# Beam Squint Due to Circular Polarization in a Beam-Waveguide Antenna 

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#### Abstract

A short study was performed to demonstrate the beam-squint effect due to the circular polarization in the beam-waveguide system of DSS 24 and to obtain quantitative values for this squint. Beam squint occurs when a circularly polarized feed illuminates a reflector system in an asymmetric or offset manner. It occurs in the plane transverse to the plane of asymmetry, and its direction changes with the sense of polarization. The beam-squint effect for the nonbeam-waveguide DSN antennas is minimal or nonexistent in the nearly symmetrical configuration of the reflectors. In the beam-waveguide systems, however, there are three asymmetric or offset-fed mirrors, M5, M3, and M2, that cause beam squint. It is shown that the squint is caused primarily by the M5 mirror, and the squint caused by the M3-M2 pair of mirrors is mostly canceled due to their mirror-image symmetry. The maximum amount of the calculated squint in the beam-waveguide system is about 2.75 mdeg , and this translates into a swing value of 5.5 mdeg when a feed switch from right to left polarization is made. The resulting beam-pointing error can cause a gain loss of about 0.07 dB and must be taken into account in the beam-calibration procedures. Suggestions are made for future work on the ways to either reduce or entirely remove the squint effects.


## I. Introduction

Beam squint occurs in conic-section reflector antennas illuminated by circularly polarized feeds. The squint occurs when the reflector system causes a depolarization of the incident linear vector field components, and these components are out of phase with respect to each other, as is the case with circularly polarized fields. This situation causes the phase front radiated from the reflector to be nonuniform, "squinting" from the bore sight. Thus, squint exists in circularly symmetric reflector systems if the circularly polarized feed is laterally offset from the focus and, more significantly, in offset systems regardless of whether the circularly polarized feed is on or off axis.

The squint occurs in the plane transverse to the principal offset plane and is toward the "left" as an observer looks in the direction of main-beam wave propagation, when the main beam of the antenna system is right circularly polarized (RCP), and to the "right" for a left circularly polarized (LCP) main beam. Naturally, due to the change in polarization upon reflection, the feed and main beam pattern have the same sense of polarization for an even number of reflectors and opposite senses for an odd number of reflectors. It can be shown $[1,2]$ that, in general, an approximate value for the squint angle $\theta_{s}$ is given by

$$
\sin \left(\theta_{s}\right)=\frac{\sin \left(\theta_{0}\right)}{2 F k}
$$

in which $\theta_{0}$ is the angle between the incident beam and the reflected beam from the main reflector, $F$ is the focal length, and $k$ is the wave number. This formula is valid for single or multiple reflectors where an equivalent focal length, $F_{e}$, is used. This formula can be further approximated to give an estimate of the amount of squint in terms of the $3-\mathrm{dB}$ or half-power beam width of the antenna, $\theta_{h p}$. It is obtained as

$$
\frac{\theta_{s}}{\theta_{h p}}=\frac{\sin \left(\theta_{0}\right)}{4 \pi(F / D)}
$$

As can be seen, the beam-squint angle normalized with respect to the main beam width is directly dependent on the offset angle, $\theta_{0}$, and inversely dependent on the $F / D$ ratio. For $\theta_{0}$ less than unity and $F / D$ of the order of unity, the squint is less than $1 / 10$ of the main beam width. Thus, for larger $F / D$, the squint becomes smaller, especially in dual-reflector systems where equivalent focal length typically is much larger than unity.

In this study, we will concentrate on the analysis of the beam squint in a beam-waveguide system, specifically the beam waveguide of DSS 24 in the Deep Space Network (DSN). The beam-waveguide antenna under consideration consists of a circularly polarized feed illuminating three offset conic reflectors and two flat mirrors, as the field is guided to the focal point of the circularly symmetric Cassegrain reflector system. It is expected, then, that a differential shift off bore sight between the right- and lefthand circularly polarized radiated beams will be found. This article summarizes the conclusions of a short study examining the differential beam squint between a right- and a left-hand circularly polarized fed beam-waveguide antenna. The beam squint was found using a physical optics calculation ${ }^{1}$ of the fields radiated from the feed and through the beam waveguide of DSS 24.

## II. The Beam-Waveguide Antenna

Figure 1 shows the geometry of the beam-waveguide antenna, DSS $24 .{ }^{2}$ This study is conducted at the X-band receive frequency of 8.45 GHz . The beam pattern reflected off the ellipsoidal mirrors M5, M3, and M2 will be examined initially, followed by the main-beam pattern calculation for differing azimuth and elevation directions.

## A. M5 Ellipsoid

Shown in Fig. 2 are plots of the center portion of the beam peak at $\mathrm{F}_{2}$ as it is reflected from the ellipsoidal mirror, M5, for right- and left-circular polarization feed illuminations. The data include the physical optics calculation of reflection off the flat mirror, M6, and are shown in $u-v$ coordinates. The plots indicate two different beam offsets from the center of the pattern coordinate system. The first is an offset to the left (toward $\phi=270 \mathrm{deg}$ ) for both polarizations, due to the offset ellipsoidal geometry of M5. The upward offset (toward $\phi=90 \mathrm{deg}$ ) for right-circular polarization and downward offset (toward $\phi=180 \mathrm{deg}$ ) for left-circular polarization are the beam squints due to the circular polarized illumination.

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Fig. 1. Optics layout of DSS 24.


Fig. 2. Beam plots for (a) left and (b) right circularly polarized feeds for the M6-M5 mirror combination of DSS 24.

## B. M3-M2 Ellipsoids

To isolate the squint due to the $\mathrm{M} 3-\mathrm{M} 2$ combination of mirrors, the X -band horn is placed at $\mathrm{F}_{2}$. Ideally, in a ray-optic limit, the horn-M6-M5 system of mirrors will focus at $\mathrm{F}_{2}$, imaging the horn pattern. By placing the horn at $\mathrm{F}_{2}$, the effects of the $\mathrm{M} 3-\mathrm{M} 2$ system can be examined. Again, ideally in the ray-optic limit, the horn at $\mathrm{F}_{2}$ is imaged through this system to $\mathrm{F}_{1}$. Shown in Fig. 3 are plots of the beam peak at $\mathrm{F}_{1}$ as it is reflected through the M4-M3-M2 system. The elevation angle is 90 deg; the azimuth angle does not enter the calculation since the horn is at $\mathrm{F}_{2}$, above the azimuth rotation axis. As in the single ellipsoid system, an offset of the beam can be separated from the squint due to this mirror combination. The offset is common to both polarizations, shifting the beam toward the local pattern coordinate $\phi=180 \mathrm{deg}$. The squint then shifts the beam upwards and downwards (towards $\phi=90 \mathrm{deg}$ and $\phi=270 \mathrm{deg}$ ) for left- and right-polarized feed patterns, respectively.


Fig. 3. Beam plots for (a) left and (b) right circularly polarized feeds located at $F_{2}$ for the M4-M3-M2 mirror combination of DSS 24.

## III. Main-Beam Calculation

A series of calculations has been performed to predict the squint of the main beam as a function of azimuth and elevation angles. Using the required geometry descriptions in the standard input sections of the physical optics software, ${ }^{3}$ a functional description for the location and orientation of the M2-M1 mirrors (for elevation rotation) and the M5-M4 mirrors (for azimuth rotation) is given in the Appendix. This information can be used as input to the physical optics software for calculating the main-beam pattern for arbitrary elevation and azimuth angles.

Shown in Fig. 4 are the main-beam patterns (about the peak) for left and right circularly polarized feeds located in the pedestal room. The local azimuth angle is 0 deg , and the elevation angle is 90 deg. From these plots, it is seen that there is a beam shift towards $\phi=0 \mathrm{deg}$ and a squint up and down on the plot. By a simple translation of the coordinate center to the peak of the right-circularized beam, the left-circular polarized beam is found to be offset by 5.5 mdeg . The decrease in gain due to this offset is about 0.07 dB .

To examine the effect of the beam offset as a function of azimuth and elevation, beam patterns at different angles were computed. Shown in Fig. 5 is a calculation for a local azimuth of 0 deg and an

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Fig. 4. Main beam plots for (a) left and (b) right circularly polarized feeds located in the pedestal room of DSS 24 for a 90 -deg elevation and a 0 -deg local azimuth.


Fig. 5. Main beam plots for (a) left and (b) right circularly polarized feeds located in the pedestal room of DSS 24 for a 45 -deg elevation and a 0 -deg local azimuth.
elevation angle of 45 deg. Plotted in Figure 6 are the patterns for a local azimuth of 45 deg and an elevation angle of 22.5 deg . Both plots show the same relationship as in the first ( $0-\mathrm{deg}, 90-\mathrm{deg}$ ) example. By translating the pattern coordinate system to the peak of the right circularly polarized beam, the offset of the left circularly polarized beam is 5.5 mdeg , and the gain loss is 0.07 dB for both cases. It should be mentioned that gravity distortion effects as a function of elevation are not included in any of the calculations in this work.

Calculations for other angles gave similar results. The constant decrease in gain and beam offset is explained by the rotation of the flat mirrors, M4 and M1, for azimuth and elevation angle changes, respectively. Flat mirror M1 is rotated about the elevation axis when the antenna is scanned in elevation. This rotation of the flat mirror causes the beam to rotate without refocusing or adding an offset or squint. A similar rotation about the azimuth rotation axis of flat mirror M4 with respect to M5 causes the beam reflected from M5 to rotate but will not add an offset or squint.


Fig. 6. Main beam plots for (a) left and (b) right circularly polarized feeds located in the pedestal room of DSS 24 for a $22.5-$ deg elevation and a 45 -deg local azimuth.

These rotations cause the main beams to rotate about the pointing direction but are constant for both polarizations. The magnitude of the offset of the left circularly polarized beam from the right circularly polarized beam will, therefore, be constant, independent of pointing directions. The direction of the offset, though, will vary as a function of the pointing angle. The decrease in gain is constant with pointing direction.

## IV. Conclusion and a Possible Compensation Method

In conclusion, if the feed is switched from an RCP to an LCP horn, the beam shift due to the squint can be as much as 5.5 mdeg . Since the 3 -dB beam width of the antenna system at the X-band receive frequency of 8.45 GHz is about 60 mdeg , using a simple Gaussian approximation, $(L / 3.0) \approx\left(\theta / \theta_{h p}\right)^{2}$, this beam shift can result in a gain loss of less than 0.1 dB . Therefore, when switching polarization, care must be taken in recalibrating the beam direction to eliminate this beam-pointing error.

As an alternative, a modification of the system may be undertaken to remove the squint effects. Squint in multireflector antenna systems may be compensated for by tilting the feedhorn and relative angles of different mirrors to specifically calculated values. This is similar to providing a squint-free condition in dual-reflector systems [2], which is based on providing a zero cross-polarization condition as first introduced in [3]. To examine if this concept is useful for a beam-waveguide antenna, the amount of main-beam squint produced by the M3-M2 mirror system will be isolated.

In Fig. 3, fields reflected from the $\mathrm{M} 3-\mathrm{M} 2$ mirrors due to the feed at $\mathrm{F}_{2}$ were plotted. Now, in Fig. 7, the main-beam patterns for this case (feed at $\mathrm{F}_{2}$ ) are shown for an elevation angle of 90 deg. It is seen that the squint is minimal, indicating that the dominant portion of main-beam squint is due to the ellipsoid M5. If, somehow, the squint due to M5 is removed, the overall antenna system squint can be reduced substantially. A general solution to this problem will require extending the concept of equivalent paraboloid and equivalent axis to a multireflector system. This can be done using the concept of a "closed path" [4]. It is found that the equivalent axis lies on the closed path. A ray-tracing algorithm can be devised to find this equivalent axis, and this equivalent axis will provide the zero cross-polarization as well as the squint-free condition. This could be the subject of a future study.


Fig. 7. Main beam plots for (a) left and (b) right circularly polarized feeds located at $F_{2}$ of DSS 24 for a 90-deg elevation.

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## Appendix

## Coordinate Transformations for Azimuth and Elevation Rotations of the Mirrors

Figure A- $1^{4}$ is the geometry of the M5-M4 mirror system for use in deriving the coordinate transformations needed for azimuth rotation of the antenna. The azimuth rotation axis is shown and, for use with the physical optics software, the subreflector M5 is rotated an angle - $\phi$ with respect to the main reflector, M4. Since M5 is offset from the rotation axis, a translation and rotation are needed. The new location of the origin of the subreflector system for an arbitrary rotation $\phi$ is

$$
\begin{aligned}
& x_{0_{s^{\prime}}}=\Delta x+R \cos \phi \\
& y_{0_{s^{\prime}}}=R \sin \phi \\
& z_{0_{s^{\prime}}}=\Delta z
\end{aligned}
$$

The unit vectors of the rotated subreflector origin are

$$
\begin{aligned}
& \hat{x}_{s^{\prime}}=\left(-\hat{x}_{m} \cos \phi-\hat{y}_{m} \sin \phi\right) \cos \theta+\hat{z}_{m} \sin \theta \\
& \hat{y}_{s^{\prime}}=-\hat{x}_{m} \sin \phi-\hat{y}_{m} \cos \phi \\
& \hat{z}_{s^{\prime}}=\left(-\hat{x}_{m} \cos \phi-\hat{y}_{m} \sin \phi\right) \sin \theta+\hat{z}_{m} \cos \theta
\end{aligned}
$$

which, using the formulation given by Jamnejad, ${ }^{5}$ result in new Euler angles to describe the rotated subreflector system:

$$
\begin{aligned}
& \alpha=\phi-90 \mathrm{deg} \\
& \beta=180 \operatorname{deg}-\phi \\
& \gamma=90 \mathrm{deg}
\end{aligned}
$$

Figure A-2 shows the geometry of the M2-M1 mirror system for use in deriving the coordinate transformation needed for elevation pointing. The elevation rotation axis is shown and, for use with the physical optics software, subreflector M2 is rotated an angle $-\theta$ with respect to the main reflector, M1. Since the M2 axis coincides with the rotation axis, only a rotation is needed. The unit vectors of the rotated subreflector origin are

[^2]\[

$$
\begin{aligned}
& \hat{x}_{s^{\prime}}=-\hat{x}_{m} \cos \theta+\hat{y}_{m} \sin \theta \\
& \hat{y}_{s^{\prime}}=+\hat{x}_{m} \sin \theta+\hat{y}_{m} \cos \theta \\
& \hat{z}_{s^{\prime}}=-\hat{z}_{m}
\end{aligned}
$$
\]

which result in a new set of Euler angles to describe the rotated subreflector system:

$$
\begin{aligned}
& \alpha=90 \mathrm{deg}-\theta \\
& \beta=180 \mathrm{deg} \\
& \gamma=-90 \mathrm{deg}
\end{aligned}
$$



Fig. A-1. Geometry of the M5-M4 mirror system for azimuth rotation.


Fig. A-2. Geometry of the M2-M1 mirror system for elevation rotation.


[^0]:    ${ }^{1}$ R. Hodges and W. Imbriale, Computer Program POMESH for Diffraction Analysis of Reflector Antennas, JPL unpublished report (internal document), Jet Propulsion Laboratory, Pasadena, California, February 1992.
    2 Additional details on mirror specifications, horn patterns, and geometry can be found in W. Veruttipong, RF Design and Expected Performances of a 34-Meter Multifrequency Beam-Waveguide Antenna, TDA/DSN 890-261, JPL D-11853 (internal document), Jet Propulsion Laboratory, Pasadena, California, July 1994.

[^1]:    ${ }^{3}$ R. Hodges and W. Imbriale, op cit.

[^2]:    ${ }^{4}$ See Fig. B-2 in W. Veruttipong, op cit.
    ${ }^{5}$ V. Jamnejad, "Calculating Eulerian Angles for Coordinate Transformations in Reflector Antenna Design and Analysis," JPL Interoffice Memorandum 3327-94-158 (internal document), Jet Propulsion Laboratory, Pasadena, California, September 17, 1994.

