# Cassini as a Narrowband Detector of Gravitational Radiation

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*A filtering technique allowing the reduction of two-way Doppler frequency fluctuations from spatially localized noise sources (like the frequency fluctuations introduced by the troposphere, the ionosphere, or the mechanical vibrations of the ground antenna) is presented in the context of the Cassini Doppler tracking searches for gravitational radiation. This method takes advantage of the sinusoidal behavior of the transfer function to the Doppler observable of these noise sources, which displays sharp nulls at selected Fourier components.*

*The gravitational wave signal remaining at these frequencies makes this Doppler tracking technique the equivalent of a series of narrowband detectors of gravitational radiation [1], distributed across the low-frequency band. Estimates for the sensitivities achievable with the future Cassini Doppler tracking experiments are presented for different classes of gravitational wave signals.*[2]

## I. Introduction

Doppler tracking of interplanetary spacecraft is the only existing technique that allows searches for gravitational radiation in the millihertz frequency region [2]. The frequency fluctuations induced by the intervening media have severely limited the sensitivities of these experiments. Among all the propagation noise sources (ionosphere, troposphere, and interplanetary plasma) observed at high radio frequencies, the troposphere is the largest and the hardest to calibrate to a reasonably low level. The frequency fluctuations due to this noise source have been observed with water vapor radiometers to be a few parts in $10^{-14}$ at a 1000-second integration time [3,4].

Mechanical vibrations of the ground antenna also introduce frequency fluctuations in the Doppler data, and on some occasions represent the limits of the sensitivity of these gravitational wave experiments. Recent frequency stability measurements performed on the antenna of the Deep Space Network (DSN) that will track the Cassini spacecraft at Ka-band (32 GHz) indicate that the frequency fluctuations it introduces are as large as $3.0 \times 10^{-15}$ at a 1000-second integration time.[3] This number is comparable to

[1] Communications Systems and Research Section.

[2] M. Tinto and J. W. Armstrong, "Spacecraft Doppler Tracking as a Narrow-Band Detector of Gravitational Radiation," Jet Propulsion Laboratory, Pasadena, California, in preparation.

[3] M. Gatti, M. Tinto, G. Morris, R. Perez, S. Petty, P. Cramer, P. Kuhnle, M. Gudim, D. Rogstad, L. Riley, and M. Marina, "Cassini Radio Science Ground System Cost and Schedule Review," Viewgraph Presentation (internal document), Jet Propulsion Laboratory, Pasadena, California, 1997.

the requirement in frequency stability requested by the experimenters for the *overall* performance of the DSN during the Cassini gravitational wave experiments at Ka-band. Although work is in progress for identifying the causes of this performance, it might turn out to be intrinsically impossible to compensate for or calibrate the mechanical vibrations of the antenna.

It was first pointed out by Estabrook [5] that the explicit frequency-domain transfer function to the Doppler observable of tropospheric and antenna mechanical noise is sinusoidal. This article shows how this modulation of the noise spectrum can be exploited in a very robust way when the magnitude of these frequency fluctuations is larger than the remaining noise sources. Since Cassini will be tracked coherently at Ka-band, with auxiliary links at X-band (≈8 GHz), the frequency fluctuations introduced by the interplanetary plasma in the Doppler data during opposition will be negligible with respect to those due to the mechanical vibrations of the ground antenna and the troposphere.

An overview of the article is now presented. The transfer functions of the different noise sources entering into the Doppler observable are derived in Section II. In particular, it is shown that the power spectral density of the overall noise affecting the Cassini Doppler data will have sharp minima at selected Fourier frequencies [1]. This is because the transfer function of the noise associated with the residual uncalibrated tropospheric noise and the noise due to the mechanical vibrations of the antenna are sinusoidal. The strain sensitivity that Cassini will be able to achieve with this technique is derived in Section III. A value of $5 \times 10^{-17}$ at $2.0 \times 10^{-3}$ Hz is estimated under the assumptions that Cassini will be tracked simultaneously at X- and Ka-bands for an observing time of 14 days. Finally, Section IV provides the conclusions.

## II. The Two-Way Doppler Response

In the Doppler tracking technique used in searches for gravitational radiation, a distant interplanetary spacecraft is monitored from Earth through a radio link, and the Earth and the spacecraft act as free-falling test particles. A radio signal of nominal frequency $\nu_0$ is transmitted to the spacecraft and coherently transponded back to Earth, where the received signal is compared to a signal referenced to a highly stable clock. Relative frequency changes, $\Delta\nu/\nu_0$, as functions of time, are measured. When a gravitational wave crossing the solar system propagates through the radio link, it causes small perturbations in $\Delta\nu/\nu_0$, which are replicated three times in the Doppler data, with maximum spacing given by the two-way light propagation time between the Earth and the spacecraft [2]. These three events in the time series can be thought of as being due to the gravitational wave buffeting the Earth and the spacecraft and to the original Earth buffeting event being transponded back to the Earth at a time $2L/c$ later, where $L$ is the distance between the Earth and the spacecraft and $c$ is the speed of light.

Detection of gravitational wave signals in this millihertz band is complicated by various noises. The principal noise sources are frequency-standard (clock) and frequency-distribution noise, unmodeled motion of the antenna, thermal noise, noise due to the electronic components on board the spacecraft, unmodeled motion of the spacecraft, and frequency scintillation as the radio beams pass through irregularities in the troposphere, ionosphere, and solar wind [3,6,7].[4] Since it is expected that the Cassini Doppler data will have very low levels of electronic and thermal noise, that the charged-particle scintillations will be removed through simultaneous tracking at X- and Ka-bands, and, furthermore, that the tropospheric scintillation will be partially calibrated by using an advanced water vapor radiometer, it follows that residual uncalibrated troposphere and antenna mechanical noise will be the leading noise sources in the gravitational wave experiments.[5]

If a set of Cartesian orthogonal coordinates $(X, Y, Z)$, in which the wave is propagating along the $Z$-axis and $(X, Y)$ are two orthogonal axes in the plane of the wave, is introduced, then the Doppler response at time $t$ can be written in the following form [1–3]:

[4] Ibid.

[5] Ibid.

$$y(t) \equiv \left(\frac{\Delta\nu(t)}{\nu_0}\right) = -\frac{(1-\mu)}{2}h(t) - \mu\, h(t-(1+\mu)L) + \frac{(1+\mu)}{2}h(t-2L)$$

$$+ C(t-2L) - C(t) + 2B(t-L) + T(t-2L) + T(t)$$

$$+ A_E(t-2L) + A_{sc}(t-L) + TR(t-L) + EL(t) + P(t) \qquad (1)$$

where $h(t)$ is equal to

$$h(t) = h_+(t)\cos(2\phi) + h_\times(t)\sin(2\phi) \qquad (2)$$

Here, $h_+(t)$ and $h_\times(t)$ are the wave's two amplitudes with respect to the $(X, Y)$ axis, $(\theta, \phi)$ are the polar angles describing the location of the spacecraft with respect to the $(X, Y, Z)$ coordinates, and $\mu$ is equal to $\cos\theta$. Also, the units are such that the speed of light, $c$, is equal to 1.

In Eq. (1), $C(t)$ is the random process associated with the frequency fluctuations of the clock on the Earth; $B(t)$ is the joint effect of the noise from buffeting of the probe by nongravitational forces and from the antenna of the spacecraft; $T(t)$ represents the joint frequency fluctuations due to the troposphere, ionosphere, and ground antenna; $A_E(t)$ is the noise of the radio transmitter on the ground; $A_{sc}(t)$ is the noise of the radio transmitter on board; $TR(t)$ is the noise due to the spacecraft transponder; $EL(t)$ is the noise from the electronics on the ground; and $P(t)$ takes into account the fluctuations due to the interplanetary plasma. Since the link frequency is very large with respect to the plasma frequency, the plasma-induced frequency fluctuations are inversely proportional to the square of the radio frequency. By using high-frequency radio signals or by monitoring two different radio frequencies transmitted to the spacecraft and coherently transmitted back to Earth, this noise source can be either suppressed to very low levels or entirely removed from the data, respectively [8]. The Cassini spacecraft will be tracked simultaneously at X- and Ka-bands by only one of the three DSN complexes over the 40 days of each of the three experiments. During the remaining two-thirds of the time, Doppler tracking will be performed at X-band only.

From Eq. (1), it is easy to see that gravitational wave pulses of durations longer than the round-trip light time, $2L$, give a Doppler response, $y(t)$, that, to first order, tends to zero. The tracking system essentially acts as a passband device, in which the low-frequency limit, $f_l$, is roughly equal to $(2L)^{-1}$ Hz, and the high-frequency limit, $f_H$, is set by the thermal noise in the receiver. Since the reference clock and some electronic components are most stable at integration times around 1000 seconds, Doppler tracking experiments are performed when the distance to the spacecraft is of the order of a few astronomical units (AU). This sets the value of $f_l$ for a typical experiment to about $10^{-4}$ Hz, while the thermal noise gives an $f_H$ of about $3 \times 10^{-2}$ Hz.

It is important to note the characteristic time signatures of the clock noise, $C(t)$; of the probe antenna and buffeting noise, $B(t)$; of the troposphere, ionosphere, and ground antenna noise, $T(t)$; and of the transmitters, $A_E(t)$ and $A_{sc}(t)$. The time signature of the clock noise can be understood by observing that the frequency of the signal received at time $t$ contains clock fluctuations transmitted $2L$ seconds earlier. By subtracting from the received frequency the frequency of the radio signal transmitted at time $t$, the clock frequency fluctuations also are subtracted [1–3,5], with the net result shown in Eq. (1).

As far as the fluctuations due to the troposphere, ionosphere, and ground antenna are concerned, the frequency of the received signal is affected at the moment of reception as well as $2L$ seconds earlier. Since the frequency of the signal generated at time $t$ does not yet contain any of these fluctuations, it follows that $T(t)$ is positive correlated at the round-trip light time, $2L$ [1–3,5]. The time signature of the noises $B(t)$, $A_E(t)$, $A_{sc}(t)$, and $TR_{sc}(t)$ in Eq. (1) can be understood through similar considerations.

Denoting the Fourier transform of the Doppler response, $y(t)$, by $\widetilde{y}(f)$,

$$\widetilde{y}(f) = \int_{-\infty}^{\infty} y(t) e^{2\pi i f t} dt \tag{3}$$

Eq. (1) can be rewritten in the Fourier domain as follows:

$$\begin{aligned}\widetilde{y}(f) = & \left[ -\frac{(1-\mu)}{2} - \mu e^{2\pi i f (1+\mu) L} + \frac{(1+\mu)}{2} e^{4\pi i f L} \right] \widetilde{h}(f) + \widetilde{C}(f) \left[ e^{4\pi i f L} - 1 \right] \\ & + \widetilde{T}(f) \left[ e^{4\pi i f L} + 1 \right] + 2\widetilde{B}(f) e^{2\pi i f L} + \left[ \widetilde{A_{sc}}(f) + \widetilde{TR}(f) \right] e^{2\pi i f L} \\ & + \widetilde{A_E}(f) e^{4\pi i f L} + \widetilde{EL}(f) + \widetilde{P}(f) \end{aligned} \tag{4}$$

If the noise due to the troposphere, ionosphere, and mechanical vibrations, $T$, will dominate over the remaining noise sources, as might be the case during the Cassini experiments, then the spectra of the noise will appear modulated and will display minima at the following frequencies:

$$f_k = \frac{(2k-1)}{4L}, \qquad k = 1, 2, 3, \ldots \tag{5}$$

At these frequencies, Eq. (4) can be rewritten in the following approximate form [1]:[6]

$$\begin{aligned}\widetilde{y}(f_k) \approx & \left[ -1 + i(-1)^k \mu e^{(\pi/2) i (2k-1) \mu} \right] \widetilde{h}(f_k) - 2\widetilde{C}(f_k) + \widetilde{T}(f_k)(\pi i L \Delta f) \\ & + i(-1)^{k+1} \left[ 2\widetilde{B}(f_k) + \widetilde{TR}(f_k) + \widetilde{A_{sc}}(f_k) \right] - \widetilde{A_E}(f_k) + \widetilde{EL}(f_k) + \widetilde{P}(f_k) \end{aligned} \tag{6}$$

where $\Delta f$ is the frequency resolution in the Fourier domain.

For current-generation precision Doppler experiments, utilizing approximately 8-GHz radio links, observations in the antisolar hemisphere have significant contributions from tropospheric and extended solar wind scintillation, while ionospheric, frequency standard, and antenna mechanical noise are secondary disturbances in the Doppler link.[7] As an example, the temporal autocorrelation function of 10-second time-resolution Mars Global Surveyor (MGS) data taken on April 17, 1997, when the two-way light-time $2L/c$ was equal to 504 seconds, is shown in Fig. 1. The clear positive correlation at a time lag of 504 seconds indicates that tropospheric scintillation and fluctuations induced by mechanical vibrations of the ground antenna dominate the noise in this data set.

The inset plot in Fig. 1 shows the power spectrum of the MGS data, with the frequency scale marked in units of $1/2L$. Although the spectrum has not been averaged [9], the cosine-squared modulation is evident, showing clearly identifiable nulls at odd multiples of $1/(4L)$. At these frequencies, fluctuations

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[6] M. Tinto and J. W. Armstrong, op cit.

[7] J. W. Armstrong, "Radiowave Phase Scintillation and Precision Doppler Tracking of Spacecraft," submitted for publication to *Radio Science*, 1998.

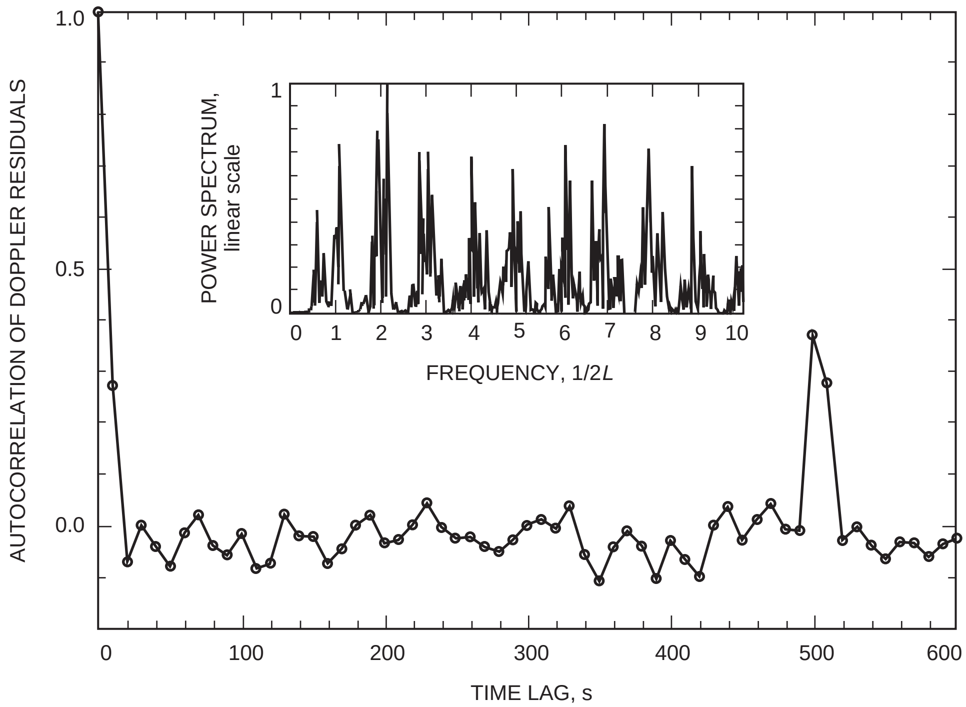


**Fig. 1. The temporal autocorrelation function of 10-s time-resolution MGS data taken on April 17, 1997, when the two-way light-time, 2*L*, was equal to 504 s. The inset plot shows the power spectrum, with the frequency scale marked in units of 1/2*L*.**

from other noise sources will dominate the power spectrum. If the spectral level of these secondary noises is low, there is a potentially large improvement in signal-to-noise ratio (SNR) for gravitational wave signals having Fourier power at the nulls of the troposphere/antenna mechanical transfer function. In its simplest form, filtering of the data to pass a comb of narrow bands centered on the nulls of the cosine-squared transfer function blocks the troposphere/antenna mechanical noise while passing gravity wave power at these frequencies. This is robust in that nothing needs to be known about the signal except that it must have power at odd multiples of $1/(4L)$.

The estimated one-sided power spectrum of the noise that will affect the Cassini Doppler data in their frequency band of observation is presented in Fig. 2.[8] The lower curve represents the configuration in which 80 percent of the noise due to the troposphere is calibrated out by means of water vapor radiometry, while the upper curve corresponds to the configuration without calibration of the troposphere. Note that the minima of both curves coincide at the nulls of the transfer function of the random process $T$.

In order to derive the Fourier transform of the Doppler response in Eq. (6), the assumption was made that the distance to the spacecraft remains constant for the duration of the experiment. In the case of the MGS experiment, it turns out that one can integrate coherently over a time scale equal to about 8 hours before the frequencies $f_k$ change by an amount larger than the frequency resolution, $\Delta f$. More relevant, however, is the magnitude of the round-trip light-time correlated noise level with respect to magnitude of the remaining noises near the frequencies $f_k$. A variation in the distance to the spacecraft smears the spectrum of the round-trip light-time correlated noise, reducing the depth of the nulls at the frequencies $f_k$. Although an *exact* null, therefore, is not achieved, the noise level still is reduced to that of the remaining noise sources over some range of frequencies near the nominal frequencies $f_k$. This is

[8] M. Gatti et al., op cit.

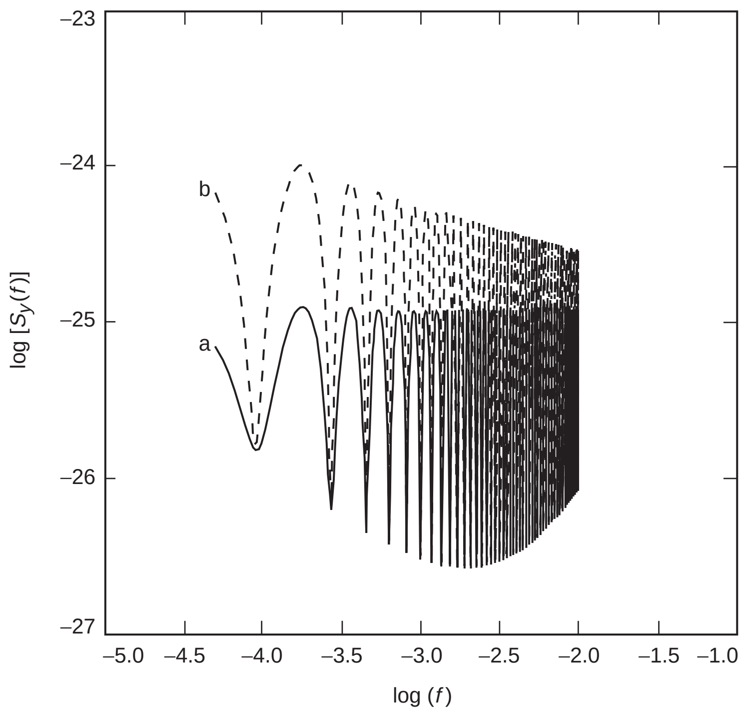


**Fig. 2. The estimated one-sided power spectral density of the noise that will affect the Cassini Doppler data. Curve "a" represents the configuration in which 80 percent of the noise due to the troposphere is calibrated out by means of water vapor radiometry, and curve "b" corresponds to the configuration without calibration of the troposphere.**

the case for the Cassini 2001–2002 solar opposition, when the time variation of the round-trip light-time[9] is sufficiently small that the round-trip light-time correlated noise still is suppressed to below the remaining noise level near the frequencies $f_k$. It is important to point out that, if we assume a frequency resolution $\Delta f = 10^{-6}$ Hz and the main distance $L = 5.5$ AU, corresponding to the first Cassini opposition, the Fourier components of the noise, $\widetilde{T}(f_k)$, will be reduced by a factor $\pi L \Delta f = 1.7 \times 10^{-2}$ at the frequencies $f_k$.

## III. Sensitivities With Cassini

From the plot given in Fig. 2 of the estimated one-sided power spectral density of the noise affecting the Cassini Doppler data, it is possible to calculate the root-mean-squared (rms) noise level, $\sigma(f_k)$, of the frequency fluctuations in the bins of width $\Delta f$, around the frequencies $f_k (k = 1, 2, 3, \ldots)$. This is given by the following expression:

$$\sigma(f_k) = [S_y(f_k)\Delta f]^{1/2} , \qquad k = 1, 2, 3, \ldots \tag{7}$$

where $S_y(f_k)$ is the one-sided power spectral density of the noise sources in the Doppler response, $y(t)$, at the frequency $f_k$.

This measure of the Doppler sensitivity is appropriate for sinusoidal or stochastic gravitational wave signals, while it overestimates the sensitivity to bursts. A detailed and quantitative analysis for various

[9] *Cassini Mission Plan*, JPL D-5564, Rev. F (internal document), Jet Propulsion Laboratory, Pasadena, California, 1995.

burst waveforms will be investigated in a future article. One should keep this observation in mind, therefore, when considering the quantitative results implied by the formula given in Eq. (7).

Figure 3 gives the sensitivity curve achievable with this technique when one applies it to the Cassini Doppler data during the first solar opposition. This curve corresponds to a Earth–spacecraft distance equal to $L = 5.5$ AU, with the fundamental frequency, $f_1$, equal to $9.0 \times 10^{-5}$ Hz. It also assumes an integration time of about 14 days, since only one of the three DSN complexes will have the capability of supporting simultaneous two-way tracking at X- and Ka-bands. If one assumes entire calibration of the plasma noise with the use of dual frequencies, then at $f_1 = 9.0 \times 10^{-5}$ we find a sensitivity equal to about $2.0 \times 10^{-16}$. The best sensitivity, however, is at $2.0 \times 10^{-3}$, at a level of about $5.0 \times 10^{-17}$.

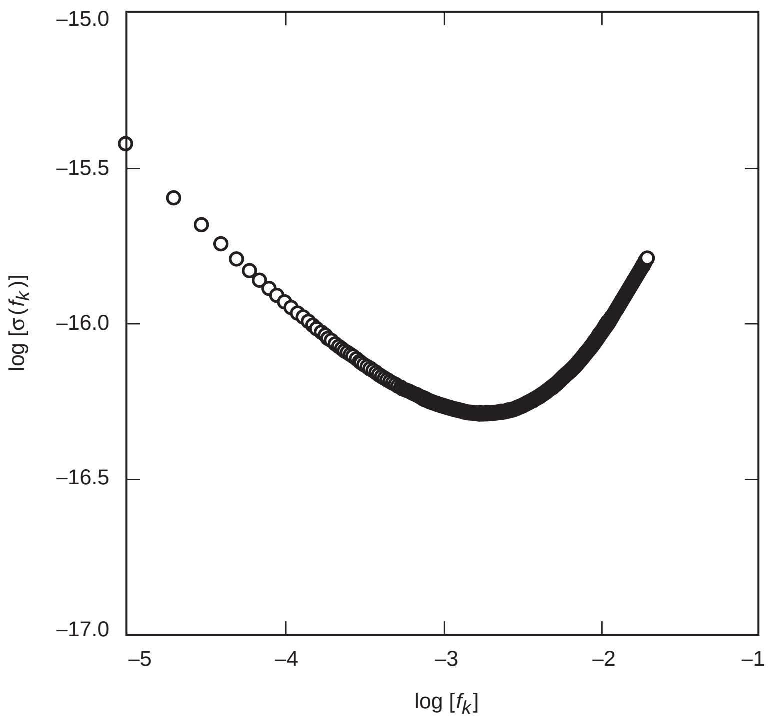


**Fig. 3. The sensitivity curve achievable with the Cassini Doppler data during the first solar opposition. The distance to the spacecraft is about $L$ = 5.5 AU, and the corresponding fundamental frequency is equal to $9.0 \times 10^{-5}$ Hz.**

## IV. Conclusions

The main result of this analysis is to show that it is possible to reduce the effects of the troposphere, ionosphere, and mechanical vibrations of the ground antenna at selected Fourier components of the power spectrum of two-way Doppler tracking data used for gravitational wave searches. A sensitivity equal to about $5.0 \times 10^{-17}$ at a frequency of $2.0 \times 10^{-3}$ has been estimated for the future Cassini gravitational wave experiments, which will first be performed in December 2001.

The experimental technique presented in this article can be extended to a configuration with two spacecraft tracking each other through microwave or laser links. In this case, the dominant noise source is the reference clock driving the coherent two-way link. As shown in Eq. (4), the transfer function of the random process, $\widetilde{C}(f)$, has nulls at frequencies that are multiple integers of the inverse of the round-trip light time, $2L/c$, making this spacecraft-to-spacecraft Doppler tracking a narrowband interferometer detector of gravitational waves.[10] Future space-based laser interferometric detectors of gravitational waves [10], for instance, could implement this technique as a backup option if failure of some of their components made normal interferometric operation impossible. This will be investigated in a future article.

## Acknowledgment

It is a great pleasure to thank Dr. J. W. Armstrong for many helpful comments and his constant and continuous encouragement.

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[10] M. Tinto, "Spacecraft-to-Spacecraft Doppler Tracking as a Xylophone Interferometer Detector of Gravitational Radiation," Jet Propulsion Laboratory, Pasadena, California, in preparation.