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Determination of the Tropospheric Fluctuation Coefficients in VLBI Parameter Estimates

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In radiometric very long baseline interferometry (VLBI), the tropospheric fluctuation of the refractivity alters the estimated parameters, e.g., baselines and radio source positions. To alleviate the effect of the tropospheric fluctuation on the estimated parameters, a covariance matrix based on atmospheric fluctuation theory may be used to weight the delay and delay-rate observables. In general, the application of this covariance matrix reduces the baseline-length scatter about a time–linear model of tectonic motion. This covariance matrix is a linear function of site-dependent Kolmogorov fluctuation coefficients C_n^2*. Here, a method is presented for determining these coefficients from the VLBI data themselves in an iterative process of least-squares parameter estimation. A variant of this method also could be applied to Global Position System (GPS) data reduction.*

I. Introduction

In lieu of actual measurements, there are two ways to reduce the effects of observables with random characteristics such as the wet tropospheric delay fluctuation in very long baseline interferometry (VLBI) and Global Positioning System (GPS) observations. In the first method, the observable is modeled and its parameters are estimated in the statistical least-squares fit; thus, the effect is reduced in the residuals of the statistical estimation. This can be accomplished either by simply estimating the model parameters sequentially at certain time intervals or by using a Kalman-filtering method. In the latter method, characteristics of temporal random walk are assumed for the model parameters in order to achieve a much higher temporal resolution of estimation in contrast with the simpler estimation of model parameters. In the second method, we do not estimate the values of the observables but only their statistical correlation and variance. Thus, the effect itself is not reduced but only its biasing influence on other statistically estimated parameters. This method allows for taking account of not only temporal but also spatial characteristics of the random process and is the subject of this article.

About 10 years ago, the idea was developed that a covariance matrix of tropospheric fluctuation could weight the VLBI observables in the least-squares statistical parameter fit [1]. Soon after it was recognized that the site-dependent Kolmogorov fluctuation coefficients ${C_{n,i}}^2$ could be determined from the VLBI parameter estimation itself and then reapplied in a statistical fit using the tropospheric delay-fluctuation covariance matrix in the observation weighting algorithm. The tropospheric fluctuation covariance matrix was incorporated into the radiometric VLBI parameter estimation program MODEST [2–4] in 1990

[1] Tracking Systems and Applications Section.

[2] Referred to station i.

with fixed coefficients $C_{n,i}$. However, the adjustment of the coefficients $C_{n,i}$ was not fully developed and implemented until late 1993.[3] The purpose of this article is two-fold. First, a description of the iterative method of fluctuation-coefficient determination is given. Second, results of analyses of the JPL radiometric VLBI data are shown that support this statistical weighting algorithm. The presence of systematic errors can make it very difficult to evaluate and tune the tropospheric covariance weighting algorithm. Indeed, the tuning process helped to discover and eliminate problems caused by peculiar data points and inadequacies in estimation methods, e.g., an inadequate treatment of the baseline shift caused by the Landers earthquake in California in 1992.

II. A Determination of C_n From VLBI Parameter Estimation

The primary experimental result of [1] is the relatively good agreement between the root-mean-square (rms) scatter of the VLBI delay-rate residuals and the prediction based on tropospheric water vapor fluctuation theory. For this prediction, a mean Deep Space Network (DSN) Kolmogorov fluctuation coefficient C_n was determined from water vapor radiometer (WVR) measurements at Goldstone. For the calculation of the three-station average, the other station coefficients were obtained by scaling the Goldstone (subscript-1) values by the ratios of the mean station water-vapor delay values, i.e.,

$$C_{n,2} = \frac{< Z_{w,2} >}{< Z_{w,1} >} C_{n,1} \tag{1}$$

where the mean wet tropospheric delays $< Z_{w,i} >$ were chosen to be the mean values of the Chao table of monthly tropospheric delay averages. Equation (1) assumes that the fluctuation of refractivity is proportional to the value of refractivity itself and, thus, the fluctuation of the tropospheric delay is proportional to the delay itself. For an evaluation of this hypothesis used for GPS-based determination of the tropospheric fluctuation coefficients, see [5].

The determination of $C_{n,i}$ from VLBI data uses the inverse process of the above considerations. First, a VLBI parameter estimation is performed from which the wet zenith delays Z_w are determined.[4] In this statistical fit, no fluctuation covariance matrix applied; only empirical weights are used to set the normalized χ^2 of each observing session to approximate unity. For a single baseline, the application of the squared form of Eq. (1) gives us the first equation for the two unknown $C_{n,1}$ and $C_{n,2}$:

$$Z_{w,1}^2 C_{n,2}^2 - Z_{w,2}^2 C_{n,1}^2 = 0 \tag{2}$$

The VLBI parameter estimation also can provide the values of delay-rate residuals for single baselines. Assuming that the water vapor fluctuation dominates the delay rate, then the variance of the delay-rate residuals at given elevation and azimuth angles are approximately given by the following:

$$\sigma_{\frac{d\tau}{dt}}^2 (\theta_1, \phi_1, \theta_2, \phi_2) = \frac{D_{\tau,\theta_1,\phi_1} \left(\frac{2}{3} v_1 T\right) + D_{\tau,\theta_2,\phi_2} \left(\frac{2}{3} v_2 T\right)}{(\frac{2}{3} T)^2} \tag{3}$$

[3] G. Lanyi, "Determination and Adjustment of the Tropospheric Fluctuation Coefficients C_n in VLBI Parameter Estimates and an Evaluation of Normalized χ_ν^2 in the Presence of A Priori Parameter Constraints," JPL Interoffice Memorandum 335.1-95.002 (internal document), Jet Propulsion Laboratory, Pasadena, California, February 17, 1994.

[4] The dry zenith delay Z_d is determined from the barometric surface pressure or from the fit itself by the application of a dry tropospheric mapping function. Since the dry and wet tropospheric delays are not absolutely separable, the actual physical wet zenith delay is approximately 4 percent larger than Z_w, while the actual physical dry zenith delay is smaller than Z_d by the same amount. Since Eq. (1) involves only the ratios of the wet zenith delays, this effect is ignorable here.

where D_τ is the delay structure function, T is the duration of observation, v_1 and v_2 are the magnitudes of wind velocity, and θ and ϕ are the elevation and azimuth angles of the radio source referred to the wind directions at the respective sites. Equation (3) is an approximation of Eq. (10) of [1] applied to the VLBI delay of two distant sites. The actual delay-rate fluctuation variance is a linear combination of structure functions corresponding to all measured points in the radio-source observation scan. The error of approximation is less than 10 percent for Eq. (3). If necessary, the ignored terms could be evaluated and Eq. (5) below would not change fundamentally, since the delay-rate variance remains linear in $C_{n,i}^2$. The computer code used for the parameter estimation includes some of the additional corrections included in Eq. (10) of [1]. Rewriting Eq. (3) by using the definition of the normalized (dimensionless) structure function, i.e.,

$$\hat{D}(\rho) = \frac{D(\rho)}{C_n^2 h^{8/3}} \tag{4}$$

and after averaging over all observations included in the determination of Z_w, we get the second equation for $C_{n,1}$ and $C_{n,2}$:

$$h^{8/3}\left(\frac{1}{N}\sum_{Stat.1}\frac{\hat{D}_{\tau,\theta_1,\phi_1}\left(\frac{2}{3}vT\right)}{\left(\frac{2}{3}T\right)^2}\right)C_{n,1}^2 + h^{8/3}\left(\frac{1}{N}\sum_{Stat.2}\frac{\hat{D}_{\tau,\theta_2,\phi_2}\left(\frac{2}{3}vT\right)}{\left(\frac{2}{3}T\right)^2}\right)C_{n,2}^2 = \sigma_{\frac{d\tau}{dt}}^2 \tag{5}$$

In Eq. (5), the number of observations N, the summation $\sum$, and the delay-rate variance $\sigma_{d\tau/dt}^2$ may correspond to the observations included in the estimation of Z_w for both stations. Note, however, that the tropospheric covariance described in [1] is an ensemble average over the deviation from the ensemble average of the tropospheric delay. Therefore, it is not well suited for short time intervals because the determined tropospheric delay does not coincide with its ensemble average. Also, global averaging is more satisfactory, since Eq. (3) refers to the ensemble average of delay-rate fluctuation at the given pair of elevation and azimuth angles. Thus, repetition of observations with approximately the same elevation and azimuth gives a better statistical estimate of the delay-rate fluctuations. On the other hand, $C_{n,i}$ can change with time in the sense that the Kolmogorov statistics may not hold for infinite times. The long-term non-Kolmogorov fluctuation, however, is semiempirically included in the fluctuation covariance expression by saturating the Kolmogorov formula for large distances. This long-term fluctuation should correspond to the mean DSN wet zenith delay fluctuation. However, for extreme levels of fluctuation, the wet tropospheric delay value must be estimated individually or the empirical saturation length must be adjusted properly for each site to yield a proper value of the long-term zenith delay fluctuation at the corresponding site.

The current implementation[5] of the delay and delay-rate fluctuation covariance matrix in MODEST utilizes the Fortran code TROPCOV.[6] This code evaluates the delay fluctuation covariance matrix $R_{cov}(\tau,\rho)$ corresponding to Eq. (15) of [1] (a subroutine called TROPCOV0) and then, using this result, calculates the approximate delay-rate fluctuation variance (Eq. 10 of [1]) ignoring the delay-rate fluctuation correlation between subsequent observations of radio sources. Thus, the structure function $\hat{D}_{\tau,\theta,\phi}([2/3]vT)$ can be evaluated as TROPCOV does by calling TROPCOV0 twice to reconstruct the structure function from the covariance matrix:[7]

[5] The computer code interface is by C. Jacobs and O. Sovers, Jet Propulsion Laboratory, Pasadena, California, 1990; the computer code optimization by is by C. Naudet, Jet Propulsion Laboratory, Pasadena, California, 1995.

[6] The computer code and its description are by C. D. Edwards, "The Effect of Spatial and Temporal Wet Troposphere Fluctuations on Connected Element Interferometry," JPL Interoffice Memorandum 335.3-88.98 (internal document), Jet Propulsion Laboratory, Pasadena, California, September 6, 1988.

[7] The covariance $R_{cov}(\tau,0)$ is a simple analytic expression. Thus, the current numeric formulation is not very efficient; an extra integration is carried out to evaluate the analytically known expression $R_{cov}(\tau,0)$.

$$D_\tau(\rho) = 2(R_{cov}(\tau, 0) - R_{cov}(\tau, \rho)) \quad (6)$$

III. Experimental Results

Table 1 contains a selection of the current test results[8] for the scatter of baseline length about its time–linear model of tectonic motion. In addition, the scatter of nutation angles about their values given by the nutation model employed in the VLBI parameter estimation also is shown.

Table 1. A comparison of baseline length and nutation angle scatter.

Solution ID	Method	DSS 15–DSS 45		DSS 15–DSS 65		Nutation angles		
		χ^2_ν [a]	σ_B, mm	χ^2_ν [a]	σ_B, mm	χ^2_ν [a]	σ_ψ, μas	σ_ϵ, μas
613	Z_w/2 h [b]	2.33	26.21	2.70	13.53	5.99	254.3	235.0
614	Z_w/12 h [b,c]	2.23	35.90	1.74	14.16	16.41	454.5	373.4
619	Z_w/12 h, C_n/24 h [d]	1.88	20.70	2.29	11.95	5.38	250.0	224.0
620	global Z_w, C_n/24 h [e]	1.82	20.63	2.29	12.29	5.60	265.3	221.6
617	global Z_w, C_n/24 h [f]	1.57	20.15	1.84	11.58	4.78	252.3	222.6
618	global Z_w, global C_n [g]	1.48	20.05	1.95	11.56	4.64	254.7	231.7
621	Z_w/12 h, global C_n [h]	1.44	20.13	1.91	11.59	4.60	256.8	238.2

[a] The formal χ^2_ν error is $0.22\chi^2_\nu$ at $\nu = 40$.

[b] Z_w is estimated once every 2 or 12 h to optimize the χ^2_ν of observing sessions.

[c] The peculiar drop and increase in values of χ^2_ν are only partially resolved, and they are under investigation.

[d] C_n adjustment with one iteration per observing session (24 h); Z_w is estimated once every 12 h per station.

[e] C_n adjustment with one iteration per observing session (24 h); one Z_w estimate per station.

[f] C_n adjustment with one iteration per observing session (24 h); Z_w is estimated once every 24 h per station, but Z_w is separately estimated on August 24, 1991.

[g] Single C_n adjustment per station with one iteration; one Z_w estimate per station; separate estimation on August 24, 1991.

[h] Single C_n a priori values from 618; Z_w estimated once every 12 h per station.

The data set consists of the radiometric Mark III JPL VLBI observations between 1978 and 1996 (85 observing sessions; a total of 17,371 radio source observations with 34-m antennas only). In solutions 613 and 614, the statistical observable weights are diagonal and an inverse quadratic combination of a constant and a tropospheric mapping-function proportional term, i.e., $1/(A^2 + B^2 m^2(E))$, where $m(E)$ is the tropospheric mapping function. In these solutions, A and B are empirically adjusted until the χ^2_ν of the particular session becomes nearly unity. In fits 617 to 621, the weight matrix is the inverse of the sum of the tropospheric covariance and the diagonal squared-system-noise matrices. The dry zenith delay Z_d is estimated; however, it is statistically constrained to the value obtained from the surface value of barometric pressure at the station by a 2-mm standard deviation. In fit 619, the wet zenith delay Z_w is estimated twice a day for each site; however, according to the theoretical foundation of the currently used tropospheric covariance, only one Z_w should be estimated for each station for the entire solution, as was done in solutions 617, 618, and 620. The number of estimated parameters is much smaller for covariance

[8] O. Sovers, personal communication, Jet Propulsion Laboratory, Pasadena, California, 1998.

weighted solutions (1780) in comparison with solutions in which Z_w is estimated once every 2 and 12 h (4586 and 2074).

In solutions 617 and 618, in addition to the global estimate of the wet zenith tropospheric delay at each site, Z_w is individually estimated in the observing session on August 24, 1991. This is because solutions 613 and 614 indicate a statistically extreme Z_w value of 14 cm at Goldstone, which is about 7 standard deviations away from the mean value of 5.5 cm given in Table 2. The mean long-term wet tropospheric zenith-delay standard deviation for DSS 15 is 1.2 cm, given by the semiempirically saturated Kolmogorov structure function with the fluctuation coefficient of 1.2×10^{-7} $m^{-1/3}$ from Table 2 and a saturation length of 3000 km. This most likely is due to actual (high) water vapor content, because there is no obvious fault in barometric surface pressure values constraining the dry zenith tropospheric delay. An estimate of the fluctuation coefficients functions only poorly in such an extreme case.

Table 2. Wet tropospheric parameters from VLBI solutions.

Parameter (unit)	DSS 15	DSS 45	DSS 63
$< Z_w >$ (cm)[a]	5.5 ± 0.04	9.6 ± 0.08	7.8 ± 0.11
C_n $(10^{-7} m^{-1/3})$[b]	1.2 ± 0.3	1.9 ± 0.3	2.0 ± 0.2
v (m/s)[c]	8.0	8.0	8.0
ϕ_v (deg)[d]	100	60	60

[a] One estimate per station.

[b] One optimized value per station; C_n is the average of individual session values.

[c] A priori values, since the optimization of v at DSS 15 currently is inconclusive; but at the other stations, results $v \approx 10$ m/s.

[d] The azimuth angle of the wind is clockwise with respect to north.

IV. Conclusion

Table 1 shows that the application of the tropospheric covariance does improve the VLBI solution in the sense of decreasing baseline and nutation angle scatters. Perhaps what is more significant is that the χ^2_ν values decrease, which indicates an improvement in the estimation of observable errors with a reduction of the bias in parameter estimates. Solutions 617, 618 and 621 provide the best performances at about an equal level. A similar analysis with GPS data also could be performed to check or even correct the potential biasing effect of the Kalman filtering.

It is rather interesting that the VLBI analysis also leads to the estimate of certain meteorological parameters. For the analysis results presented in Table 1, the wind directions were chosen from VLBI measurements by minimizing the baseline scatter and its respective χ^2_ν values in an iterative search procedure. Using the model parameters, the effective slab height of the water vapor layer, $h = 2$ km, and the lateral fluctuation saturation distance of $L = 3000$ km, the meteorological parameter values are obtained as given in Table 2.

Note that the mean DSS-15 C_n value of $1.2{\times}10^{-7}$ $m^{-1/3}$ compares very well with the value of $1.1{\times}10^{-7}$ $m^{-1/3}$ obtained independently from GPS measurements, the mean value for the 1994 DSS-15 GPS data, in [5]. The obtained wind velocity vectors agree to a certain degree with the meteorological radiosonde measurements taken at a height of about 1.2 km. However, there are potential problems with the radiosonde data: For the DSS-15 site, the averages of two independent sets of radiosonde measurements yield a discrepancy of 30 deg in wind directions and, thus, only one of the data sets agrees with the VLBI estimate.

References

[1] R. N. Treuhaft and G. E. Lanyi, "The Effect of the Dynamic Wet Troposphere on Radio Interferometric Measurements," *Radio Science*, vol. 22, pp. 251–265, 1987.

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[3] O. J. Sovers, J. L. Fanselow, and C. S. Jacobs, "Astrometry and Geodesy With Radio Interferometry: Experiments, Models, Results," *Rev. Mod. Phys.*, vol. 70, October 1998.

[4] C. S. Jacobs, O. J. Sovers, C. J. Naudet, R. F. Coker, and R. P. Branson, "The JPL Extragalactic Radio Reference Frame: Astrometric Results of 1978–96 Deep Space Network VLBI," *The Telecommunications and Mission Operations Progress Report 42-133, January–March 1998*, Jet Propulsion Laboratory, Pasadena, California, pp. 1–55, May 15, 1998.
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[5] C. J. Naudet, "Estimation of Tropospheric Fluctuations Using GPS Data," *The Telecommunications and Data Acquisition Progress Report 42-126, April–June 1996*, Jet Propulsion Laboratory, Pasadena, California, pp. 1–19, August 15, 1996.
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