

# A New Model for Yaw Attitude of Global Positioning System Satellites

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*Proper modeling of the Global Positioning System (GPS) satellite yaw attitude is important in high-precision applications. A new model for the GPS satellite yaw attitude is introduced that constitutes a significant improvement over the previously available model in terms of efficiency, flexibility, and portability. The model is described in detail, and implementation issues, including the proper estimation strategy, are addressed. The performance of the new model is analyzed, and an error budget is presented. This is the first self-contained description of the GPS yaw attitude model.*

## I. Introduction

On June 6, 1994, the U.S. Air Force implemented a yaw bias on most Global Positioning System (GPS) satellites. By January 1995, the implementation was extended to all the satellites except SVN 10. The yaw bias was introduced as a way to make modeling of the yaw attitude of the GPS satellites during shadow crossings possible [2]. The yaw attitude of a biased GPS satellite during eclipse seasons is markedly different from the yaw attitude of a noneclipsing satellite or from that of an unbiased satellite. The yaw attitude of the GPS satellite has a profound effect on precise applications. Mismodeling the satellite attitude can cause decimeter-level error in the positioning of ground stations with certain GPS-based techniques and skew media calibrations. This required the development of a special attitude model for biased GPS satellites. In addition to the yaw bias effects, that model also corrected other mismodeling that existed in the old model, namely, that of the “noon turn.”

The first attitude model written for the biased constellation was made freely available to the GPS community in the form of a collection of FORTRAN routines [1]. For simplicity, this model is referred to in this article as GYM94 (for GPS Yaw Attitude Model—94). GYM94 was implemented in JPL’s GIPSY software and, in various forms, in other high-precision geodetic packages. The model was successfully used within JPL’s routine processing of daily GPS orbits and ground station coordinates for the International Global Positioning System Service (IGS). The model had some drawbacks, though. Mainly, it was cumbersome to implement and very demanding of computer resources, namely, memory and central processing unit (CPU) time.

In this article, we describe a new model for the GPS satellite attitude, referred to as GYM95. The model is analytic, in contrast to the numerical nature of GYM94, which required sequential processing in time. A time series of yaw rates estimated by the routine GPS processing at JPL will be analyzed to demonstrate the need to estimate the yaw rates.

## II. Background

The analysis that led to the implementation of the yaw bias on GPS satellites is described in Bar-Sever et al. [2]. A general description of the first yaw attitude model can also be found there. For completeness, we give here a brief summary.

The nominal yaw attitude of a GPS satellite is determined by satisfying two constraints: first, that the navigation antennas point toward the geocenter and, second, that the normal to the solar array surface will be pointing at the Sun. To meet these two conditions, the satellite has to yaw constantly. The resulting yaw attitude algorithm is singular at two points—the intersections of the orbit with the Earth–Sun line. At these points, the yaw attitude is not single-valued, as any yaw angle allows optimal view of the Sun. In the vicinity of these singular points, the yaw rate of the spacecraft, required to keep track of the Sun, is unbounded. This singularity problem was largely ignored prior to the release of GYM94. While this mismodeling problem could be fixed easily through the realization of a finite limit on the spacecraft yaw rate, a bigger problem existed that could only be addressed by changing the attitude control subsystem (ACS) on board the spacecraft. The ACS determines the yaw attitude of the satellite by using a pair of solar sensors mounted on the solar panels. As long as the Sun is visible, the signal from the solar sensors is a true representation of the yaw error. During shadow, in the absence of sunlight, the output from the sensors is essentially zero and the ACS is driven in an open-loop mode by the noise in the system. It turns out that even a small amount of noise can be enough to trigger a yaw maneuver at maximum rate. To make it possible to model the yaw attitude of the GPS satellites, the ACS had to be biased by a small but fixed amount. Biasing the ACS means that the Sun sensor’s signal is superposed with another signal (the bias) equivalent to an observed yaw error of 0.5 deg (the smallest bias possible). As a result, during periods when the Sun is observed, the satellite yaw attitude will be about 0.5 deg in error with respect to the nominal orientation. During shadow, this bias dominates the open-loop noise and will yaw the satellite at full rate in the direction of the bias. Upon shadow exit, the yaw attitude of the satellite can be calculated, and the Sun recovery maneuver can also be modeled.

GYM94 accounted for the yaw bias as well as the limit on the yaw rate. It computed the satellite yaw angle through numerical integration of a control law. Its output was a large file containing the yaw attitude history and, optionally, partial derivatives of the yaw attitude with respect to the yaw rate parameter. This file could later be interpolated to retrieve a yaw angle at the requested time. This process required relatively large amounts of computer memory and CPU time. In addition, the model’s complex control law—a simulation of the onboard attitude determination algorithm—did not allow much physical insight into the problem and was hard to tune. To overcome all these deficiencies, the GYM95 model was created. GYM94 was used in studies of GPS calibration for the DSN since September 1994, and the design of the new attitude model drew on the experience accumulated with GYM94. GYM95 is simple enough to be described by a small set of formulas, allowing easy implementation in different computing environments. Its analytic nature, as opposed to the numerical nature of GYM94, allows queries at arbitrary time points with great savings in computer resources. Finally, it allows more flexibility in tuning and adapting it to the changing conditions of the GPS constellation.

## III. The New Yaw Attitude Model (GYM95)

### A. Overview

The yaw attitude of a GPS satellite can be divided into four regimes: nominal attitude, shadow crossing, postshadow maneuver, and noon turn. Most of the time (and for noneclipsing satellites all the time), the satellite is in the nominal attitude regime. The postshadow maneuver begins immediately after emerging from the Earth’s shadow and lasts until the satellite has regained its nominal attitude. This phase can last from 0 to 40 min. The noon-turn maneuver does not occur until the beta angle goes below about 5 deg and can last between 0 and 40 min.

We will start by defining a few important terms in Table 1 and the notation used, and then describe the yaw attitude during each of the four regimes, including the governing formulas. Finally, we will describe how to tie all the regimes together into one functional model and analyze any built-in errors.

**Table 1. Definition of terms.**

Term	Definition
Orbit midnight	The point on the orbit furthest from the Sun.
Orbit noon	The point on the orbit closest to the Sun.
Orbit normal	The unit vector along the direction of the satellite’s angular momentum, treating the satellite as a point mass (equals position $\times$ velocity, where the order of the cross-product is important).
Sun vector	The direction from the spacecraft to the Sun.
Beta angle	The acute angle between the Sun vector and the orbit plane. It is defined as positive if the Sun vector forms an acute angle with the orbit normal and negative otherwise.
Orbit angle	The angle formed between the spacecraft position vector and orbit midnight, growing with the satellite’s motion.
Yaw origin	A unit vector that completes the spacecraft position vector to form an orthogonal basis for the orbit plane and is in the general direction of the spacecraft velocity vector.
Spacecraft-fixed z-axis	The direction of the GPS navigation antennas.
Nominal spacecraft-fixed x-axis	A unit vector orthogonal to the spacecraft-fixed z-axis such that it lies in the Earth–spacecraft–Sun plane and points in the general direction of the Sun (note that this definition is not single valued when the Earth, spacecraft, and Sun are collinear).
Spacecraft-fixed x-axis	A spacecraft-fixed vector, rotating with the spacecraft, such that far enough from orbit noon and orbit midnight, it coincides with the nominal spacecraft-fixed x-axis. Elsewhere, it is a rotation of the nominal spacecraft-fixed x-axis around the spacecraft-fixed z-axis.
Nominal yaw angle	The angle between the nominal spacecraft-fixed x-axis and the yaw-origin direction, restricted to be in $[-180,180]$ . It is defined to have a sign opposite to that of the beta angle.
Yaw angle	The angle between the spacecraft-fixed x-axis and the yaw-origin direction, restricted to be in $[-180,180]$ , also termed “actual yaw angle.”
Yaw error	The difference between the yaw angle and the nominal yaw angle, restricted to be in $[-180,180]$ .
Midnight turn	The yaw maneuver the spacecraft is conducting from shadow entry until it resumes nominal attitude sometime after shadow exit.
Noon turn	The yaw maneuver the spacecraft is conducting in the vicinity of orbit noon when the nominal yaw rate would be higher than the yaw rate the spacecraft is able to maintain. It ends when the spacecraft resumes nominal attitude.
Spin-up/down time	The time it takes for the spacecraft to spin up or down to its maximal yaw rate. The spacecraft is spinning down when it has to reverse its yaw rate.

The notation used is as follows:

$\mu$  = orbit angle

$\beta$  = beta angle

$E$  = Earth-spacecraft-Sun angle

$b$  = yaw bias inserted in the satellite ACS

$B$  = actual yaw angle induced by  $b$   
 $\Psi$  = actual yaw angle  
 $\Psi_n$  = nominal yaw angle  
 $t$  = current time, s  
 $t_i$  = time of shadow entry  
 $t_e$  = time of shadow exit  
 $t_n$  = start time of the noon-turn maneuver  
 $t_1$  = spin-up/-down time  
 $\Psi_i$  = yaw angle upon shadow entry  
 $\Psi_e$  = yaw angle upon shadow exit  
 $R$  = maximal yaw rate of the satellite  
 $RR$  = maximal yaw-rate rate of the satellite

Angle units, i.e., radians or degrees, will be implied by context. Radians will usually be used in formulas, and degrees will usually be used in the text. FORTRAN function names are used whenever possible with the implied FORTRAN functionality, e.g., ATAN2(a,b) is used to denote arc-tangent( $a/b$ ) with the usual FORTRAN sign convention.

## B. The Nominal Attitude Regime

The realization of the two requirements for satellite orientation mentioned above yields the following formula for the nominal yaw angle:

$$\Psi_n = \text{ATAN2}(-\text{TAN}(\beta), \text{SIN}(\mu)) + B(b, \beta, \mu) \quad (1)$$

where  $\beta$  is the beta angle,  $\mu$  is the orbit angle, measured from orbit midnight in the direction of motion, and  $B$  is the yaw bias (see below). It follows from this formula that the sign of the yaw angle is always opposite that of the beta angle.

Ignoring the time variation of the slow-changing beta angle leads to the following formula for the yaw rate (there are simpler formulas, but they contain removable singularities that are undesirable for computer codes):

$$\dot{\Psi}_n = \text{TAN}(\beta) \times \text{COS}(\mu) \times \frac{\dot{\mu}}{\text{SIN}(\mu)^2 + \text{TAN}(\beta)^2} + \dot{B}(b, \beta, \mu) \quad (2)$$

where  $\dot{\mu}$  varies little in time and can safely be replaced by 0.0083 deg/s. Notice that the sign of the nominal yaw rate is the same as the sign of the beta angle in the vicinity of orbit midnight ( $\mu = 0$ ).

The singularity of these two formulas when  $\beta = 0$  and  $\mu = 0, 180$  is genuine and cannot be removed.

## C. The Yaw Bias

Like any medicine, the yaw bias has its side effects. Outside shadow, it introduces yaw errors that are actually larger than 0.5 deg. To fully understand this, we have to describe the ACS hardware, which

is beyond the scope of this article. The underlying reason is that the output of the solar sensor is proportional not to the yaw error but to its sine, and it is also proportional to the sine of the Earth–spacecraft–Sun angle,  $E$ . So, in order to offset a bias of  $b$  deg inserted in the ACS, the satellite has to actually yaw  $B$  deg, where  $B$  is given by:

$$B(b, \beta, \mu) = B(b, E) = \text{ASIN} \frac{0.0175 \times b}{\text{SIN}(E)} \quad (3)$$

The hardware-dependent proportionality factor is 0.0175, and the Earth–spacecraft–Sun angle,  $E$ , the beta angle,  $\beta$ , and the orbit angle,  $\mu$ , satisfy the following approximate relationship:

$$\text{COS}(E) = \text{COS}(\beta) \times \text{COS}(\mu) \quad (4)$$

and  $E$  is restricted to  $[0, 180]$ . Equation (3) becomes singular for  $E$  less than 0.5013 deg. This has no effect on the actual yaw because a small value of  $E$  implies that the spacecraft is in the middle of a midnight turn or a noon turn and is already yawing at full rate. The value of  $B$  does have a significant effect, though, on the timing of noon-turn entry and on the yaw angle shortly before that. For example, for  $E = 5$  deg, which is the typical threshold value for noon-turn entry, the actual yaw bias is  $B \approx 6$  deg.

The bias rate,  $\dot{B}$ , is given by

$$\dot{B}(b, \beta, \mu) = -0.0175 \times b \times \text{COS}(E) \times \text{COS}(\beta) \times \text{SIN}(\mu) \times \frac{\dot{\mu}}{\text{COS}(B) \times \text{SIN}(E)^3} \quad (5)$$

The ACS bias,  $b$ , can be  $\pm 0.5$  deg or 0 deg. With few exceptions, to be discussed below, the bias is always set to  $b = -\text{SIGN}(0.5, \beta)$  since this selection was found to expedite the Sun recovery time after shadow exit.

#### D. The Shadow-Crossing Regime

As soon as the Sun disappears from view, the yaw bias alone is steering the satellite. On most satellites, the yaw bias has a sign opposite to that of the beta angle. To correct for the bias-induced error, the satellite has to reverse its yaw rate upon shadow entry. For those satellites with bias of equal sign to that of the beta angle, there is no yaw reversal. The bias is large enough to cause the satellite to yaw at full rate until shadow exit, when the bias can be finally compensated. The yaw angle during shadow crossing depends, therefore, on three parameters: the yaw angle upon shadow entry,  $\Psi_i$ , the yaw rate upon shadow entry,  $\dot{\Psi}_i$ , and the maximal yaw rate,  $R$ . Let  $t_i$  be the time of shadow entry and let  $t$  be the current time, and define

$$t_1 = \frac{\text{SIGN}(R, b) - \dot{\Psi}_i}{\text{SIGN}(RR, b)} \quad (6)$$

to be the spin-up/-down time. Then the yaw angle during shadow crossing is given by

$$\Psi = \begin{cases} \Psi_i + \dot{\Psi}_i \times (t - t_i) + 0.5 \times \text{SIGN}(RR, b) \times (t - t_i)^2 & t < t_i + t_1 \\ \Psi_i + \dot{\Psi}_i \times t_1 + 0.5 \times \text{SIGN}(RR, b) \times t_1^2 + \text{SIGN}(R, b) \times (t - t_i - t_1) & \text{else} \end{cases} \quad (7)$$

Using this formula, we avoid the singularity problem of the nominal attitude at midnight.

## E. The Postshadow Maneuver

This is the trickiest part of the yaw attitude model. The postshadow maneuver depends critically upon the yaw angle at shadow exit. The ACS is designed to reacquire the Sun in the fastest way possible. Upon shadow exit, the ACS has two options: One is to continue yawing at the same rate until the nominal attitude is resumed; the second is to reverse the yaw rate and yaw at full rate until the nominal attitude is resumed. In this model, we assume that the decision is based on the difference between the actual yaw angle and the nominal yaw angle upon shadow exit, and we denote this difference by  $D$ . If  $t_e$  is the shadow-exit time, then

$$D = \Psi_n(t_e) - \Psi(t_e) - \text{NINT} \left( \frac{\Psi_n(t_e) - \Psi(t_e)}{360} \right) \times 360 \quad (8)$$

and the yaw rate during the postshadow maneuver will be  $\text{SIGN}(R, D)$ .

Given the yaw angle upon shadow exit, the yaw rate upon shadow exit,  $\text{SIGN}(R, b)$ , and the yaw rate during the postshadow maneuver, we can compute the actual yaw angle during the postshadow maneuver by using Eq. (7) with the appropriate substitutions. This yields

$$t_1 = \frac{\text{SIGN}(R, D) - \text{SIGN}(R, b)}{\text{SIGN}(RR, D)} \quad (9)$$

$$\Psi = \begin{cases} \Psi(t_e) + \text{SIGN}(R, b) \times (t - t_e) + 0.5 \times \text{SIGN}(RR, D) \times (t - t_e)^2 & t < t_e + t_1 \\ \Psi(t_e) + \text{SIGN}(R, b) \times t_1 + 0.5 \times \text{SIGN}(RR, D) \times t_1^2 + \text{SIGN}(R, D) \times (t - t_e - t_1) & \text{else} \end{cases} \quad (10)$$

The postshadow maneuver ends when the actual yaw attitude, derived from Eq. (10), becomes equal to the nominal yaw attitude. The time of this occurrence is computed in GYM95 by an iterative process that brackets the root of the equation  $\Psi(t) = \Psi_n(t)$ , where the time dependence of  $\Psi_n(t)$  is introduced by substituting  $\mu = \mu_e + 0.0083 \times (t - t_e)$  in Eq. (1). This equation can be solved as soon as the satellite emerges from shadow. Once the time of resuming nominal yaw is reached, the satellite switches back to that regime.

## F. The Noon-Turn Regime

The noon-turn regime starts in the vicinity of orbit noon, when the nominal yaw rate reaches its maximal allowed value, and ends when the actual yaw attitude catches up with the nominal regime. First, we have to identify the starting point, and this can be done by finding the root,  $t_n$ , of the equation  $\dot{\Psi}_n(t) = -\text{SIGN}(R, \beta)$ , where  $\dot{\Psi}_n(t)$  is the nominal yaw rate from Eq. (2). After the start of the noon turn, the yaw angle is governed by Eq. (7), again with the proper substitutions. This yields

$$\Psi = \Psi_n(t_n) - \text{SIGN}(R, \beta) \times (t - t_n) \quad (11)$$

The end time is found by the same procedure that is used to find the end time of the postshadow maneuver.

## G. The Complete Model

Satellite position and velocity, as well as the timing of shadow crossings, are required inputs to GYM95. The model is able to bootstrap, though, if these input values are unavailable far enough into the past. For example, if the satellite is potentially in the postshadow regime upon first query, there is a need to know the shadow-entry time so that all the inputs to Eqs. (9) and (10) be known. If this shadow-entry

time is missing from the input, the model can compute it approximately, as well as the shadow-exit time. Once all the timing information is available, yaw angle queries can be made at arbitrary time points. The model will decide the relevant yaw regime and compute the yaw angle using the correct formula. Given the above formulas, it is an easy matter to compute the partials of the yaw angle with respect to any parameter of the problem, the most important of which is the maximal yaw rate,  $R$ .

## H. Model Fidelity

The fidelity of the model is a measure of how accurately it describes the true behavior of the satellite. This is hard to measure because there is no high-quality telemetry from the satellite and because the estimated value of the main model parameter, namely, the yaw rate, depends on many other factors beside the attitude model itself: data, estimation strategy, and other models for the orbit and the radiometric measurements. Nevertheless, based on the experience accumulated thus far with this model and its predecessor, GYM94, it is possible to come up with an educated guess of the inaccuracy of GYM95.

The nominal attitude regime is believed to be very accurate. The only source of error is mispointing of the satellite, which is poorly understood and relatively small (of the order of 1 deg around the pitch, yaw, and roll axes). Compensations for the dynamic effect of this error source are discussed in [3] and [4], where they are treated, properly, within the context of the solar pressure model.

Modeling the midnight turn accurately is difficult. Inherent uncertainties like the exact shadow-entry and -exit times are a constant error source. Inaccuracies in shadow-entry time are more important than inaccuracies in shadow-exit time because errors in the former are propagated by the model throughout the midnight-turn maneuver. In contrast, error in the shadow-exit time will affect the postshadow maneuver only. Either way, the inaccuracy will be manifested through a constant error in the yaw angle, something that can be partially compensated through the estimation of the yaw rate. The length of the penumbra region is usually about 60 s. Sometime during this period, the yaw bias kicks in. GYM95 puts that time midway into penumbra. The maximum timing error is, therefore, less than 30 s. A worst-case scenario, ignoring the short spin-up/-down period and using a yaw rate of 0.13 deg/s, will give rise to a constant yaw error of  $30 \times 0.13 \approx 4$  deg throughout the midnight turn. A more realistic estimate is 3 deg, even before applying yaw rate compensation, after which the rms error will remain the same, but the mean is expected to vanish. Another error source is the uncertainty in the value of the maximal yaw-rate rate,  $RR$ . This parameter is weakly observable and, therefore, hard to estimate. The nominal value used in GYM95 is 0.00165 deg/s<sup>2</sup> for Block IIA satellites and 0.0018 deg/s<sup>2</sup> for Block II satellites, and it should be at least 70-percent accurate. The long-term effects of a yaw-rate rate error can be computed from the second part of Eq. (7) as

$$\Psi(RR) = \frac{\dot{\Psi}_i \times \text{SIGN}(R, b) - 0.5 \times \dot{\Psi}_i^2 - 0.5 \times \text{SIGN}(R, b)^2}{\text{sign}(RR, b)}$$

A worst-case scenario assuming  $\dot{\Psi}_i = -\text{SIGN}(R, b) = 0.13$  and 30-percent error in the yaw-rate rate would give rise to a yaw error of about 5 deg. These assumptions also imply a very short shadow duration so the error will not be long lasting. For long shadow events,  $\dot{\Psi}_i \approx 0$ , and the resulting yaw error is about 1 deg. Again, this error can be partially offset by estimating the yaw rate.

The main error source for the noon turn is the timing uncertainty of the onset of the maneuver. This uncertainty is not expected to be larger than 2 min. A 2-min error will cause a constant yaw error of about 15 deg, assuming a yaw rate of 0.13 deg/s. The relatively short duration of the noon turn diminishes somewhat the effects of such a large error. Estimating the yaw rate will decrease the error further.

The value of the yaw rate is not considered here as an error source. Any nominal value stands to be at least 10 percent in error (see below). Since errors due to yaw rate grow in time, this parameter must

be estimated or, alternatively, a previously estimated value should be used. For example, an error of 0.01 deg/s in the yaw rate will give rise to a 30-deg error in yaw at the end of a 50-min shadow event.

Although unlikely, errors from different sources can add up. In that case, the maximal error for each regime is as follows: 2 deg for the nominal yaw regime, 9 deg for the midnight-turn regime, and 15 deg for the noon-turn regime. Typical errors are expected to be less than half of these values.

#### IV. The Estimated Yaw Rates

As part of the implementation of the GYM models at JPL, the yaw rates of all eclipsing satellites are estimated for every midnight turn and every noon turn. In JPL's GIPSY software, this is done by treating the yaw rate as a piece-wise constant parameter for each satellite. The parameter value is allowed to change twice per revolution, midway between noon and midnight. Since a small error in the yaw rate can cause a large yaw error over time and, since our a priori knowledge of the yaw rate is not sufficiently accurate, we found it necessary to iterate on the yaw rate value. JPL routinely publishes the final estimates for the yaw rates as daily text files. Unfortunately, due to a software bug, the archived yaw rates for dates prior to February 16, 1995, were in error. This leaves a period of about 2 months when the estimated yaw rates are available. Figure 1 depicts the estimated yaw rates for each eclipsing satellite, for each midnight turn, and for each noon turn, from February 16 to April 26, 1995. The accuracy of the estimates depends on the amount of data available during each maneuver and this, in turn, is proportional to the duration of the maneuver. The longer the maneuver, the better the estimate. The effect of a reduced estimation accuracy during short maneuvers is mitigated by the fact that the resulting yaw error is also proportional to the duration of the maneuver. For long maneuvers, e.g., a midnight turn at the middle of the eclipse season, the estimates are good to 0.002 deg/s, which leads to a maximal yaw error of about 6 deg. A similar error level is expected for short maneuvers. Noon turns occur only during the middle part of the eclipse season. In Fig. 1, they can be distinguished from midnight-turn rates by the larger formal error associated with them, since they are typically short events of 15- to 30-min duration. As a result, the scatter of the noon-turn rates is larger than that of the midnight-turn rates. Toward the edges of the eclipse season, the quality of the yaw rate estimates drops, again because of the short duration of the shadow events. The most striking feature in Fig. 1 is the discontinuity of the estimated yaw rates in the middle of the eclipse season, corresponding to the beta angle crossing zero. No plausible explanation is currently available for this jump. SVN 29 is the only satellite that does not have a jump discontinuity; this is also the only satellite that does not undergo a bias switch in the middle of the eclipse season. SVN 31 is the only satellite with a jump from high yaw rates to low yaw rates as the beta angle transitions from positive to negative. There is nothing otherwise special about SVN 31. The ratio of the high yaw-rate values to the low yaw-rate values is about 1.3 for all satellites.

Within each half of the eclipse season, the midnight yaw rates are fairly constant, varying by 10 percent or less. The noon-turn yaw rates seem to be more variable. This is not only a consequence of the weak observability but also of the fact that the spacecraft is subject to a varying level of external torque during the noon turn as the eclipse season progresses.

The modeling of the postshadow maneuver is a problem for which a satisfactory solution has not yet been found. The source of the problem is that the presence of the postshadow regime makes the estimation of the yaw rate into a nonlinear problem. There is always a critical value of the yaw rate such that, for higher values, the spacecraft will reverse its yaw upon shadow exit and, for lower values, the spacecraft will retain its yaw rate until the end of the midnight turn. If this critical value falls in the range of feasible yaw rates—which it often does—it becomes very hard to figure out what kind of maneuver the satellite is doing upon shadow exit. To avoid this postshadow ambiguity, we have been rejecting measurement data from shadow exit until about 30 min thereafter.

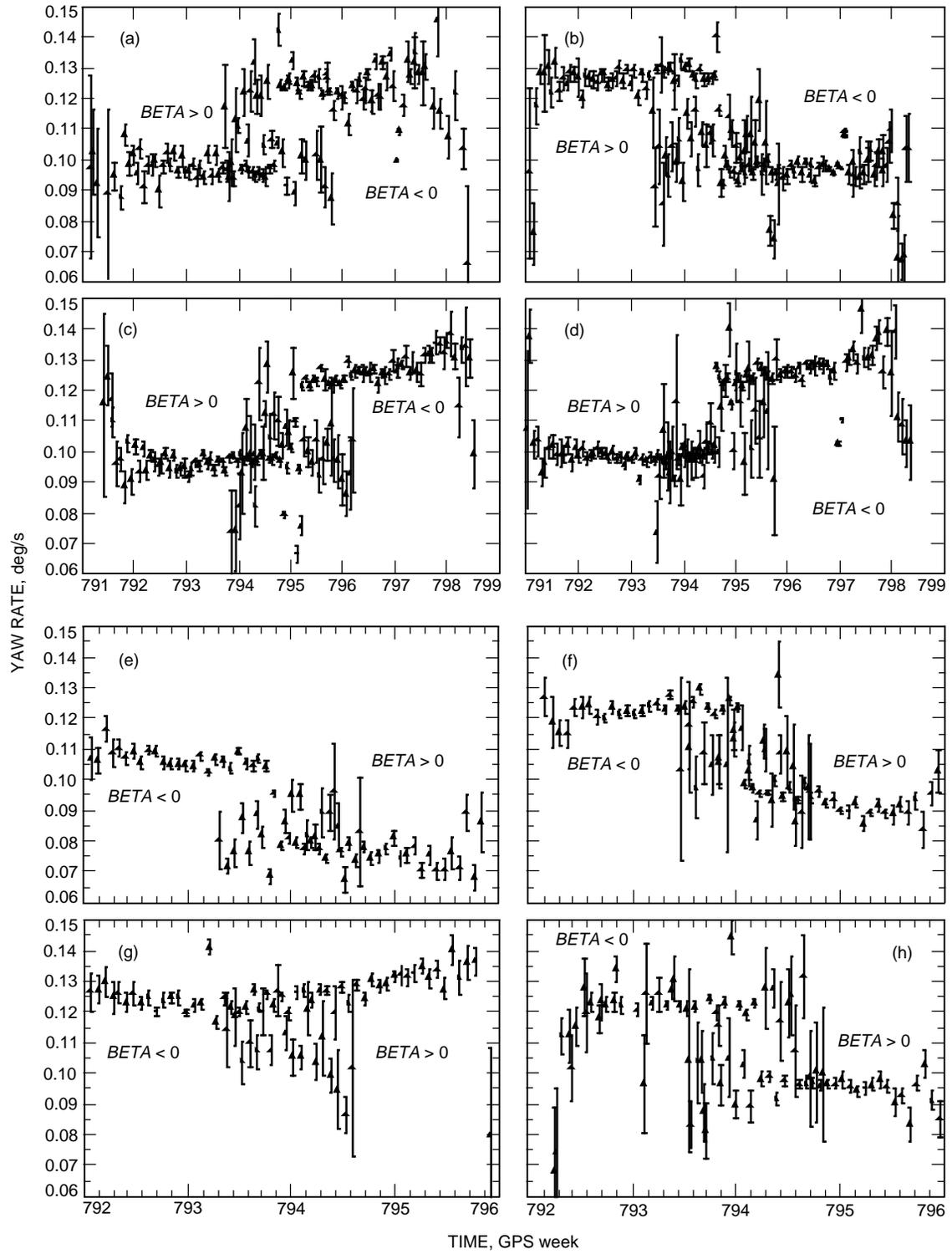


Fig. 1. Estimated yaw rates with their formal errors versus GPS week for coplanar (C-plane) (a) SVN 28, (b) SVN 31, (c) SVN 36, and (d) SVN 37 and for F-plane (e) SVN 18, (f) SVN 26, (g) SVN 29, and (h) SVN 32.

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