative decisions are not independent of one another*. More about this shortly.

After respreading/remodulation, interference cancellation, and despreading/demodulation, the normalized output of the I&D corresponding to the *final* decisions is given by

$$x_{11} = a_{01}\sqrt{E_b} + n_1 e^{-j\phi_1} + \overbrace{\sqrt{E_b}\sum_{i=2}^{M} \beta_i \gamma_{1i} e^{j(\phi_i - \phi_1)}}^{I_1} \tag{3}$$

where

[4] For convenience, we shall use complex notation to represent the various signals in the receiver.

[5] Since we are working with a baseband model, the term "remodulation" or "demodulation" refers to complex multiplication by the particular user's carrier phase or its complex conjugate, respectively.

[6] The normalized interference noises $\gamma_{1i}; i = 2, 3, \cdots, M$ have variance equal to the reciprocal of the spreading ratio, i.e., $\eta^{-1} = T_c/T_b$.

$$\beta_i = a_{0i} - \text{sgn}\left[Re\left\{\sqrt{E_b}\left(a_{0i} + \sum_{\substack{m=1\\m\neq i}}^{M} a_{0m}\gamma_{im}e^{j(\phi_m-\phi_i)}\right) + n_i e^{-j\phi_i}\right\}\right] \tag{4}$$

is a three-valued (0, ±2) indicator random variable whose magnitude represents whether or not a correct tentative decision is made on the $i$th user's bit. It is tempting to model the $\beta_i$'s as independent random variables. Unfortunately, this leads to optimistic results (when compared with the true performance results obtained from simulation). In addition to the fact that the $\beta_i$'s are not themselves independent, they are also dependent on the PN cross-correlations, i.e., the $\gamma_{1i}$'s. Fortunately, however, the $\beta_i$'s are not strongly dependent, i.e., the only terms that preclude *complete* independence of, say, $\beta_i$ and $\beta_j$, are $a_{0j}\gamma_{ij}$ in $\beta_i$ and $a_{0i}\gamma_{ji} = a_{0i}\gamma_{ij}$ in $\beta_j$. Hence, for sufficiently large $M$, it is reasonable to assume a Gaussian model for the total residual (after cancellation) interference term in Eq. (3). The accuracy of this model will improve as $M$ increases (actually, as the number of nonzero terms in $I_1$ increases, which implies a high tentative-decision error rate). We shall be more detailed about this issue later on when comparing the performance results derived from this analytical model with those obtained from a true computer simulation of the receiver.

Assuming then a Gaussian model for $I_1$ (note that $I_1$ is not zero mean), then the average probability of error associated with the final decisions is given by

$$\begin{aligned} P_b(E) &= \frac{1}{2}Pr\left\{Re\{x_{11} > 0|_{a_{01}=-1}\}\right\} + \frac{1}{2}Pr\left\{Re\{x_{11} < 0|_{a_{01}=1}\}\right\} \\ &= Pr\left\{Re\{x_{11} > 0|_{a_{01}=-1}\}\right\} = Pr\left\{N_t > \sqrt{E_b} - \sqrt{E_b}\sum_{i=2}^{M}\overline{\beta_i\gamma_{1i}\cos(\phi_i-\phi_1)}\right\} \end{aligned} \tag{5}$$

where[7]

$$N_t = Re\{n_1 e^{-j\phi_1} + I_1 - \overline{I_1}\} = N_1 + \sqrt{E_b}\sum_{i=2}^{M}\beta_i\gamma_{1i}\cos(\phi_i-\phi_1) - \sqrt{E_b}\sum_{i=2}^{M}\overline{\beta_i\gamma_{1i}\cos(\phi_i-\phi_1)} \tag{6}$$

is the effective noise seen by user 1 after cancellation, which in view of the above, is modeled as a real zero-mean Gaussian noise random variable whose thermal noise component $N_1$ has variance $\sigma_{N_1}^2 = N_0/2$. It is straightforward to compute the variance of $N_t$ as

$$\begin{aligned} \sigma_{N_1}^2 = {} & E_b(M-1)\overline{\beta_i^2\gamma_{1i}^2\cos^2(\phi_i-\phi_1)} - E_b(M-1)^2\left(\overline{\beta_i\gamma_{1i}\cos(\phi_i-\phi_1)}\right)^2 \\ & + E_b(M-1)(M-2)\overline{\beta_i\gamma_{1i}\beta_j\gamma_{1j}\cos(\phi_i-\phi_1)\cos(\phi_j-\phi_1)} \end{aligned} \tag{7}$$

where $i$ can take on any value from the set $2, 3, \cdots, M$. Hence, from Eq. (5), the average probability can be obtained as

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[7] To simplify the notation here and in what follows, it is understood that the statistical mean $\overline{\beta_i\gamma_{1i}\cos(\phi_i-\phi_1)}$ is computed under the hypothesis $a_{01} = -1$.

$$P_b(E) = Q\left(\sqrt{\frac{2E_b}{N_0}\Lambda}\right) \tag{8}$$

where

$$\Lambda \triangleq \frac{\left(1-(M-1)\overline{\xi_{1i}}\right)^2}{1+2(E_b)/(N_0)(M-1)\left[\overline{\xi_{1i}^2}-(M-1)\left(\overline{\xi_{1i}}\right)^2+(M-2)\overline{\xi_{1i}\xi_{1j}}\right]}; \xi_{1i} \triangleq \beta_i \gamma_{1i}\cos(\phi_i - \phi_1) \tag{9}$$

is an SNR degradation factor (relative to the performance of a single BPSK user transmitting alone) and $Q(x)$ is the Gaussian probability integral defined by

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}}\int_x^\infty \exp\left(-\frac{y^2}{2}\right)dy \tag{10}$$

Thus, the evaluation of $P_b(E)$ reduces to the evaluation of the various statistical averages (moments) of $\xi_{1i}$ required in Eq. (9). These statistical averages, which must be performed over the Gaussian noise and interference random variables as well as the uniformly distributed carrier phases, are not trivial to compute. Nevertheless, they can be obtained in the form of definite integrals of tabulated functions with the following results:

$$\overline{\xi_{1i}} = \frac{1}{2\pi}\int_0^{2\pi}\sqrt{\frac{2\sigma^2\cos^2\phi}{\pi}\left(\frac{\alpha^2\sigma^2\cos^2\phi}{1+\alpha^2\sigma^2\cos^2\phi}\right)}\exp\left\{-\frac{\alpha^2}{2(1+\alpha^2\sigma^2\cos^2\phi)}\right\}d\phi \tag{11a}$$

$$\overline{\xi_{1i}^2} = \frac{1}{2\pi}\int_0^{2\pi}\sqrt{\frac{8}{\pi}}\sigma^4\cos^4\phi\left(\frac{\alpha^2}{1+\alpha^2\sigma^2\cos^2\phi}\right)^{3/2}\exp\left\{-\frac{\alpha^2}{2(1+\alpha^2\sigma^2\cos^2\phi)}\right\}d\phi$$

$$+\frac{1}{2\pi}\int_0^{2\pi}(4\sigma^2\cos^2\phi)Q\left(\frac{\alpha}{\sqrt{1+\alpha^2\sigma^2\cos^2\phi}}\right)d\phi \tag{11b}$$

$$\overline{\xi_{1i}\xi_{1j}} = \left(\frac{1}{2\pi}\right)^2\int_0^{2\pi}\int_0^{2\pi}\frac{2}{\pi}\frac{\sigma^4\cos^2\phi_1\cos^2\phi_2\sqrt{B_1B_2}}{\sqrt{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}}$$

$$\times\exp\left\{-\frac{1}{2}\left[\frac{B_1+B_2}{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}\right]\right\}d\phi_1 d\phi_2 \tag{11c}$$

where

$$B_i = \frac{\alpha'^2}{1+\alpha'^2\sigma^2\cos^2\phi_i}; \quad i=1,2$$

with

$$\alpha \triangleq \sqrt{\frac{2(E_b/N_0)_R}{1 + ((M-2)/\eta)\,(E_b/N_0)_R}}$$

$$\alpha' \triangleq \sqrt{\frac{2(E_b/N_0)_R}{1 + ((M-3)/\eta)\,(E_b/N_0)_R}} \tag{12}$$

$$\sigma^2 = \frac{1}{\eta} = \frac{T_c}{T_b}$$

where $(E_b/N_0)_R$ denotes the *required* bit energy-to-noise spectral density ratio for $M$ users communicating simultaneously, each of which operates at an average bit error rate $P_b(E)$.

It is common in analyses of CDMA systems [5] to define a degradation factor (loss), $D$, as the ratio (in dB) of the $E_b/N_0$ required to achieve a given bit error rate in the presence of $M$ users, namely, $(E_b/N_0)_R$, to that which would be required to achieve the same level of performance if only a single user were communicating, namely, $(E_b/N_0)_1$. By the definition of $(E_b/N_0)_1$, we have

$$P_b(E) = Q\left(\sqrt{2(E_B/N_0)_1}\right) \tag{13}$$

To obtain the degradation factor for a given value of $P_b(E)$, we substitute $D(E_b/N_0)_1 = D \times \left[(1/2)[Q^{-1}(P_b(E))]^2\right]$ for $(E_b/N_0)_R$ in Eq. (12), which in turn is substituted in Eq. (11). Then, using the given value of $P_b(E)$, one can solve for $D$. Unfortunately, a closed-form expression for $D$ cannot be obtained, so the results will be obtained numerically. Before presenting these numerical results, however, we briefly review the analogous results for conventional CDMA and the successive (serial) interference-cancellation scheme proposed by Viterbi [3] (later patented by Dent [6]), since we shall use these as a basis of comparison to demonstrate the increased effectiveness of parallel cancellation.

**1. Comparison With Conventional CDMA and Successive Interference Cancellation.** In a conventional CDMA system, there is no attempt made to cancel the other user interference. Hence, $(E_b/N_0)_1$ is given by

$$\left(\frac{E_b}{N_0'}\right)_1 = \frac{(E_b)_R}{N_0 + (M-1)ST_c} = \frac{(E_b/N_0)_R}{1 + (M-1)_\eta^{-1}(E_b/N_0)_R} \tag{14}$$

Thus, the degradation factor, $D$, is [5]

$$D = \frac{(E_b/N_0)_R}{(E_b/N_0)_1} = \frac{1}{1 - (M-1)\eta^{-1}(E_b/N_0)_1} \tag{15}$$

For the successive cancellation scheme [3], Viterbi showed that to guarantee that each user in the system sees the same amount of interference from the other users, the user powers should be assigned as

$$S_k = S_1\left(1 + \frac{S_1 T_b}{N_0}\eta^{-1}\right)^{k-1}, \quad k = M, M-1, \cdots, 2 \tag{16}$$

where $S_1$ is the power of the user to be processed last (the weakest one) and $S_M$ is the power of the user to be processed first (the strongest one). Distributing the powers as in Eq. (16) ideally guarantees that all users see the same ratio of signal power to effective noise spectral density and, thus, the user to be processed first (the one that sees all the user interference) is not at any SNR disadvantage relative to the user to be processed last (the one for which all interference has been removed). In view of the above, the degradation factor for the $k$th user is given by

$$D_k = \frac{(E_b/N_0)_{R_k}}{(E_b/N_0)_1} = \frac{S_k}{S_1} = \left(1 + \eta^{-1}(E_b/N_0)_1\right)^{k-1} \tag{17}$$

where $(E_b/N_0)_{R_k}$ denotes the required bit energy-to-noise spectral density ratio for the $k$th user. The *average* degradation factor, $D$, for the $M$ user system is obtained by averaging Eq. (17) over $k$, which yields

$$D = \frac{1}{M}\sum_{k=1}^{M} D_k = \frac{\left(1 + \eta^{-1}(E_b/N_0)_1\right)^M - 1}{M\eta^{-1}(E_b/N_0)_1} \tag{18}$$

It should be emphasized that the result in Eq. (18) ignores the effect of decision errors made at the various successive interference-cancellation stages; that is, the interference cancellation is assumed perfect. As a result, numerical results derived from Eq. (18) will be optimistic when compared to the actual performance of the scheme.

**2. Numerical Results.** To illustrate the significant performance advantage of the parallel interference-cancellation scheme in Fig. 1, we consider a plot of $D$ versus $M$ for an average bit error probability,[8] $P_b(E) = 10^{-2}$, and a spreading ratio, $\eta = 100$. Figure 2 shows the analytical performance of the three schemes (conventional, successive interference cancellation, and parallel interference cancellation) as well as computer simulation results for the latter. We see that for the conventional and parallel interference-cancellation schemes there exists a user capacity limit in that regardless of how much one is willing to increase $(E_b/N_0)_R$ (for a given $(E_b/N_0)_1$, or equivalently, a given $P_b(E)$), the required bit error rate cannot be achieved if more than $M_{max}$ users simultaneously access the system. For conventional CDMA,

$$M_{max} = 1 + \frac{\eta}{(E_b/N_0)_1} = 1 + \frac{\eta}{(1/2)[Q^{-1}(P_b(E))]^2} \tag{19}$$

whereas for the parallel interference-cancellation scheme, the solution is determined from

$$10^{-2} = Q\left(\frac{1 - (M_{max}-1)\overline{\xi_{1i}}}{\sqrt{(M_{max}-1)\left[\overline{\xi_{1i}^2} - (M_{max}-1)\left(\overline{\xi_{1i}}\right)^2 + (M_{max}-2)\overline{\xi_{1i}\xi_{1j}}\right]}}\right) \tag{20}$$

together with the moments in Eq. (11), where now

$$\alpha \triangleq \sqrt{\frac{2}{\eta^{-1}(M_{max}-2)}}, \quad \alpha' \triangleq \sqrt{\frac{2}{\eta^{-1}(M_{max}-3)}} \tag{21}$$

[8] The value of $P_b(E) = 10^{-2}$ is chosen to allow for obtaining computer simulation results in a reasonable amount of time.

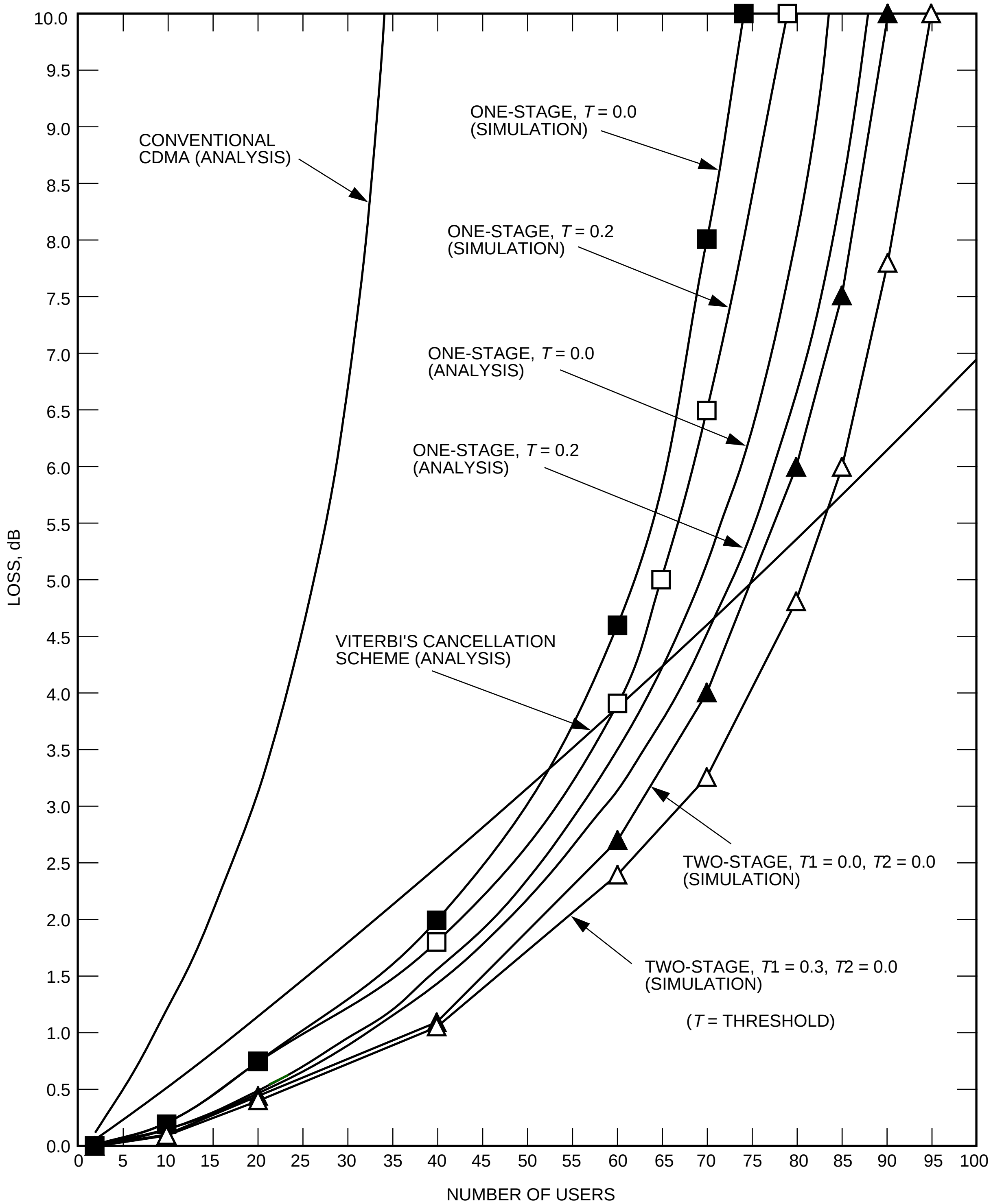


**Fig. 2. Performance of interference cancellation schemes with equal power users (Viterbi's scheme with unequal power users is also shown).**

It is emphasized that the user capacity limit for the parallel interference-cancellation scheme comes about entirely because of the finite probability of error associated with the tentative decisions. From Fig. 2, it appears that the successive interference cancellation does not have a user capacity limit. This is because in [3] it was assumed for this scheme that the interference cancellation is perfect, i.e., the effect of decision errors at the various interference-cancellation stages was not accounted for.

Comparing the analytical and simulation results for the parallel interference-cancellation scheme, we observe that the analytical results are somewhat optimistic. The discrepancy between the two stems from the assumption of an analytical Gaussian model for the total residual user interference in Eq. (3), whereas the computer simulation makes no such assumption and, thus, predicts the exact performance.

### B. Tentative Hard Decisions—Unequal Power, Synchronous Users

The results of the previous section can be generalized to the case where the users have unequal powers, i.e., $S_i; i = 1, 2, \cdots, M$. Let $\alpha_{ij} = S_i/S_j$ denote the ratio of the power of the $i$th user to that of the $j$th user, who is arbitrarily considered to be the desired user. After interference cancellation, the normalized output of the I&D corresponding to the *final* decisions of user $j$ is, by analogy with Eq. (3),

$$x_{1j} = a_{0j}\sqrt{E_{bj}} + n_j e^{-j\phi_j} + \overbrace{\sum_{\substack{i=1\\i\neq j}}^{M} \sqrt{E_{bi}}\beta_i \gamma_{ji} e^{j(\phi_i - \phi_j)}}^{I_j}$$

$$= a_{0j}\sqrt{E_{bj}} + n_j e^{-j\phi_j} + \sqrt{E_{bj}}\overbrace{\sum_{\substack{i=1\\i\neq j}}^{M} \sqrt{\alpha_{ij}}\beta_i \gamma_{ji} e^{j(\phi_i - \phi_j)}}^{I_j} \qquad (22)$$

where $n_j = (1/\sqrt{T_b})\int_0^{T_b} n(t)PN_j(t)dt, j = 1, 2, \cdots, M$ is a zero-mean complex Gaussian random variable with variance $N_0$ representing the thermal noise of the $j$th user; $\gamma_{ji} \triangleq (1/T_b)\int_0^{T_b} PN_j(t)PN_i(t)dt, i \neq j$ are the normalized interference noises of the other $M-1$ users as seen by user $j$ ($\gamma_{ji}$ has variance $\eta^{-1}$; see Footnote 6), and $E_{bi} \triangleq S_iT_b$ is the bit energy of the $i$th user. Also, analogous to Eq. (4), $\beta_i$ is now defined by

$$\beta_i = a_{0i} - \text{sgn}\left[Re\left\{\sqrt{E_{bi}}\left(a_{0i} + \sum_{\substack{m=1\\m\neq i}}^{M} \sqrt{\alpha_{mi}}a_{0m}\gamma_{im}e^{j(\phi_m - \phi_i)}\right) + n_i e^{-j\phi_i}\right\}\right] \qquad (23)$$

Following steps analogous to Eqs. (5) through (7), we arrive at the desired result for the bit error probability of the desired (the $j$th) user, namely,

$$P_{bj}(E) = Q\left(\sqrt{\frac{2E_{bj}}{N_0}\Lambda_j}\right) \qquad (24)$$

where (for $a_{0j} = -1$),

$$\Lambda_j \triangleq \frac{\left(1-\sum_{\substack{i=1\\i\neq j}}^{M}\sqrt{\alpha_{ij}}\,\overline{\xi_{ji}}\right)^2}{1+2(E_{bj}/N_0)\left[\sum_{\substack{i=1\\i\neq j}}^{M}\alpha_{ij}\,\overline{\xi_{ji}^2}-\left(\sum_{\substack{i=1\\i\neq j}}^{M}\sqrt{\alpha_{ij}}\,\overline{\xi_{ji}}\right)^2+\sum_{\substack{i=1\\i\neq j}}^{M}\sum_{\substack{m=1\\m\neq j,i}}^{M}\sqrt{\alpha_{ij}\alpha_{mj}}\,\overline{\xi_{ji}\xi_{jm}}\right]};\quad \xi_{ji}\triangleq\beta_i\gamma_{ji}\cos(\phi_i-\phi_j) \tag{25}$$

As an example, consider a group of $M$ users with powers exponentially distributed (linearly distributed on a dB scale) over a range of 10 dB between the minimum and the maximum. This model might correspond to a distribution of users that are exponentially distant from the base station within a cell. Assume that we fix the error probability of the *lowest power user* (assumed to be user 1 for convenience of notation) equal to $10^{-2}$ (all others would then obviously have a lower error probability). Then, Fig. 3 illustrates the degradation factor, $D_1$, of user 1 versus $M$. For comparison, the results corresponding to conventional CDMA with the same user power distribution are also shown in this figure. By comparing Fig. 3 with Fig. 2, we observe that in the unequal power case, parallel interference cancellation offers more of an advantage over conventional CDMA. The reason behind this observation is that the larger power of the other users (which are producing the user interference to user 1) produces tentative decisions with a smaller error probability, which in turn results in a better degree of cancellation with regard to the final decisions.

## III. Parallel Interference Cancellation Using Null Zone Tentative Decisions

Much like the idea of including erasures in conventional data detection to eliminate the need for making decisions when the SNR is low, one can employ a null zone hard-decision device [see Eq. (27)] for the tentative decisions to further improve the fidelity of the interference-cancellation process. The idea here is that when a given user's signal-to-interference ratio is low, it is better not to attempt to cancel the interference from that user than to erroneously detect his data bit and, thus, enhance his interference. Following the development in Section II.A for a single-stage scheme with equal-power synchronous users, then the normalized output of the I&D corresponding to the final decision on user 1's bit $a_{01}$ is still given by Eq. (3), with $\beta_i$ now defined by

$$\beta_i = a_{0i} - \mathrm{nsgn}\left[Re\left\{\sqrt{E_b}\left(a_{0i}+\sum_{\substack{m=1\\m\neq i}}^{M}a_{0m}\gamma_{im}e^{j(\phi_m-\phi_i)}\right)+n_ie^{-j\phi_i}\right\}\right] \tag{26}$$

where "nsgn" denotes the null zone signum function defined by

$$\mathrm{nsgn}\,x = \begin{cases} 1, & x>\zeta \\ 0, & -\zeta\le x\le\zeta \\ -1, & x<-\zeta \end{cases} \tag{27}$$

Here $\beta_i$ takes on possible values $(0,\pm1,\pm2)$, and its magnitude is an indicator of whether a correct decision is made ($i$th user's interference is perfectly cancelled), no decision is made ($i$th user's interference is unaltered), or an incorrect decision is made ($i$th user's interference is enhanced). Once again, a Gaussian assumption is made on the total residual interference; then, since the final decisions are still made as hard decisions, the average bit error probability is still given by Eq. (8) together with Eq. (9), with the statistical moments of $\xi_{1i}$ now given by

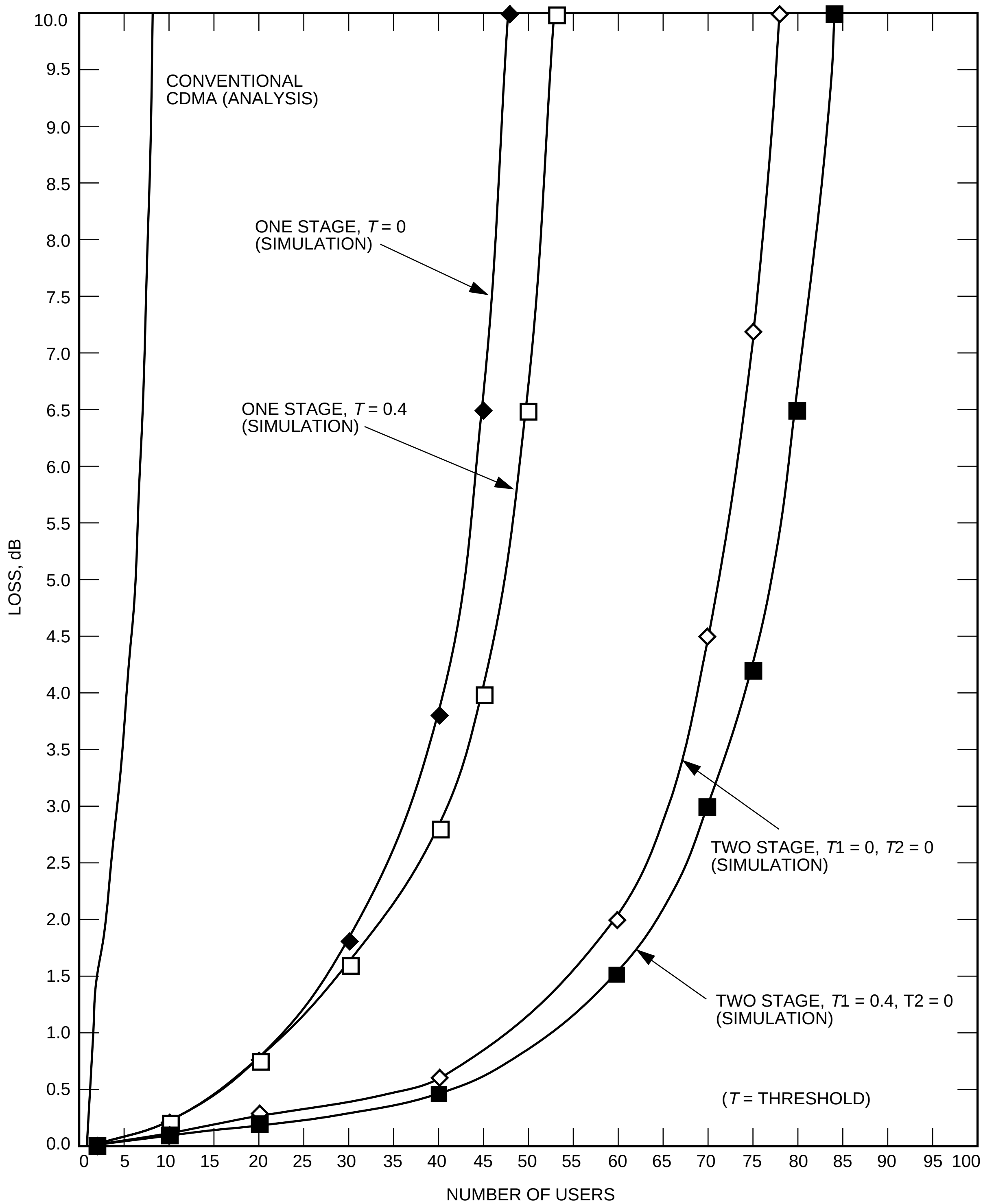


**Fig. 3. Performance of interference cancellation schemes with unequal power users.**

$$\overline{\xi_{1i}} = \frac{1}{2\pi}\int_0^{2\pi}\sqrt{\frac{\sigma^2\cos^2\phi}{2\pi}\left(\frac{\alpha^2\sigma^2\cos^2\phi}{1+\alpha^2\sigma^2\cos^2\phi}\right)}$$

$$\times\left[\exp\left\{-\frac{\alpha^2(1+\zeta')^2}{2(1+\alpha^2\sigma^2\cos^2\phi)}\right\}+\exp\left\{-\frac{\alpha^2(1-\zeta')^2}{2(1+\alpha^2\sigma^2\cos^2\phi)}\right\}\right]d\phi \tag{28a}$$

$$\overline{\xi_{1i}^2} = \frac{1}{2\pi}\int_0^{2\pi}\sqrt{\frac{1}{2\pi}}\sigma^4\cos^4\phi\left(\frac{\alpha^2}{1+\alpha^2\sigma^2\cos^2\phi}\right)^{3/2}$$

$$\times\left[3(1+\zeta')\exp\left\{-\frac{\alpha^2(1+\zeta')^2}{2(1+\alpha^2\sigma^2\cos^2\phi)}\right\}+(1-\zeta')\exp\left\{-\frac{\alpha^2(1-\zeta')^2}{2(1+\alpha^2\sigma^2\cos^2\phi)}\right\}\right]d\phi$$

$$+\frac{1}{2\pi}\int_0^{2\pi}(\sigma^2\cos^2\phi)\left[3Q\left(\frac{\alpha(1+\zeta')}{\sqrt{1+\alpha^2\sigma^2\cos^2\phi}}\right)+Q\left(\frac{\alpha(1-\zeta')}{\sqrt{1+\alpha^2\sigma^2\cos^2\phi}}\right)\right]d\phi \tag{28b}$$

$$\overline{\xi_{1i}\xi_{1j}} = \left(\frac{1}{2\pi}\right)^2\int_0^{2\pi}\int_0^{2\pi}\frac{1}{2\pi}\frac{\sigma^4\cos^2\phi_1\cos^2\phi_2\sqrt{B_1B_2}}{\sqrt{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}}$$

$$\times\left[\exp\left\{-\frac{1}{2}\left[\frac{(B_1+B_2)(1+\zeta')^2}{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}\right]\right\}\right.$$

$$+\exp\left\{-\frac{1}{2}\left[\frac{4\zeta'^2B_1B_2\cos^2(\phi_1-\phi_2)+(1+\zeta')^2B_1+(1-\zeta')^2B_2}{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}\right]\right\}$$

$$\left.+\exp\left\{-\frac{1}{2}\left[\frac{4\zeta'^2B_1B_2\cos^2(\phi_1-\phi_2)+(1-\zeta')^2B_1+(1+\zeta')^2B_2}{1+\sigma^2\cos^2(\phi_1-\phi_2)(B_1+B_2)}\right]\right\}\right]d\phi_1d\phi_2 \tag{28c}$$

where

$$B_i = \frac{\alpha'^2}{1+\alpha'^2\sigma^2\cos^2\phi_i};\quad i=1,2$$

and $\zeta' = \zeta/\sqrt{E_b}$ is the normalized decision threshold that should be chosen to minimize $D$ for a given $P_b(E)$ and $(E_b/N_0)_1$ determined from Eq. (13). Superimposed on the performance results for the hard limiter previously given in Fig. 2 are the results for the null zone limiter. For the specified processing gain and average bit error probability, we see that using a null zone limiter allows the maximum number of users that can be supported to be increased by about 10 percent. For convenience, the normalized threshold has been fixed at $\zeta' = 0.2$. For an unequal (exponentially distributed) power distribution among the users, the corresponding results using null zone tentative decisions are superimposed on those previously discussed in Fig. 3. For convenience, the normalized threshold has been fixed at $\zeta' = 0.4$. Here again we see a modest improvement in performance.

## IV. Multiple-Stage Interference Cancellation

The single-stage scheme of Fig. 1 can be improved upon by cascading multiple stages of parallel interference cancellation. The idea here is to repeatedly improve the fidelity of the $M$ tentative decisions since each successive stage sees less and less interference. Note that in principle this idea is similar to what Viterbi accomplishes in the serial interference-cancellation scheme except that here at each stage we simultaneously act on the interference from the most reliable users rather than one user at a time. An analysis of the performance of such a multistage scheme is difficult if not impossible to obtain due to the fact that the tentative decisions at the $i$th interference-cancellation stage depend on the tentative decisions at the $(i-1)$st stage. Because of this difficulty, numerical results for the performance of the multistage parallel interference scheme will be obtained from computer simulation. Illustrated in Figs. 2 and 3 are performance results for a two-stage parallel interference canceller with hard and null zone[9] tentative decisions, respectively. We observe that there is significant gain to be achieved by going to more than one stage.

## V. Conclusions

A parallel interference-cancellation scheme was proposed that uses tentative decision devices with different optimum thresholds at the multiple stages to produce the most reliably received data for generation and cancellation of user interference. The one-stage interference cancellation was analyzed for two types of tentative decision devices, namely, hard and null zone decision. Simulation results are given for one- and two-stage interference cancellation for equal as well as unequal power users. The results indicate that, by using multiple stages with optimum thresholds at each stage, performance can be significantly improved relative to conventional CDMA. Although linear tentative decisions can be used, the performance of such a scheme is inferior to one with nonlinear tentative decisions, as our simulations have shown. However, this scheme with noncoherent detection does not require amplitude and phase estimation.

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[9] In the null zone results of Fig. 2, the normalized threshold in the first stage has been fixed at $\xi' = 0.3$ and, in the second stage, it has been set equal to 0, i.e., a hard-limited tentative decision.