

# Maximum-Likelihood-Based Acquisition and Tracking Scheme for Downlink Optical Telemetry

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*Free-space laser communication is a promising technology to meet the demand for cost-effective wideband support for future NASA missions. JPL has begun construction at the Table Mountain Facility in California of a 1-meter telescope system, known as the Optical Communication Telescope Laboratory (OCTL) and capable of tracking spacecraft from low Earth orbit to deep space, for development and validation of vital optical communication technologies. One of them is accurate pointing for both up (ground-to-space) and down (space-to-ground) links. This article describes an acquisition and tracking scheme for downlink optical telemetry that is based on a correlation-type technique developed for extended sources covering several elements of the detector array. The purpose of developing such a scheme is to enable simultaneous beam tracking and data detection using a small number of optical sensors. The open-loop acquisition derived from the maximum-likelihood criterion involves a transform-domain correlation between the received laser image and the reference image derived from the known intensity profile of the transmitting laser. The optimal acquisition algorithm requires solving two nonlinear equations, or iteratively solving their linearized variants, to estimate the coordinate of the transmitting laser when a rotation-invariant movement is considered. A maximum-likelihood-based closed-loop tracking algorithm is developed as well, in which the loop feedback signals are formulated as weighted transform-domain correlations between the received laser image and the previously estimated reference image. This scheme is expected to be able to achieve sub-pixel resolutions in a high-disturbance environment.*

## I. Introduction

Free-space laser communication is a promising technology to meet the demand for cost-effective wideband support for future NASA missions. JPL has begun construction at the Table Mountain Facility in California of a 1-meter telescope system, known as the Optical Communication Telescope Laboratory (OCTL) and capable of tracking spacecraft from low Earth orbit to deep space, for development and validation of vital optical communication technologies. One of them is the acquisition and tracking of the laser signal to ensure accurate pointing of the receiving telescope to the laser transmitter onboard the spacecraft for an optical space-to-ground link (known as an optical downlink). It is well-known that various atmospheric propagation effects [1], such as scintillation, image dancing, and blurring, can cause poor pointing and significantly impair the communication quality. These adverse optical turbulence effects nor-

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mally cause less serious problems for an optical downlink than for an uplink and, therefore, it is possible to use the highly sensitive, small-sized detector array intended for data detection to perform acquisition and tracking as well in a ground-based optical receiving system. This article presents an acquisition and tracking scheme for optical downlink that is based on a correlation-type technique developed for extended sources covering several elements of the detector array [2]. The reason for adopting the extended-source model, even though the pulse-position-modulated downlink laser beam usually is characterized as from a point source, is to reflect the fact that, for data detection, it is a common practice to intentionally defocus the incoming laser beam to ensure that at least a portion of the detector array can capture the signal under optical turbulence. This intentional defocusing is similar to other blurring effects introduced by atmosphere and imperfect optics, spreading the signal over more than one pixel on the focal plane of the detector array like an image of an extended source. Furthermore, the unique and typically asymmetric intensity profile (see Fig. 1) of a laser used for optical communications makes it particularly viable to use a correlation-type scheme, which is able to discriminate the pattern within a laser image, instead of other centroid-type schemes intended for symmetric point sources with high signal-to-noise ratios. Indeed, by including random disturbances, the correlation-type open-loop acquisition scheme presented in this article represents an extension to the studies in terms of target registration and moving target indication for military applications [3,4].

The development of this new scheme is based on the maximum-likelihood estimation, in which the uncertainties between the received image and the reference image are modeled as independent additive white Gaussian disturbances. It also is assumed that the relative position between the spacecraft and the ground-based optical terminal is changing with time, but only in a rotation-invariant fashion. As suggested by this study, the received laser image obtained from a detector array first is correlated in

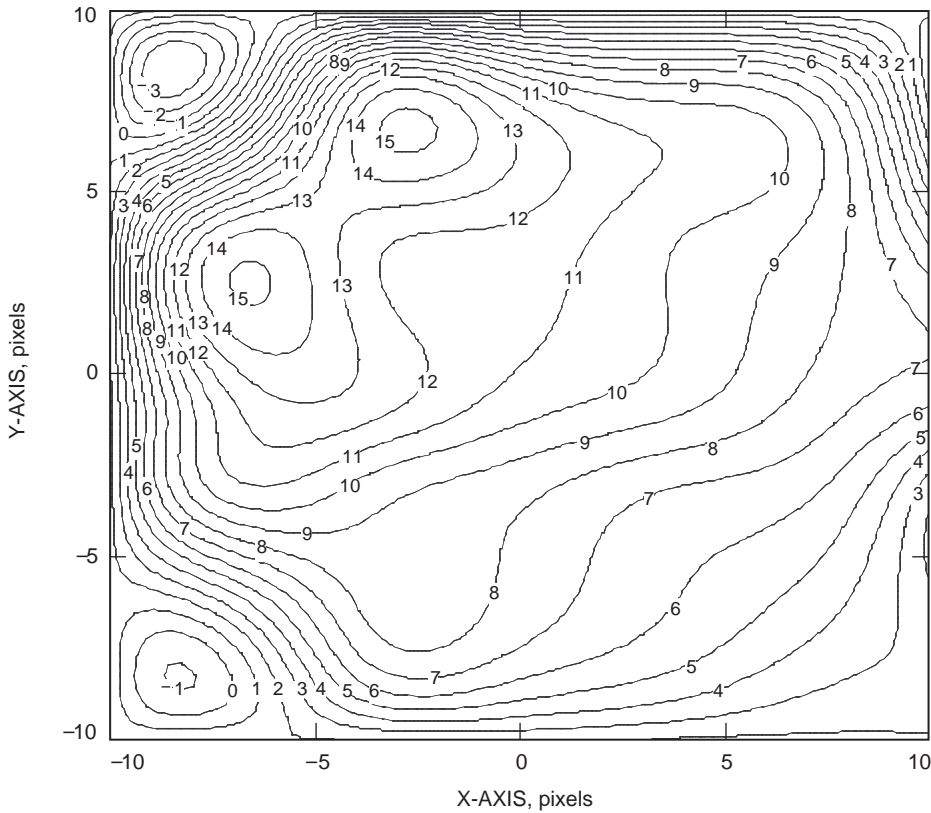


Fig. 1. The contour plot of a typical laser intensity profile.

the transform domain with the reference image derived from the priorly known intensity profile of the transmitting laser. The coordinate of the onboard optical transmitter then is estimated and tracked using the derived open-loop acquisition and closed-loop tracking algorithms, respectively. The optimal acquisition requires solving two nonlinear equations to estimate the coordinate. A suboptimal estimate exists by solving a linearized version of the maximum-likelihood criterion when computation complexity becomes a concern. A closed-loop tracking algorithm motivated by the maximum-likelihood criterion is developed as well for continuous tracking of the translation movement between the spacecraft and ground terminal. The closed-loop tracking structure has its loop feedback signals formulated as weighted transform-domain correlations between the received laser image and the previously estimated reference image. Only two linear equations are involved in each iteration of a continuous tracking. The closed-loop tracking scheme has many potential applications, including free-space optical communications and astronomy in which accurate and stabilized optical pointing is important. This scheme is expected to be able to achieve sub-pixel resolutions in a high-disturbance environment.

In this article, Section II provides a general description of the mathematical model and the discrete Fourier transform of an image using the lexicographic representation. The effect of rotation-invariant translation movement in the transform domain also is discussed. Section III lays the groundwork for the maximum-likelihood estimate of the translation vector, which leads to the open-loop acquisition algorithm. The derivation of the closed-loop tracking scheme is provided in Section IV, followed by the summary of this study and a brief description of possible future research efforts in Section V.

## II. Mathematical Model

### A. Representation of an Image

Based on the assumption of additive white Gaussian random disturbance mentioned previously, the laser image detected by an  $M \times N$  array at time  $t_l$ , denoted as  $r_l(m, n)$ , can be represented by a sum of the source image,  $s_l(m, n)$ , and the random disturbance,  $n_l(m, n)$ , as follows:

$$r_l(m, n) = s_l(m, n) + n_l(m, n), \quad m = 0, 1, \dots, M-1, \quad n = 0, 1, \dots, N-1 \quad (1)$$

where  $n_l(m, n)$  is an independent zero-mean Gaussian random variable with variance  $\sigma_l^2$  for all  $m$  and  $n$ .

The discrete Fourier transform of the received image at time  $t_l$  becomes

$$\mathcal{R}_l(m, n) = \mathcal{S}_l(m, n) + \mathcal{N}_l(m, n) \quad (2)$$

where the transform-domain source image and random disturbance are

$$\mathcal{S}_l(m, n) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} s_l(p, q) e^{-i2\pi([m/M]p + [n/N]q)} \quad (3)$$

and

$$\mathcal{N}_l(m, n) = \sum_{p=0}^{M-1} \sum_{q=0}^{N-1} n_l(p, q) e^{-i2\pi([m/M]p + [n/N]q)} \quad (4)$$

For notational convenience, the image matrix is represented in the lexicographic form such that

$$\vec{\mathcal{S}}_l = \begin{bmatrix} \mathcal{S}_l(0,0) \\ \mathcal{S}_l(0,1) \\ \vdots \\ \mathcal{S}_l(0,N-1) \\ \mathcal{S}_l(1,0) \\ \vdots \\ \mathcal{S}_l(1,N-1) \\ \vdots \\ \mathcal{S}_l(M-1,0) \\ \vdots \\ \mathcal{S}_l(M-1,N-1) \end{bmatrix} = \mathcal{F}\{\vec{s}_l\} = \mathcal{F} \left\{ \begin{bmatrix} s_l(0,0) \\ s_l(0,1) \\ \vdots \\ s_l(0,N-1) \\ s_l(1,0) \\ \vdots \\ s_l(1,N-1) \\ \vdots \\ s_l(M-1,0) \\ \vdots \\ s_l(M-1,N-1) \end{bmatrix} \right\} \quad (5)$$

where  $\mathcal{F}\{\cdot\}$  denotes the discrete Fourier transform. Using the vector notation, from Eqs. (1) and (2) we have

$$\vec{r}_l = \vec{s}_l + \vec{n}_l$$

and

$$\vec{\mathcal{R}}_l = \vec{\mathcal{S}}_l + \vec{\mathcal{N}}_l$$

## B. Translation Movement

If the laser image makes a translation movement within the field of view of the detector array between  $t_l$  and  $t_{l+1}$  by the amount of  $x_l$  and  $y_l$  pixels along the x-axis and y-axis, respectively, the image at  $t_{l+1}$  is related to the original image at  $t_l$  by

$$s_{l+1}(m, n) = s_l(m - x_l, n - y_l) \quad (6)$$

or, in the lexicographic form, as

$$\vec{s}_{l+1} \triangleq \mathbb{L}_{x_l, y_l} \{\vec{s}_l\} \quad (7)$$

where  $\mathbb{L}_{x,y}\{\cdot\}$  is defined as a translation operator that moves the operand by a translation vector  $(x, y)$ .

In the transform domain, it can easily be shown that the effect of translation movement becomes

$$\mathcal{S}_{l+1}(m, n) = \mathcal{S}_l(m, n) e^{i\theta_{m,n,l}} \quad (8)$$

where

$$\theta_{m,n,l} = -2\pi \left( \frac{m}{M} x_l + \frac{n}{N} y_l \right) \quad (9)$$

is the phase introduced to the pixel  $(m, n)$  of the transform-domain image due to the translation of the coordinate from  $t_l$  to  $t_{l+1}$ .

Hence, from Eq. (2), the transform-domain received image at time  $t_{l+1}$  can be expressed by

$$\mathcal{R}_{l+1}(m, n) = \mathcal{S}_l(m, n)e^{i\theta_{m,n,l}} + \mathcal{N}_{l+1}(m, n) \quad (10)$$

or, in the lexicographic form, as

$$\vec{\mathcal{R}}_{l+1} = \vec{\mathcal{S}}_l \cdot e^{i\Theta(x_l, y_l)} + \vec{\mathcal{N}}_{l+1} \quad (11)$$

where “ $\cdot$ ” denotes the entry-by-entry product of two vectors and

$$e^{i\Theta(x_l, y_l)} = \begin{bmatrix} e^{i\theta_{0,0,l}} \\ e^{i\theta_{0,1,l}} \\ \vdots \\ e^{i\theta_{0,N-1,l}} \\ e^{i\theta_{1,0,l}} \\ \vdots \\ e^{i\theta_{1,N-1,l}} \\ \vdots \\ e^{i\theta_{M-1,0,l}} \\ \vdots \\ e^{i\theta_{M-1,N-1,l}} \end{bmatrix} \quad (12)$$

with  $\theta_{m,n,l}$  being given in Eq. (9).

### III. The Maximum-Likelihood Estimate of the Translation Vector

From Eq. (1), the maximum-likelihood estimator will compare the received image,  $\vec{r}_{l+1}$ , against the reference image,  $\vec{s}_l$ , and declare the estimated translation vector,  $(\hat{x}_l, \hat{y}_l)$ , if

$$p\{\vec{r}_{l+1} \mid \vec{s}_l, (\hat{x}_l, \hat{y}_l)\} = \max_{\{(x_l, y_l)\}} p\{\vec{r}_{l+1} \mid \vec{s}_l, (x_l, y_l)\} \quad (13)$$

where

$$p\{\vec{r}_{l+1} \mid \vec{s}_l, (x_l, y_l)\} = \frac{1}{(\sqrt{2\pi}\sigma_l)^{MN}} e^{-[1/2\sigma_l^2]\|\vec{r}_{l+1} - \mathbb{L}_{x_l, y_l}\{\vec{s}_l\}\|^2} \quad (14)$$

is the conditional probability density function of  $\vec{r}_{l+1}$  given that the translation vector is  $(x_l, y_l)$ . The maximum-likelihood criterion stated in Eq. (13) is equivalent to

$$\|\vec{r}_{l+1} - \mathbb{L}_{\hat{x}_l, \hat{y}_l}\{\vec{s}_l\}\|^2 = \min_{\{(x_l, y_l)\}} \|\vec{r}_{l+1} - \mathbb{L}_{x_l, y_l}\{\vec{s}_l\}\|^2 \quad (15)$$

Since the Fourier transform  $[1/MN]\mathcal{F}\{\cdot\}$  is unitary, under  $\mathcal{L}_2$  norm, we have

$$\begin{aligned}
\| \vec{r}_{l+1} - \mathbb{L}_{x_l, y_l} \{ \vec{s}_l \} \|^2 &= \frac{1}{MN} \| \mathcal{F} \{ \vec{r}_{l+1} \} - \mathcal{F} \{ \mathbb{L}_{x_l, y_l} \{ \vec{s}_l \} \} \|^2 \\
&= \frac{1}{MN} \| \vec{R}_{l+1} - \vec{S}_l \cdot e^{i\Theta(x_l, y_l)} \|^2
\end{aligned} \tag{16}$$

Expanding the above expression leads to

$$\| \vec{R}_{l+1} - \vec{S}_l \cdot e^{i\Theta(x_l, y_l)} \|^2 = \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \{ |\mathcal{R}_{l+1}(m, n)|^2 + |\mathcal{S}_l(m, n)|^2 - 2|w_l(m, n)| \cos(\xi_{m, n, l} - \theta_{m, n, l}) \} \tag{17}$$

where

$$w_l(m, n) \triangleq \mathcal{R}_{l+1}(m, n) \mathcal{S}_l^*(m, n) = |w_l(m, n)| e^{i\xi_{m, n, l}} \tag{18}$$

is the pixel-by-pixel product of the transform-domain received image,  $\mathcal{R}_{l+1}(m, n)$ , and the complex conjugate of the transform-domain reference image,  $\mathcal{S}_l(m, n)$ . By substituting Eq. (18) into Eq. (17) and removing constant terms not affected by the choice of  $(x_l, y_l)$ , the maximum-likelihood criterion stated in Eq. (15) finally can be reduced to

$$\max_{\{x_l, y_l\}} \left\{ \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} |w_l(m, n)| \cos(\xi_{m, n, l} - \theta_{m, n, l}) \right\} = \| \vec{r}_{l+1} - \mathbb{L}_{\hat{x}_l, \hat{y}_l} \{ \vec{s}_l \} \|^2 \tag{19}$$

Note that the likelihood function to be maximized in Eq. (19) can be rewritten as

$$\text{Re} \left\{ \frac{1}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \mathcal{R}_{l+1}(m, n) \mathcal{S}_l^*(m, n) e^{-i\theta_{m, n, l}} \right\} \tag{20}$$

where  $\text{Re}\{\cdot\}$  represents the real part of a complex quantity. It clearly is indicated that the likelihood function involves the average over all pixels of the pixel-wise multiplied received and reference images in the transform domain, as well as the phase to be estimated.

Taking the partial derivatives of Eq. (20) with respect to  $x_l$  and  $y_l$  and equating them to zero, we have

$$\left. \begin{aligned}
\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} m |w_l(m, n)| \sin(\xi_{m, n, l} - \theta_{m, n, l}) &= 0 \\
\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} n |w_l(m, n)| \sin(\xi_{m, n, l} - \theta_{m, n, l}) &= 0
\end{aligned} \right\} \tag{21}$$

which forms a set of nonlinear equations to be solved for the maximum-likelihood estimates of  $x_l$  and  $y_l$ .

If the extended source is close to being acquired, the phase differences,  $(\xi_{m, n, l} - \theta_{m, n, l})$ , will be small, and the approximation of  $\sin(x) \approx x$  can be applied to Eq. (21), rendering the suboptimal linear estimator

$$\left. \begin{aligned} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} m |w_l(m, n)| \xi_{m,n,l} &= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} m |w_l(m, n)| \theta_{m,n,l} \\ \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} n |w_l(m, n)| \xi_{m,n,l} &= \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} n |w_l(m, n)| \theta_{m,n,l} \end{aligned} \right\} \quad (22)$$

The estimated vector  $(\hat{x}_l, \hat{y}_l)$  obtained from solving this suboptimal linear maximum-likelihood criterion must satisfy

$$\left. \begin{aligned} -\frac{1}{2\pi} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} m |w_l(m, n)| \xi_{m,n,l} &= x_l \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} m \frac{m}{M} |w_l(m, n)| + y_l \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} m \frac{n}{N} |w_l(m, n)| \\ -\frac{1}{2\pi} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} n |w_l(m, n)| \xi_{m,n,l} &= x_l \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} n \frac{m}{M} |w_l(m, n)| + y_l \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} n \frac{n}{N} |w_l(m, n)| \end{aligned} \right\} \quad (23)$$

The solution of Eq. (23) is identical to algorithms derived by Kuglin and Pearson [3,4].

#### IV. The Image Tracking Loop

To maximize the likelihood function of acquiring an image disturbed by additive white Gaussian noise involves a comparison of the received image with the reference image. However, for image tracking, it is the correlation between the transform-domain received image  $\mathcal{R}_{l+1}(m, n)$  and the estimated translated reference image

$$\hat{\mathcal{R}}_{l+1}(m, n) = \mathcal{S}_l(m, n) e^{i\hat{\theta}_{m,n,l}}$$

that is to be continuously maximized. The pixel-wise product of  $\mathcal{R}_{l+1}(m, n)$  and  $\hat{\mathcal{R}}_{l+1}^*(m, n)$  can be expressed by

$$\begin{aligned} C_{l+1}(m, n) &\triangleq \mathcal{R}_{l+1}(m, n) \mathcal{S}_l^*(m, n) e^{-i\hat{\theta}_{m,n,l}} \\ &= |\mathcal{S}_l(m, n)|^2 e^{i\phi_{m,n,l}} + \mathcal{N}_{l+1}(m, n) \mathcal{S}_l^*(m, n) e^{-i\hat{\theta}_{m,n,l}} \end{aligned} \quad (24)$$

where  $\hat{\theta}_{m,n,l}$  is the estimate of  $\theta_{m,n,l}$ , and the estimation error is

$$\begin{aligned} \phi_{m,n,l} &= \theta_{m,n,l} - \hat{\theta}_{m,n,l} \\ &= -2\pi \left[ \frac{m}{M} (x_l - \hat{x}_l) + \frac{n}{N} (y_l - \hat{y}_l) \right] \\ &\triangleq -2\pi \left( \frac{m}{M} \Delta_x + \frac{n}{N} \Delta_y \right) \end{aligned} \quad (25)$$

where  $\hat{x}_l$  and  $\hat{y}_l$  are estimates of  $x_l$  and  $y_l$ , respectively, and  $\Delta_x$  and  $\Delta_y$  are the associated estimate errors. The average of  $C_{l+1}(m, n)$  over all pixels results in the correlation between  $\mathcal{R}_{l+1}(m, n)$  and  $\hat{\mathcal{R}}_{l+1}(m, n)$ .

According to the maximum-likelihood criterion derived in the previous section, the real part of Eq. (24) shall be maximized over the entire detector array, rendering the maximum-likelihood estimates  $\hat{x}_l$  and  $\hat{y}_l$ , which maximize

$$\sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \text{Re} \{C_{l+1}(m, n)\} \quad (26)$$

By following the derivation given in the previous section, it turns out that  $\hat{x}_l$  and  $\hat{y}_l$  can be obtained by solving a set of two nonlinear equations that are formed by setting the partial derivatives of Eq. (26) with respect to  $\hat{x}_l$  and  $\hat{y}_l$  to zero. In deriving the image-tracking algorithm to continuously update the estimates, two loop feedback signals can be similarly formed as the partial derivatives of Eq. (26) with respect to  $\Delta_x$  and  $\Delta_y$ , rendering

$$\begin{aligned} \varepsilon_x &\triangleq \frac{\partial}{\partial \Delta_x} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \text{Re} \{C_{l+1}(m, n)\} \\ &= \frac{2\pi}{M} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} m |\mathcal{S}_l(m, n)|^2 \sin(\phi_{m,n,l}) + \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \mathcal{N}_{l,eff}^{(x)}(m, n) \end{aligned} \quad (27)$$

and

$$\begin{aligned} \varepsilon_y &\triangleq \frac{\partial}{\partial \Delta_y} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \text{Re} \{C_{l+1}(m, n)\} \\ &= \frac{2\pi}{N} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} n |\mathcal{S}_l(m, n)|^2 \sin(\phi_{m,n,l}) + \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} \mathcal{N}_{l,eff}^{(y)}(m, n) \end{aligned} \quad (28)$$

where

$$\mathcal{N}_{l,eff}^{(x)}(m, n) = \frac{\partial}{\partial \Delta_x} \text{Re} \left\{ \mathcal{N}_{l+1}(m, n) \mathcal{S}_l^*(m, n) e^{-i\hat{\theta}_{m,n,l}} \right\}$$

and

$$\mathcal{N}_{l,eff}^{(y)}(m, n) = \frac{\partial}{\partial \Delta_y} \text{Re} \left\{ \mathcal{N}_{l+1}(m, n) \mathcal{S}_l^*(m, n) e^{-i\hat{\theta}_{m,n,l}} \right\}$$

constitute the effective noises in the loop operation. Equations (27) and (28) characterize the relationship between the estimate errors,  $\Delta_x$  and  $\Delta_y$ , and the loop feedback signals,  $\varepsilon_x$  and  $\varepsilon_y$ . However, to solve for  $\Delta_x$  and  $\Delta_y$  from these nonlinear equations can be quite challenging. With a reasonable linear assumption valid when the phase error  $\phi_{m,n,l}$  remains small during the tracking mode, one can substitute Eq. (25)



for  $\sin(\phi_{m,n,l})$  in Eqs. (27) and (28). The resulting simultaneous equations are linear for  $\Delta_x$  and  $\Delta_y$  and can easily be solved, yielding

$$\Delta_x = \frac{C_n \mathbb{E}[\varepsilon_x] - C_{mn} \mathbb{E}[\varepsilon_y]}{C_m C_n - C_{mn}^2} \quad (29)$$

and

$$\Delta_y = \frac{C_{mn} \mathbb{E}[\varepsilon_x] - C_m \mathbb{E}[\varepsilon_y]}{C_{mn}^2 - C_m C_n} \quad (30)$$

where  $\mathbb{E}[\cdot]$  denotes the statistical expectation and

$$C_m \triangleq \frac{4\pi^2}{M^2} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} m^2 |\mathcal{S}_l(m, n)|^2$$

$$C_n \triangleq \frac{4\pi^2}{N^2} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} n^2 |\mathcal{S}_l(m, n)|^2$$

$$C_{mn} \triangleq \frac{4\pi^2}{MN} \sum_{m=0}^{M-1} \sum_{n=0}^{N-1} mn |\mathcal{S}_l(m, n)|^2$$

are coefficients that can be calculated from the reference image of the previous iteration at  $t_l$ .

An image tracking-loop structure can be realized based upon the above discussion and is depicted in Fig. 2. The transform-domain received image  $\{\mathcal{R}_{l+1}(m, n)\}$  first is multiplied pixel-wise with a properly translated transform-domain reference image established according to the estimate  $(\hat{x}_l, \hat{y}_l)$  from the previous iteration at  $t_l$ . After being averaged over the extended source, the correlation result is used to compute the loop feedback signals,  $\varepsilon_x$  and  $\varepsilon_y$ . The subsequent calculation of  $\Delta_x$  and  $\Delta_y$  from  $\varepsilon_x$  and  $\varepsilon_y$  is straightforward, as indicated in Eqs. (29) and (30), except that the statistical averages are replaced by time averages performed by low-pass filters. The calculated  $\Delta_x$  and  $\Delta_y$  will be used to update the movement estimates through an accumulator, such that

$$\hat{x}_{l+1} = \hat{x}_l + \Delta_x \quad (31)$$

and

$$\hat{y}_{l+1} = \hat{y}_l + \Delta_y \quad (32)$$

The updated accumulator contents will be used to calculate the estimate  $\hat{\theta}_{m,n,l+1}$  and prepare the translated reference image for the next loop iteration at  $t_{l+1}$ .

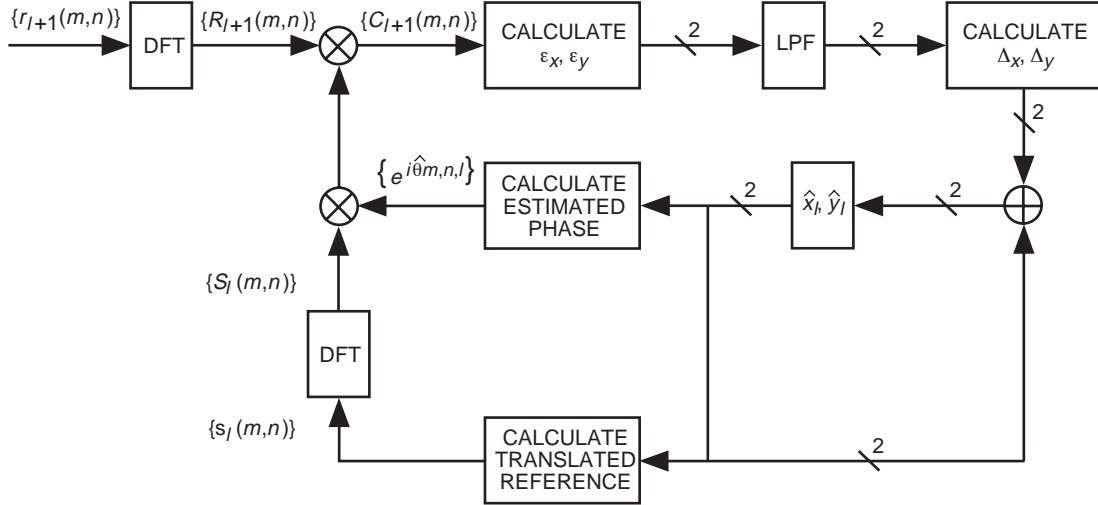


Fig. 2. The correlation-type image tracking loop.

## V. Conclusion

This article presents a correlation-type acquisition and tracking scheme for downlink optical telemetry that uses the known laser intensity profile as the reference. The uncertainties between the reference image and the received image are modeled as additive white Gaussian disturbances. It has been shown that, for rotation-invariant movement, the optimum acquisition algorithm derived from the maximum-likelihood criterion under these assumptions requires solving two nonlinear equations to estimate the coordinate of the onboard transmitting laser from the received laser image in the transform domain. The optimal solution also may be obtained iteratively using linear approximations. Similar linearization leads to the closed-loop tracking algorithm in which the loop feedback signals are formulated as weighted transform-domain correlations between the received image and the reference image.

The presented scheme is capable of supporting simultaneous data detection and target tracking. It is expected to achieve sub-pixel resolution in a high-disturbance environment. With recent advances in photonic and computing technologies, integrated acquisition and tracking becomes possible as a portion of the detector subarray can be dedicated to perform tracking, which minimizes the hand-over and possible alignment problems between acquisition and tracking. The effectiveness of this scheme will be simulated and results will be included in a future article. Other issues, such as the existence of spatially correlated disturbances in albedo variations and the image obtained from sub-pixel scanning, will be investigated.

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