Performance of Pulse Code Modulation/ Phase Modulation Receivers With Nonideal Data

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In pulse code modulation/phase modulation systems, the nonreturn-to-zero or biphase data are directly modulated onto the residual carrier. Imperfection in the data, such as unbalance between +1 and −1 data (data unbalance) and unequal transition time (data asymmetry), cause improper synchronization of the residual carrier and the data clock. The result is a degradation in the symbol-error rate (SER) performance. In this article, the impact of imperfect data on the carrier synchronization process and SER performance is assessed. Simulation results are presented to support the analysis.

I. Introduction

A study of the symbol-error rate (SER) performance of pulse code modulation/phase modulation/nonreturn-to-zero (PCM/PM/NRZ) and PCM/PM/biphase (bi-ϕ) receivers[1] with nonideal data is presented in the presence of two separate effects that degrade the performance of the receiver. These are unbalanced data (the unbalance between the +1's and −1's in the binary data stream) and data asymmetry (the unequal rise and fall times of the logic gating circuits). PCM/PM modulation has the advantage that, because the modulation index is less than 90 degrees, there exists a residual carrier component that can be tracked by the phase-locked loop (PLL) to provide a coherent phase reference. This is particularly useful when the received signal level is weak and/or contains high Doppler dynamics, since it is well known that a PLL can operate at much lower loop signal-to-noise ratios (SNRs) than can a Costas loop. Moreover, PCM/PM modulations are bandwidth efficient and require little modification to the existing National Aeronautics and Space Administration (NASA) Deep Space Network (DSN) receivers.

Since the PCM/PM data are phase modulated directly on the carrier, it is not surprising that any imperfection in the data component will interfere with the carrier tracking and ultimately impact the SER performance. Recently, there have been extensive efforts in characterizing the effects of imperfect data (unbalance and asymmetry) on the PCM/PM receivers. In [1] and [2], the authors provided the

[1] Bi-ϕ modulation typically is referred to in the literature as Manchester code.

SER performance for BPSK/NRZ and BPSK/bi-ϕ modulation[2] in the presence of data asymmetry where a Costas loop is used instead of a PLL to track the carrier phase. In [3], the PLL and SER performance in the presence of perfect data for PCM/PM modulation was provided. The performance for imperfect data, on the other hand, was presented in [4]. An attempt was made to reproduce the results given in [4] by simulation, but it was found that the simulation results did not agree with the theory. Consequently, the effects of imperfect data on the PLL and SER performance were rederived and are presented in this article along with the simulation results.

This article includes the separate and combined effects of data unbalance and asymmetry on the performance of PCM/PM/NRZ and PCM/PM/bi-ϕ receivers. The article begins with an introduction of the PCM/PM systems. Next, the conditional error probability performance of these systems is derived with separate and combined effects of data unbalance and asymmetry for both. In Section IV, the performance of the PLL is analyzed in the presence of imperfect data. Simulation results are presented along with the numerical calculations in Section V. Finally, in Section VI, important conclusions are made on the performance of PCM/PM systems with imperfect data.

II. The PCM/PM System

The mathematical representation of the received PCM/PM/NRZ or PCM/PM/bi-ϕ signal is given as

$$r(t) = \sqrt{2P_t}\sin(\omega_c t + \theta_c + m_T P(t)d(t)) + n(t) \tag{1}$$

where P_t is the total received power in watts (W); m_T is the modulation index in radians (rad); $d(t)$ is a binary random data waveform with rectangular pulse shape that takes on values ± 1 at the bit rate $R_b = 1/T_b$; ω_c and θ_c are the carrier angular frequency in radians per second (rad/s) and phase in radians (rad), respectively; and $n(t)$ is additive white Gaussian noise with single-sided power spectral density (PSD) N_0 (W/Hz). Moreover, for PCM/PM/NRZ and PCM/PM/bi-ϕ signals, $P(t)$ is defined as

$$P(t) = 1 \tag{2}$$

and

$$P(t) = \mathrm{Sqr}(2\pi R_b t) \tag{3}$$

respectively, where $\mathrm{Sqr}(2\pi R_b t)$ is a square-wave subcarrier at the bit rate R_b. Using simple trigonometry, Eq. (1) can be rewritten as

$$r(t) = \sqrt{2P_c}\sin(\omega_c t + \theta_c) + \sqrt{2P_d}d(t)P(t)\cos(\omega_c t + \theta_c) + n(t) \tag{4}$$

where the residual power $P_c = P_t\cos^2(m_T)$ and the data power in the data component $P_d = P_t\sin^2(m_T)$. The power spectral densities (PSDs) for PCM/PM/NRZ and PCM/PM/bi-ϕ are shown in Figs. 1(a) and 1(b), respectively. Observe that the carrier component for PCM/PM/bi-ϕ occurs at the null of the data spectrum.

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[2] PCM/PM modulation is identical to binary phase-shifted keying (BPSK) if the modulation index is set to 90 degrees.

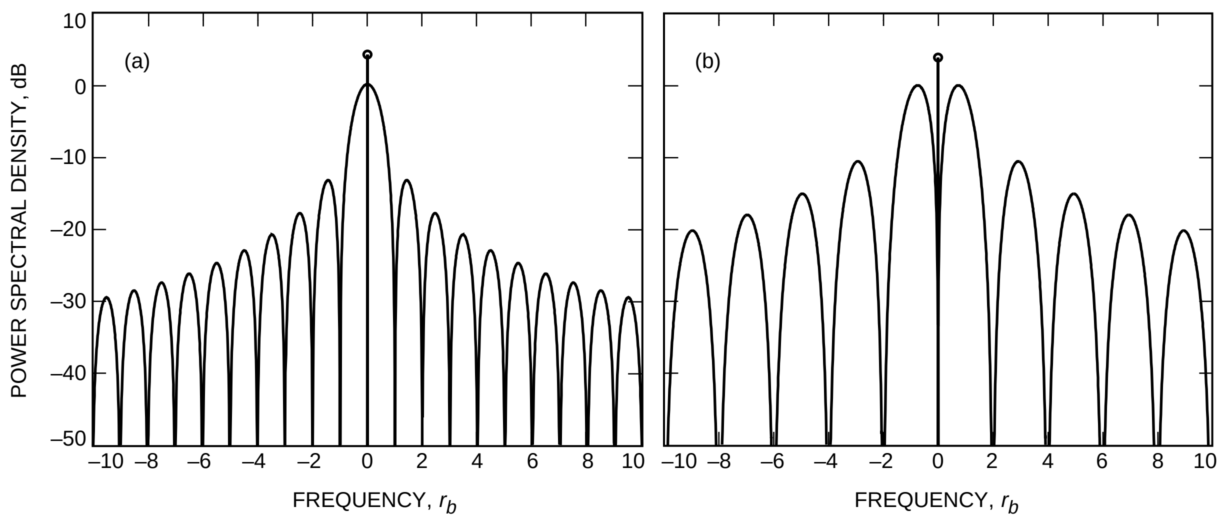

Fig. 1. Power spectral density: (a) PCM/PM/NRZ and (b) PCM/PM/bi-ϕ.

The received signal given in Eq. (3) is then demodulated using the receiver shown in Fig. 2. The PCM/PM receiver consists of a PLL to track the carrier phase and a symbol synchronizer loop to track the symbol timing. For ideal data, the PLL tracks the carrier phase and provides the carrier reference that is in quadrature to the received signal; that is, $r^{'}(t) = \sqrt{2}\cos(\omega_c t + \hat{\theta}_c)$ where $\hat{\theta}_c$ is an estimate of the received phase, θ_c. In the presence of nonideal data, however, the carrier reference becomes

$$r^{'}(t) = \sqrt{2}\cos(\omega_c t + \hat{\theta}_c + \theta_m) \tag{5}$$

where θ_m is the carrier phase bias, to be defined later. We will see that this phase bias will be a major source of SER degradation for PCM/PM modulation. The SER is defined as

$$P_s(E) = \int_{-\pi}^{\pi} P_s'(E)p(\phi)d\phi \tag{6}$$

where the carrier phase error is $\phi = \theta_c$ - $\hat{\theta}_c$; $P_s'(E)$ is the SER conditioned on ϕ; and the probability density function (pdf) of ϕ, denoted $p(\phi)$, has a Tikhonov form, namely,

$$p(\phi) = \frac{\exp(\rho\cos\phi)}{2\pi I_o(\rho)} \quad |\phi| \le \frac{\pi}{2} \tag{7}$$

where $I_k(\rho)$ denotes the modified Bessel function of order k and ρ is the carrier loop SNR, which for a linear loop model, is defined as the inverse of the carrier phase jitter; that is, $\rho = 1/\sigma_\phi^2$.

In the following sections, the explicit expressions for the conditional SER, $P_s'(E)$, will be derived for both PCM/PM/NRZ and PCM/PM/bi-ϕ receivers as a function of data unbalance, data asymmetry, and the combination of both. Afterward, the carrier loop SNR also will be derived in the presence of nonideal data. Finally, numerical and simulation results are presented for the unconditional SER, $P_s(E)$, using various combinations of data unbalance and asymmetric data.

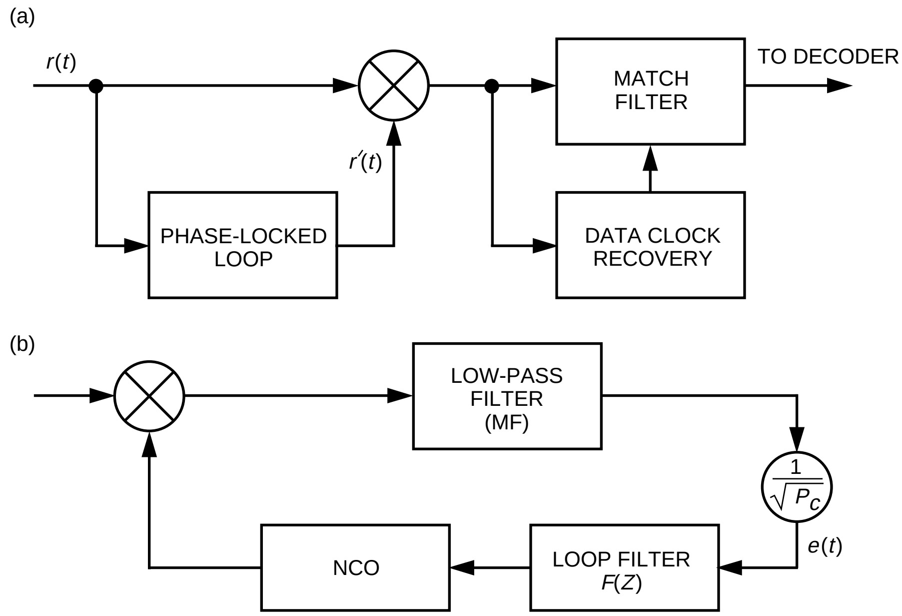

Fig. 2. PCM/PM receiver: (a) a block diagram of the receiver and (b) a digital PLL.

III. Conditional Error Probability Performance

In this section, we derive the conditional SER, $P_s'(E)$, for unbalanced data, data asymmetry, and the combination of both for PCM/PM/NRZ and PCM/PM/bi-ϕ modulation. Moreover, the phase bias is given for each data imperfection, and the concept of vectors is introduced to explain its intuitive meaning. In arriving at the conditional SER results, we assume perfect symbol timing, as was done in [1]. The impact of imperfect symbol timing will be a topic of a future article.

A. Unbalanced Data

The unbalance between +1's and −1's in the data stream can cause the PLL to track the carrier phase bias, θ_m, which for PCM/PM/NRZ and PCM/PM/bi-ϕ is given as

$$\theta_m = \tan^{-1}\left[(p-q)\tan m_T\right] \tag{8}$$

and

$$\theta_m = 0 \tag{9}$$

respectively. The phase bias for PCM/PM/bi-ϕ is zero because the bi-ϕ data sequence always has 50-percent transitions. Note that the results in Eqs. (8) and (9) also can be derived from the PSD results given in [5] by setting the frequency of the discrete component to zero. The conditional SERs for PCM/PM/NRZ and PCM/PM/bi-ϕ are given as [6]

$$P_s'(E) = \frac{p}{2}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}\cos(\phi+\theta_m) - \sqrt{\frac{E_s'}{N_0}}\sin(\phi+\theta_m)\right]$$

$$+ \frac{q}{2}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}\cos(\phi+\theta_m) + \sqrt{\frac{E_s'}{N_0}}\sin(\phi+\theta_m)\right] \tag{10}$$

and

$$P_s'(E) = \frac{1}{2}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}\right] \tag{11}$$

respectively, where $E_s/N_0 = (P_d T_b/N_0)\sin^2 m_T$, $E_s'/N_0 = (E_s/N_0)\tan^2 m_T$, and $\text{erf}(x) = 1 - \text{erfc}(x) = (2/\sqrt{\pi})\int_0^x \exp(-v^2)dv$ is the error function. Moreover, p is the probability of $+1$ data,[3] q is the probability of -1 data, and each is assumed to be generated from a purely random data source. From the above equations, it is clear that unbalanced data degrade PCM/PM/NRZ but not PCM/PM/bi-ϕ.

The impact of unbalanced data on the PLL can best be illustrated by a vector diagram, presented in Fig. 3, for various values of the probability of mark, p. The diagram shows in vector form the mathematical equation of the received signal given in Eq. (4). In Fig. 3(a), where p=0.5, the $-\sqrt{P_d}$ data vector exactly cancels the $+\sqrt{P_d}$ data vector, resulting in no phase bias, $\theta_m = 0$. Figure 3(b), on the other hand, shows the case when the probability of mark is 0.6. In this case, the resultant data vector is $+\sqrt{P_d}(p-q) = +\sqrt{P_d}(2p-1)$. Moreover, the PLL tracks the resultant vector produced by the vector addition of $+\sqrt{P_d}(p-q)$ and $\sqrt{P_c}$, denoted as $\sqrt{P_R}$. The angle θ_m can be derived here by taking the ratio of $+\sqrt{P_d}(p-q)$ to $\sqrt{P_c}$ and then taking the inverse tangent of this ratio, resulting in the phase bias given in Eq. (8). It can be observed here that the carrier loop SNR increases with unbalanced data since the magnitude of $\sqrt{P_R}$ is always greater or equal to the magnitude of $\sqrt{P_c}$. The exact amount of increase in the carrier loop SNR will be a focus of the next section. Similar to the case when $p = 0.6$, Fig. 3(c) shows the resultant vector when $p = 0.4$. As we will see, the performance of PCM/PM receivers will critically depend on the phase bias: the greater the phase bias, the more the degradation on the SER performance and vice versa. The phase bias, in this case, is the negative of the phase bias for $p = 0.6$. In summary, the phase bias is a function of the unbalance in the data as well as the modulation index. For instance, Fig. 4 shows the phase bias as a function of p for modulation indices of 1.25, 1.15, 1.05, and 0.785 rad. It is shown that for p greater than 0.5, the phase bias increases as m_T increases, and for p less than 0.5, the phase bias decreases as m_T increases. This trend also can be observed from the vector representation in Fig. 3.

B. Data Asymmetry

Similar to unbalanced data, data asymmetry also can impact the performance of the telemetry system. The data asymmetry model given in [1] (model 1) will be adopted here for the analysis. As shown in Fig. 5, the $+1$ data are elongated by ξT for NRZ data and $\xi T/2$ for bi-ϕ data when a data transition from a $+1$ to a -1 occurs; and -1 data are shortened by the same amount when a data transition from a -1 to a $+1$ occurs. The asymmetry discussed above is defined as positive data asymmetry. For negative data asymmetry, on the other hand, the elongation occurs in the -1 to $+1$ transition times.

The phase biases due to data asymmetry for PCM/PM/NRZ and PCM/PM/bi-ϕ are given as

[3] Throughout this article, the probability of $+1$ or p is also called the probability of mark.

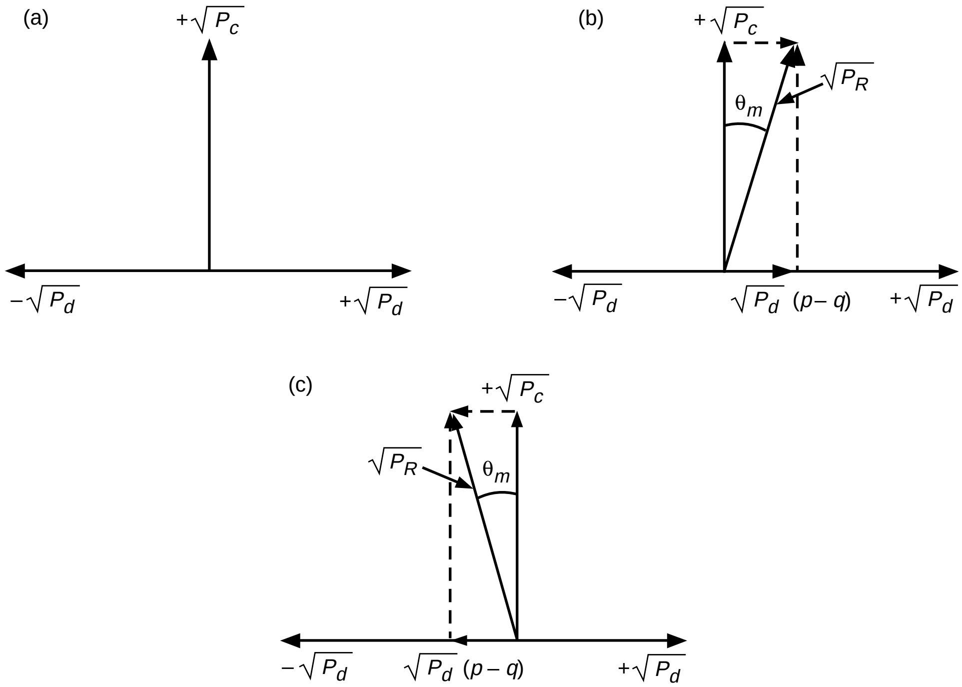

Fig. 3. Vector representation of NRZ data and carrier: (a) p = 0.5, (b) p = 0.6, and (c) p = 0.4.

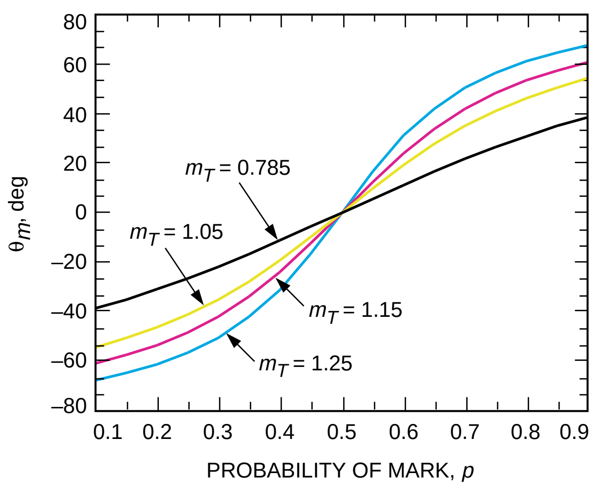

Fig. 4. The impact of data imbalance on the carrier phase for PCM/PM/NRZ.

$$\theta_m = \tan^{-1}\left[\frac{\xi}{2}\tan m_T\right] \tag{12}$$

and

$$\theta_m = \tan^{-1}\left[\frac{3\xi}{2}\tan m_T\right] \tag{13}$$

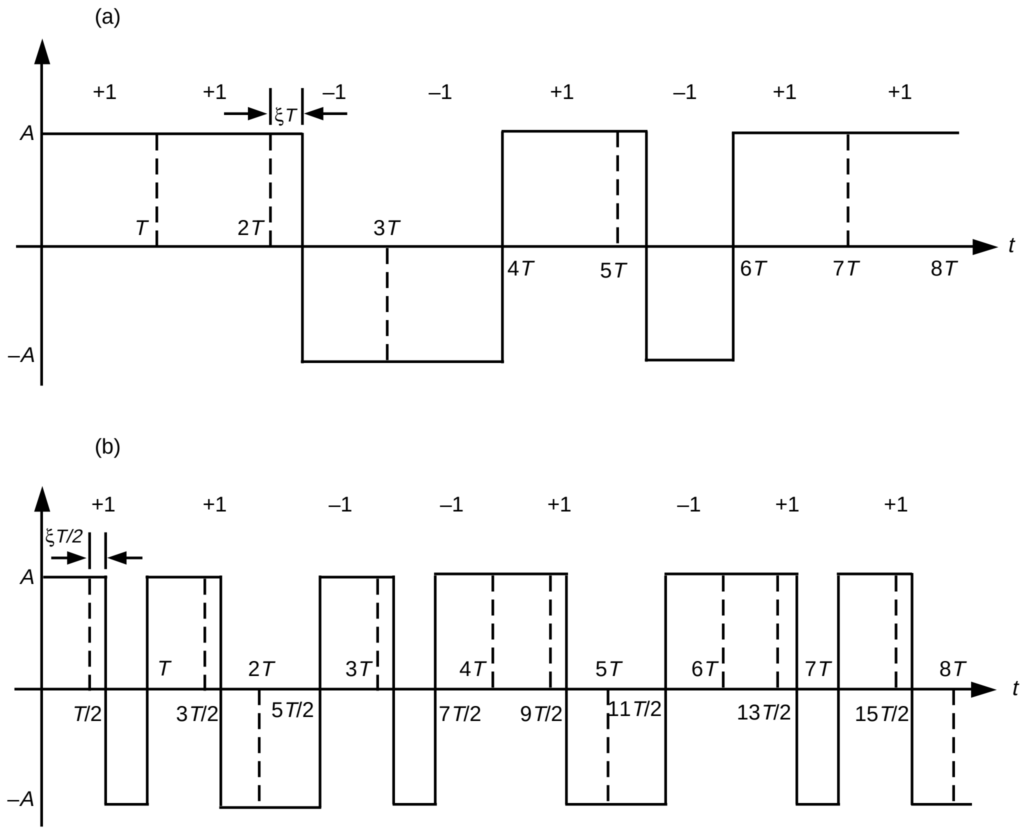

Fig. 5. Waveforms: (a) unbalanced, asymmetric NRZ and (b) unbalanced, asymmetric bi-ϕ.

respectively. Moreover, the SER performance for PCM/PM/NRZ and PCM/PM/bi-ϕ receivers due to data asymmetry can be shown to be

$$P_s'(E) = \frac{4}{16}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}\cos(\phi+\theta_m) \mp \sqrt{\frac{E_s'}{N_0}}\sin(\phi+\theta_m)\right]$$

$$+\frac{1}{16}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}(1-2|\xi|)\cos(\phi+\theta_m) \pm \sqrt{\frac{E_s'}{N_0}}\sin(\phi+\theta_m)\right]$$

$$+\frac{1}{8}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}(1-|\xi|)\cos(\phi+\theta_m) \pm \sqrt{\frac{E_s'}{N_0}}\sin(\phi+\theta_m)\right]$$

$$+\frac{1}{16}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}\cos(\phi+\theta_m) \pm \sqrt{\frac{E_s'}{N_0}}\sin(\phi+\theta_m)\right] \tag{14}$$

and

$$P'_s(E) = \frac{1}{4}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}(1-|\xi|)\cos(\phi+\theta_m)\right] + \frac{1}{4}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}\left(1-\frac{|\xi|}{2}\right)\cos(\phi+\theta_m)\right] \tag{15}$$

respectively, where ξ is the data asymmetry and $|*|$ represents the absolute value operator. For the PCM/PM/NRZ SER given in Eq. (14), use the upper sign for positive data asymmetry and the lower sign for negative data asymmetry. Note that the SER equations are symmetric for positive and negative data asymmetry; that is, data asymmetry of 10 percent will give the same SER calculation as for a -10 percent data asymmetry, as we would expect. Analogous to the unbalanced data case, the impact of data asymmetry on the PLL receiver can be illustrated by a vector diagram similar to Fig. 3 but with $\sqrt{P_d}(p-q)$ replaced by $\sqrt{P_d}(\xi/2)$ and $\sqrt{P_d}(3\xi/2)$, respectively. Figures 6(a) and 6(b) show the impact of data asymmetry on the carrier phase bias as a function of m_T for PCM/PM/NRZ and PCM/PM/bi-ϕ, respectively. It is shown that the phase bias for PCM/PM/bi-ϕ exceeds that of PCM/PM/NRZ, although only slightly.

C. Combined Effect

Similar to the separate effects of unbalanced data and data asymmetry, the combined effects can also impact the SER performance. The phase biases due to data asymmetry and unbalanced data for PCM/PM/NRZ and PCM/PM/bi-ϕ are given as

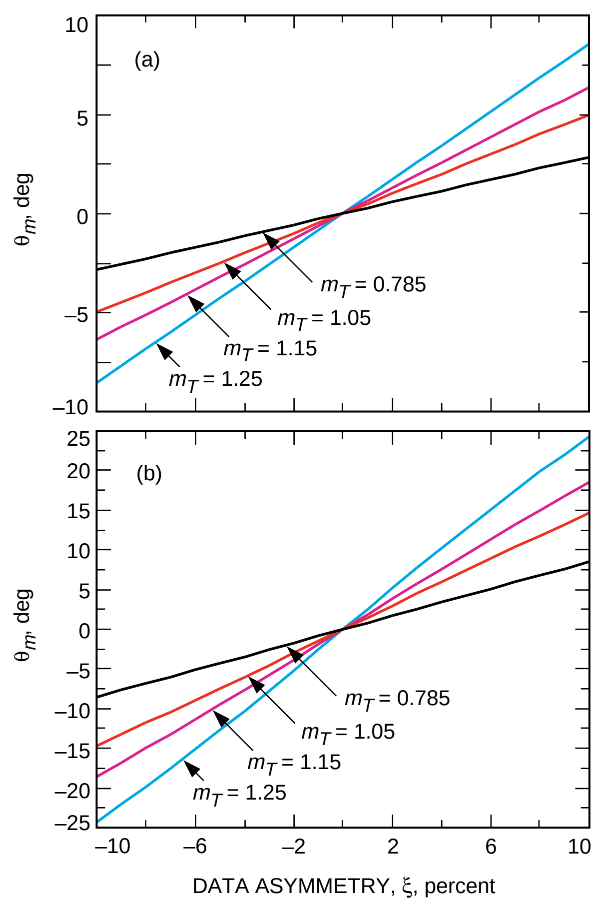

Fig. 6. The impact of data asymmetry on the carrier phase for (a) PCM/PM/NRZ and (b) PCM/PM/bi-ϕ.

$$\theta_m = \tan^{-1}\left[(p - q + 2\xi qp) \tan m_T\right] \tag{16}$$

and

$$\theta_m = \tan^{-1}\left[(2\xi(1 - pq) \tan m_T\right] \tag{17}$$

respectively. Note that setting $p = q = 0.5$ for the PCM/PM/NRZ and PCM/PM/bi-ϕ phase bias results in the phase biases given in Eqs. (12) and (13), and setting $\xi = 0$ results in the phase biases given in Eqs. (8) and (9), respectively. The SER performances of PCM/PM/NRZ and PCM/PM/bi-ϕ receivers are given as

$$\begin{aligned} P_s'(E) = {} & \frac{p}{2}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}\cos(\phi + \theta_m) \mp \sqrt{\frac{E_s'}{N_0}}\sin(\phi + \theta_m)\right] \\ & + \frac{q(1 - p_t)^2}{2} erfc\left[\sqrt{\frac{E_s}{N_0}}\cos(\phi + \theta_m) \pm \sqrt{\frac{E_s'}{N_0}}\sin(\phi + \theta_m)\right] \\ & + \frac{qp_t^2}{2}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}(1 - 2|\xi|)\cos(\phi + \theta_m) \pm \sqrt{\frac{E_s'}{N_0}}\sin(\phi + \theta_m)\right] \\ & + \frac{2q(1 - p_t)p_t}{2}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}(1 - |\xi|)\cos(\phi + \theta_m) \pm \sqrt{\frac{E_s'}{N_0}}\sin(\phi + \theta_m)\right] \end{aligned} \tag{18}$$

and

$$\begin{aligned} P_s'(E) = {} & \frac{p(1 - p_t)}{2}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}(1 - |\xi|)\cos(\phi + \theta_m)\right] + \frac{pp_t}{2}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}\left(1 - \frac{|\xi|}{2}\right)\cos(\phi + \theta_m)\right] \\ & + \frac{q(1 - p_t)}{2}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}(1 - |\xi|)\cos(\phi + \theta_m)\right] + \frac{qp_t}{2}\text{erfc}\left[\sqrt{\frac{E_s}{N_0}}\left(1 - \frac{|\xi|}{2}\right)\cos(\phi + \theta_m)\right] \end{aligned} \tag{19}$$

respectively, where the transition density is $p_t = 2pq$ for purely random data. For the PCM/PM/NRZ SER given in Eq. (18), use the upper sign for positive data asymmetry. For negative data asymmetry, on the other hand, use the lower sign and interchange the p and q. Note that setting $p = q = 0.5$ for the PCM/PM/NRZ and PCM/PM/bi-ϕ results in the SERs given in Eqs. (14) and (15), and setting ξ=0 results in the SERs given in Eqs. (10) and (11), respectively.

Analogous to the separate effects of unbalanced and asymmetric data, the combined effect on the PCM/PM/NRZ and PCM/PM/bi-ϕ receiver can be illustrated by a vector diagram similar to Fig. 3 but with $\sqrt{P_d}(p - q)$ replaced by $\sqrt{P_d}(p - q + 2\xi qp)$ and $\sqrt{P_d}(\xi(1 - pq))$, respectively. For the combined case, the impact of unbalanced data and data asymmetry can constructively add or subtract from the phase bias. Figure 7(a) illustrates that the phase bias for PCM/PM/NRZ is dominated by the

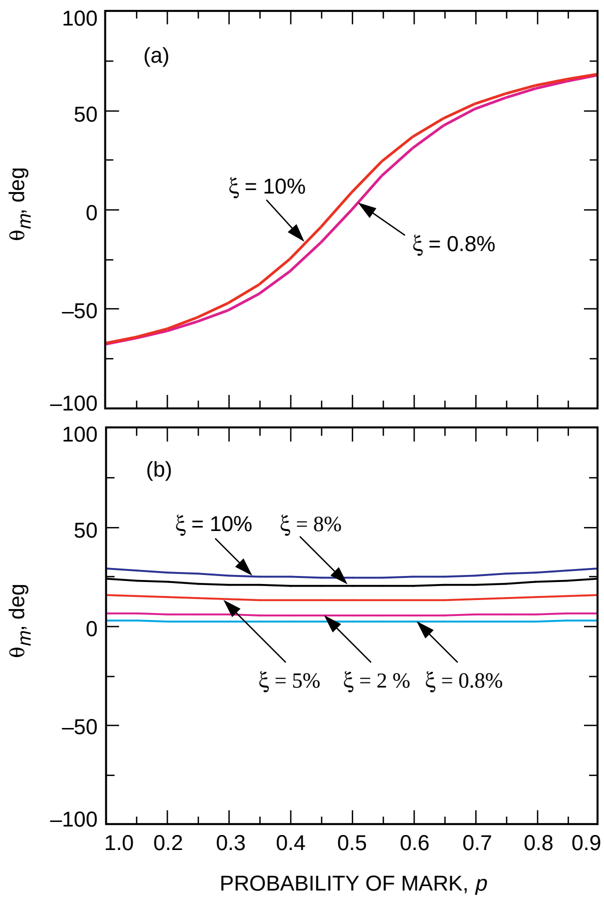

Fig. 7. The impact of data unbalance and data asymmetry on the carrier phase for (a) PCM/PM/NRZ and (b) PCM/PM/bi-ϕ with m_T = 1.25.

unbalanced data. In contrast to PCM/PM/NRZ, Fig. 7(b) shows that the phase bias for PCM/PM/bi-ϕ is dominated by the data asymmetry.

IV. PLL Performance in the Presence of Imperfect Data

In the previous section, the conditional SER probability, $P_s'(E)$, was derived for PCM/PM/NRZ and PCM/PM/bi-ϕ modulation. To obtain the unconditional SER probability, $P_s(E)$, as defined in Eq. (6), the carrier loop SNR must first be derived. In this section, we consider the impact of unbalanced and asymmetric data on the PLL synchronization. Intuitively, from the vector diagram representation point of view given in the previous sections, we anticipate the carrier loop SNR to increase with greater data imperfections. We assume a digital PLL as shown in Fig. 2(b). In order to derive the carrier loop SNR assuming linear theory, the following two parameters must first be found: (1) the S-curve and (2) the PSD of the effective noise that falls inside the PLL bandwidth.

We begin the analysis by deriving the error signal produced by multiplying Eqs. (4) and (5) and low-pass filtering. Ignoring the double frequency terms and normalizing by $\sqrt{P_c}$, the error signal becomes

$$
\begin{aligned}
e(\phi) = & \; [P(t)d(t)\tan(m_T)\sin(\theta_m) + \cos(\theta_m)]\sin(\phi) \\
& + [P(t)d(t)\tan(m_T)\cos(\theta_m) - \sin(\theta_m)]\cos(\phi) + \frac{n'(t)}{\sqrt{P_c}}
\end{aligned} \tag{20}
$$

for PCM/PM/NRZ and PCM/PM/bi-ϕ, respectively, where θ_m is given in Eqs. (16) and (17). The mean of $e(\phi)$ is referred to as the loop S-curve and is given as

$$
\begin{aligned}
g(\phi) = & \; [(p - q + 2pq\xi)\tan(m_T)\sin(\theta_m) + \cos(\theta_m)]\sin(\phi) \\
& + [(p - q + 2pq\xi)\tan(m_T)\cos(\theta_m) - \sin(\theta_m)]\cos(\phi)
\end{aligned} \tag{21}
$$

and

$$
\begin{aligned}
g(\phi) = & \; [\xi(1 - pq)\tan(m_T)\sin(\theta_m) + \cos(\theta_m)]\sin(\phi) \\
& + [\xi(1 - pq)\tan(m_T)\cos(\theta_m) - \sin(\theta_m)]\cos(\phi)
\end{aligned} \tag{22}
$$

for PCM/PM/NRZ and PCM/PM/bi-ϕ, respectively. Moreover, the slopes of the S-curves for PCM/PM/NRZ and PCM/PM/bi-ϕ are given as

$$
K_o \triangleq \frac{dg(\phi)}{d\phi}|_{\phi=0} = [(p - q + 2pq\xi)\tan(m_T)\sin(\theta_m) + \cos(\theta_m)] \tag{23}
$$

and

$$
K_o \triangleq \frac{dg(\phi)}{d\phi}|_{\phi=0} = [(\xi(1 - pq)\tan(m_T)\sin(\theta_m) + \cos(\theta_m)] \tag{24}
$$

respectively. The effective noise from Eq. (20) is the sum of the data component and the noise process: $P(t)d(t)\tan(m_T)\cos(\theta_m) + n'(t)/\sqrt{P_c}$, where the $\sin(\phi) \approx 0$ and $\cos(\phi) \approx 1$. The PSD of the effective noise is then

$$
N_{eff} = \tan^2(m_T)\cos^2(\theta_m)S(w) + \frac{N_o/2}{P_c} \tag{25}
$$

where $S(w)$ is the continuous PSD of the nonideal data, $P(t)d(t)$, which is given in [5]. Unfortunately, the continuous PSD is a complicated expression and cannot be simplified easily to show in this article.

Now the variance of the phase error can be found from

$$
\sigma_\phi^2 = \frac{N_{eff}B_{eff}}{K_o^2} \tag{26}
$$

where B_{eff} is the effective bandwidth of the PLL and is given as

$$B_{eff} = \frac{1}{2\pi j} \oint_{|z|=1} H(z) H\left(z^{-1}\right) \frac{dz}{z} \tag{27}$$

The effective bandwidth is a function of the closed-loop transfer function $H(z)$, which is given as

$$H(z) = \frac{K_o F(z) N(z)}{1 + K_o F(z) N(z)} \tag{28}$$

where $N(z) = T/[z^2(z-1)]$ is the numerically controlled oscillator (NCO) transfer function, and F(z) is the loop filter transfer function; a first-order loop filter has the form $F(z) = 4B_L$ and a second-order loop filter has the form

$$F(z) = G_1 + \frac{G_2}{(1 - z^{-1})} \tag{29}$$

where $G_1 = rd/T_s$; $G_2 = rd^2/T_s$; $d = 4B_L T_s/(r-1)$, which is typically set to 2 or 4; B_L is the designed loop bandwidth; and T_s is the PLL update time.

After proper substitution, the variance of the phase jitter in Eq. (26) can be found. Consequently, the loop SNR can now be found by taking the inverse of the phase jitter. Setting $\xi = 0$ and $p = 0.5$ in Eq. (26) results in a loop SNR

$$\rho = \tan(m_T) \frac{B_L}{R_b} + \frac{P_c}{N_o/2} \tag{30}$$

which is identical to that given in [3].

Using Eq. (26), the PCM/PM/NRZ and PCM/PM/bi-ϕ loop SNRs for various B_L/R_b ratios are shown in Figs. 8(a) and 8(b), respectively. As expected, for PCM/PM/NRZ, the carrier loop SNR increases significantly as p deviates from 0.5. This is not the case for PCM/PM/bi-ϕ since p does not impact this modulation significantly.

V. Numerical and Simulation Results

All simulation presented in this article was carried out using the Signal Processing WorkSystem (SPW) software. The numerical and simulation SER performances for unbalanced data, data asymmetry, and the combination of both are presented below. The data unbalance and data asymmetry values of $p = 0.55$ and $\xi = 2$ percent, respectively, are used. These values represent the maximum deviations from ideal data that technical studies undertaken by the Consultative Committee for Space Data Systems (CCSDS) RF and Modulation Subpanel show should be permitted [7].

Using Eq. (10), the SER performance for PCM/PM/NRZ was numerically calculated and simulated for unbalanced data, as shown in Fig. 9. This assumes that the carrier loop SNR is infinite so that $P_s(E)$ equals $P'_s(E)$. Observe that the SER degradation is symmetric around $p = 0.5$; that is, the SER degradation for $p = 0.4$ is the same as that for $p = 0.6$. Figure 9 also shows the theoretical SER

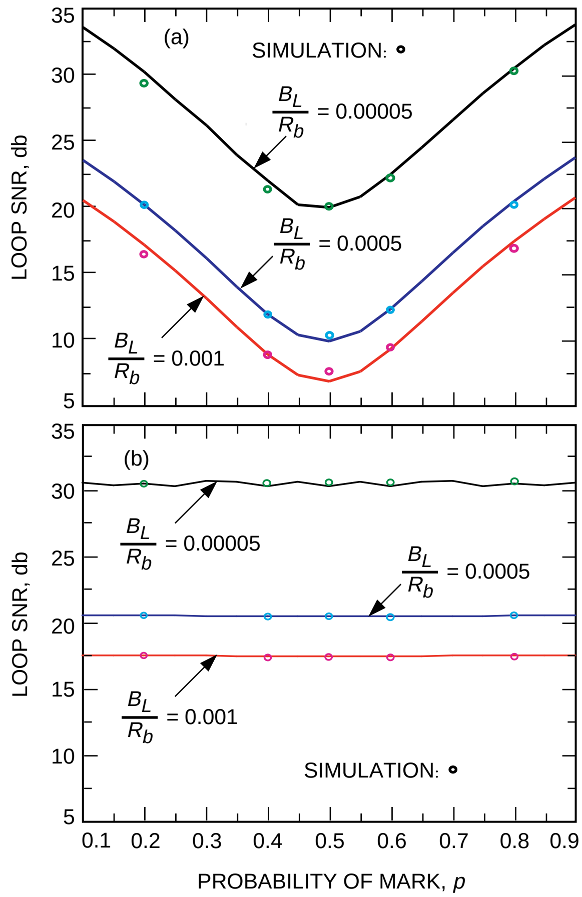

Fig. 8. Carrier loop SNR for (a) PCM/PM/NRZ with ξ = 2 percent and (b) PCM/PM/bi-ϕ with ξ = 10 percent.

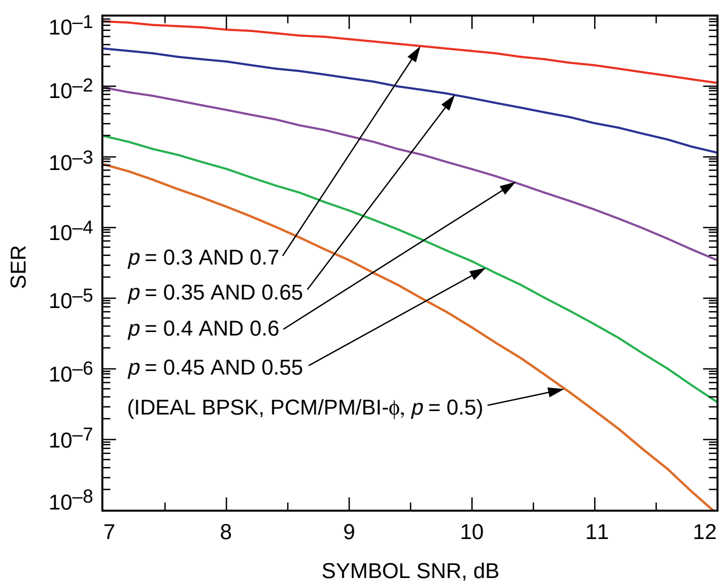

Fig. 9. PCM/PM/NRZ and PCM/PM/bi-ϕ with unbalanced data for m_T = 1.25 rad and infinite loop SNR.

for a PCM/PM/bi-ϕ receiver for unbalanced data. Essentially, unbalanced data have no impact on a PCM/PM/bi-ϕ receiver but result in significant loss for a PCM/PM/NRZ receiver. As we will see, this will not be the case for data asymmetry.

Using Eqs. (14) and (15), the SER performances for PCM/PM/NRZ and PCM/PM/bi-ϕ were evaluated for data asymmetry of 2 percent, again assuming infinite carrier loop SNR. Figure 10 shows the numerical calculation along with the simulation results. Clearly, both PCM/PM/NRZ and PCM/PM/bi-ϕ are impacted by data asymmetry, with PCM/PM/bi-ϕ being impacted more. This is due to the fact that, for PCM/PM/bi-ϕ, data transitions are forced to occur in every NRZ symbol period. Consequently, the impact on the SER performance due to data asymmetry is greater.

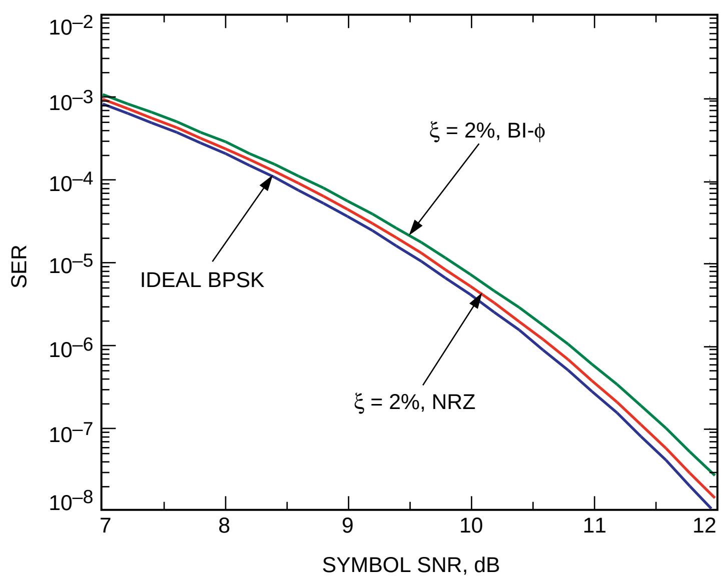

Fig. 10. PCM/PM/NRZ and PCM/PM/bi-ϕ with data asymmetry for m_T = 1.25 rad and infinite loop SNR.

Using Eqs. (18) and (19), the SER performances for PCM/PM/NRZ and PCM/PM/bi-ϕ were evaluated with different combinations of unbalanced data and data asymmetry for infinite carrier loop SNR, as shown in Fig. 11. It is shown that the maximum degradation occurs when p is greater than 0.5 and with positive data asymmetry, or when p is less than 0.5 and with negative data asymmetry. Specifically, $p = 0.6$ and $\xi = 2$ percent and $p = 0.55$ and $\xi = 2$ percent result in worst-case degradation for both receivers. Clearly, the performance of PCM/PM/NRZ is dominated by unbalanced data, while the performance of PCM/PM/bi-ϕ is dominated by data asymmetry. In summary, the performance of PCM/PM/bi-ϕ for different combinations of unbalanced data and data asymmetry ($p = 0.4, 0.45, 0.55,$ and 0.6 and $\xi = 2$ percent) results in better SER performance than that of PCM/PM/NRZ. The SER performance is symmetric for a particular combination of data imbalance and data asymmetry. For example, the degradation from data unbalance of $p_m = 0.45$ and $\xi = 2$ percent is equal to data imbalance of $p_m = 0.55$ and data asymmetry of $\xi = -2$ percent.

For finite carrier loop SNR, Figs. 12(a) and 12(b) show the SER performances for PCM/PM/NRZ and PCM/PM/bi-ϕ, respectively. The nominal ($p = 0.5, \eta = 0$) carrier loop SNRs for PCM/PM/NRZ and PCM/PM/bi-ϕ were set to 20 dB and 31.5 dB, respectively. Both simulation and theory are in agreement to within 0.2 dB. Figure 13, on the other hand, shows the SNR loss at an error probability of 10^{-3} for various data imperfections for PCM/PM/NRZ and PCM/PM/bi-ϕ. Clearly, PCM/PM/NRZ has significant degradation compared to PCM/PM/bi-ϕ.

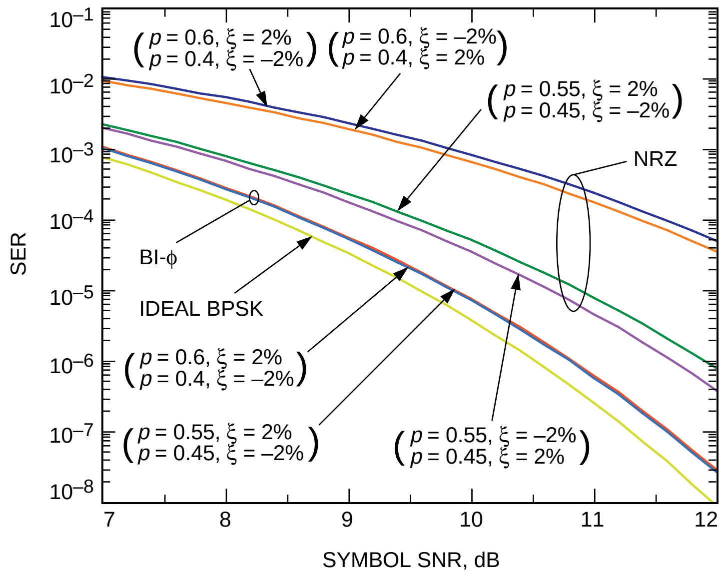

Fig. 11. PCM/PM/NRZ and PCM/PM/bi-ϕ with asymmetric and unbalanced data for m_T = 1.25 rad and infinite loop SNR.

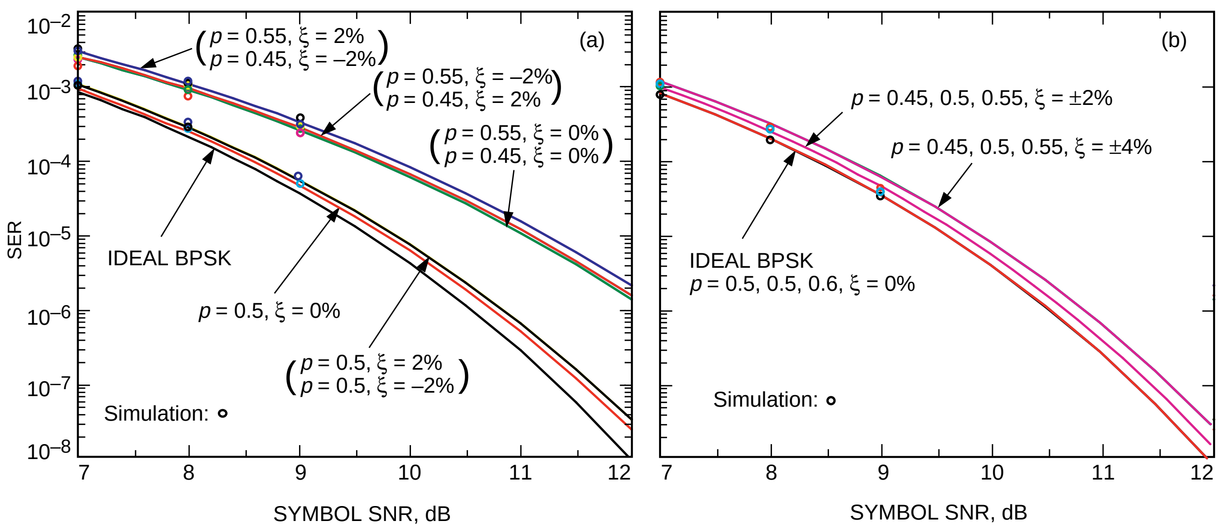

Fig. 12. SER performances for finite carrier loop SNR: (a) PCM/PM/NRZ with asymmetric and unbalanced data for m_T = 1.25 rad and a nominal loop SNR of 20 dB and (b) PCM/PM/bi-ϕ with asymmetric and unbalanced data for m_T = 1.25 rad and a nominal loop SNR of 31.5 dB.

VI. Conclusion

This article studied the impact of unbalanced data and data asymmetry on the performances of PCM/PM/NRZ and PCM/PM/bi-ϕ receivers. The PLL loop SNR and the SER performances have been derived in the presence of imperfect data. The performances of PCM/PM receivers are shown to be critically dependent on the amount of phase bias; the greater the phase bias, the more the SER degradation, and vice versa. It has been shown that unbalanced data can significantly degrade the SER performance of PCM/PM/NRZ, while the SER performance of PCM/PM/bi-ϕ is unaffected. However,

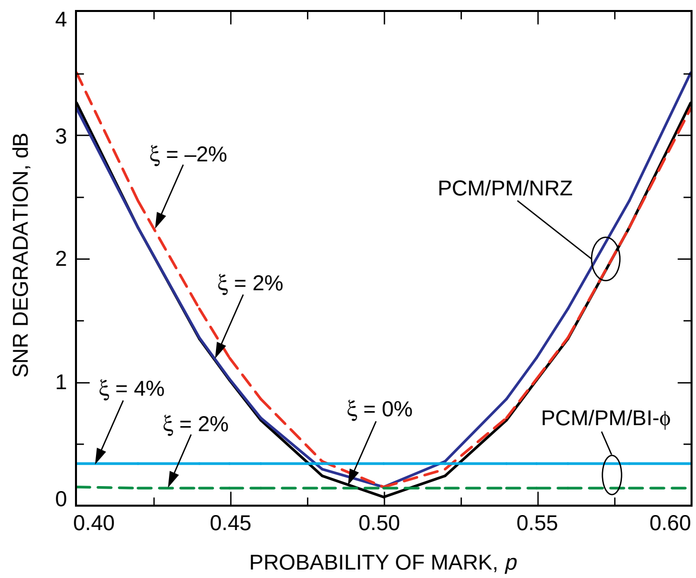

Fig. 13. SNR degradation at SER = 10^{-3} for PCM/PM/NRZ and PCM/PM/bi-ϕ.

with data asymmetry, the SER performances of both PCM/PM/NRZ and PCM/PM/bi-ϕ are impacted, with more impact on the PCM/PM/bi-ϕ receiver. For the combined effects, the SER performance for PCM/PM/NRZ is dominated by unbalanced data, while the SER performance of PCM/PM/bi-ϕ is dominated by the data asymmetry. It is shown that the maximum SER degradation for PCM/PM/NRZ and PCM/PM/bi-ϕ occurs when p is greater than 0.5 and with positive data asymmetry, or when p is less than 0.5 and with negative data asymmetry. For the values of data asymmetry and unbalanced data simulated, the SER performance of PCM/PM/bi-ϕ outperforms that of PCM/PM/NRZ. Finally, note that in this article we have not examined the impact of unbalanced data and data asymmetry on the symbol phase jitter. This issue needs to be examined in the future.

Acknowledgments

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