A Comparison of the Performances of Coherent Binary-Phase-Shift Keying (BPSK) and Offset Quadrature-Phase-Shift Keying (OQPSK) in the Presence of Interference

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The performance of offset quadrature-phase-shift keying (OQPSK) in the presence of narrowband and wideband interference signals is considered, assuming perfect carrier synchronization. Narrowband interference is modeled as an unmodulated tone at a given frequency offset and random phase with respect to the desired OQPSK signal. Wideband interference is modeled as another OQPSK signal at a given frequency offset, random phase, and random timing with respect to the desired OQPSK signal, but having the identical data rate. This model is more typical of co-channel interference than that considered in other studies of this subject, where a wideband Gaussian noise was assumed. The results obtained here for OQPSK are compared with analogous results obtained previously by one of the authors for binary-phase-shift-keyed (BPSK) modulation.

I. Introduction

In a previous article [1], we considered the performance of coherent binary-phase-shift keying (BPSK) with Costas loop tracking when, in addition to the additive white Gaussian noise (AWGN), co-channel interference (e.g., narrowband [unmodulated tone] or wideband [modulated tone]) was present. We observed in [1] that, even though the interference results in degradation taking place both in the tracking performance of the loop and in the data-detection process itself, the more dominant effect was by far the latter. With this in mind, we consider in this article the effect of the above interference types on the data-detection performance of offset quadrature-phase-shift keying (OQPSK), assuming perfect carrier synchronization, and then compare these results with those obtained under similar circumstances in [1] for BPSK so as to assess the relative sensitivity of the two modulations to this interference. Without going through the details, it is assumed that the conclusions drawn from this comparison of ideal coherent detection of BPSK and OQPSK would also carry over to the case when the demodulation references are supplied by the individual carrier tracking loops as appropriate. As in [1], the article will be structured into two major sections corresponding to the narrowband and wideband interferer cases.

[1] Communications Systems and Research Section.

II. Performance in the Presence of Narrowband Interference

Consider the OQPSK receiver illustrated in Fig. 1, where the in-phase (I) and quadrature-phase (Q) demodulation reference signals are assumed to be ideal. Input to this receiver is the sum of a desired signal, $s_s(t)$, and a narrowband (tone) interference signal, $s_I(t)$, which are mathematically modeled as

$$\left.\begin{aligned} s_s(t) &= \sqrt{P_s}\left[m_{sc}(t)\cos(\omega_c t + \theta_s) + m_{ss}(t)\sin(\omega_c t + \theta_s)\right] \\ s_I(t) &= \sqrt{2P_I}\sin\left((\omega_c + \Delta\omega)t + \theta_I\right) \end{aligned}\right\} \tag{1}$$

where P_s, ω_c, θ_s and $P_I, \omega_c + \Delta\omega, \theta_I$ are, respectively, the power, radian carrier frequency, and phase of the desired and interference signals and

$$\left.\begin{aligned} m_{sc}(t) &= \sum_{n=-\infty}^{\infty} a_n p(t - nT_s) \\ m_{ss}(t) &= \sum_{n=-\infty}^{\infty} b_n p\left(t - nT_s - \frac{T_s}{2}\right) \end{aligned}\right\} \tag{2}$$

are the binary data modulations with $\{a_n\}$ and $\{b_n\}$ each independent and identically distributed (i.i.d.) sequences (which are also independent of each other) taking on equiprobable values ± 1, and $p(t)$ is a unit amplitude rectangular pulse of duration equal to the symbol time, T_s, which is equal to twice the bit time, T_b. Adding to $s_s(t)$ and $s_I(t)$ is the WGN noise:

$$n(t) = \sqrt{2}\left[N_c(t)\cos(\omega_c t + \theta_s) - N_s(t)\sin(\omega_c t + \theta_s)\right] \tag{3}$$

where $N_c(t)$ and $N_s(t)$ are I and Q low-pass noise components that are independent and have single-sided power spectral density (PSD) N_0 W/Hz. As such, the total received signal is then

$$r(t) = s_s(t) + s_I(t) + n(t) \tag{4}$$

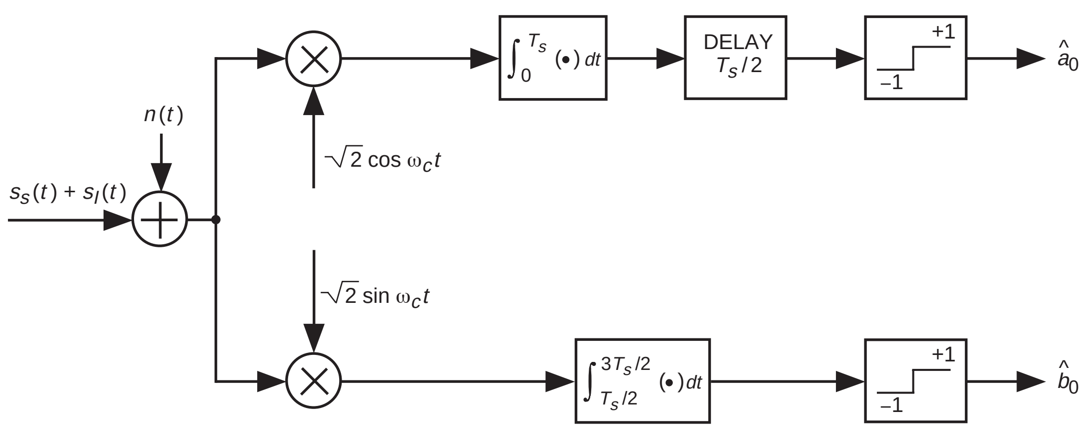

Fig. 1. The OQPSK receiver.

Demodulating $r(t)$ with the perfectly synchronized (to the desired signal phase and frequency) I and Q reference signals produces the baseband signals

$$\left.\begin{aligned}\varepsilon_c(t) &= r(t)\sqrt{2}\cos(\omega_c t+\theta_s)\\ &= \sqrt{\frac{P_s}{2}}m_{sc}(t)+N_c(t)+\sqrt{P_I}\sin(\Delta\omega t+\Delta\theta)\\ \varepsilon_s(t) &= r(t)\sqrt{2}\sin(\omega_c t+\theta_s)\\ &= \sqrt{\frac{P_s}{2}}m_{ss}(t)-N_s(t)+\sqrt{P_I}\cos(\Delta\omega t+\Delta\theta)\end{aligned}\right\} \tag{5}$$

where $\Delta\theta \triangleq \theta_I - \theta_s$ is the phase difference between the desired and interference signals. After passing through the I and Q matched filters, we obtain the sample-and-hold values at the end of the kth symbol interval:

$$\left.\begin{aligned} z_s(t) &= \int_{(k+1/2)T_s}^{(k+3/2)T_s}\varepsilon_s(t)\,dt = T_s\sqrt{\frac{P_s}{2}}b_k - N_2' + T_s\sqrt{P_I}\left\{A_{ck}'\cos\Delta\theta - A_{sk}'\sin\Delta\theta\right\},\\ &\qquad\qquad \left(k+\frac{3}{2}\right)T_s \le t \le \left(k+\frac{5}{2}\right)T_s\\ z_c(t) &= \int_{kT_s}^{(k+1)T_s}\varepsilon_c(t)\,dt = T_s\sqrt{\frac{P_s}{2}}a_k + N_1 + T_s\sqrt{P_I}\left\{A_{sk}\cos\Delta\theta + A_{ck}\sin\Delta\theta\right\},\\ &\qquad\qquad (k+1)T_s \le t \le (k+2)T_s\end{aligned}\right\} \tag{6}$$

where

$$\left.\begin{aligned} A_{sk}' &\triangleq \frac{1}{T_s}\int_{(k+1/2)T_s}^{(k+3/2)T_s}\sin\Delta\omega t dt = \frac{\cos\Delta\omega\left(k+\frac{1}{2}\right)T_s - \cos\Delta\omega\left(k+\frac{3}{2}\right)T_s}{\Delta\omega T_s}\\ A_{ck}' &\triangleq \frac{1}{T_s}\int_{(k+1/2)T_s}^{(k+3/2)T_s}\cos\Delta\omega t dt = \frac{-\sin\Delta\omega\left(k+\frac{1}{2}\right)T_s + \sin\Delta\omega\left(k+\frac{3}{2}\right)T_s}{\Delta\omega T_s}\\ A_{sk} &\triangleq \frac{1}{T_s}\int_{kT_s}^{(k+1)T_s}\sin\Delta\omega t dt = \frac{\cos\Delta\omega k T_s - \cos\Delta\omega(k+1)T_s}{\Delta\omega T_s}\\ A_{ck} &\triangleq \frac{1}{T_s}\int_{kT_s}^{(k+1)T_s}\cos\Delta\omega t dt = \frac{-\sin\Delta\omega k T_s + \sin\Delta\omega(k+1)T_s}{\Delta\omega T_s}\end{aligned}\right\} \tag{7}$$

and N_1 and N_2' are independent zero-mean Gaussian random variables with variance $\sigma^2_{N_1} = \sigma^2_{N_2'} = N_0 T_s/2$. Alternatively, defining the complex amplitudes

$$\left.\begin{aligned} A_k &= A_{ck} + jA_{sk} = \frac{1}{T_s}\int_{kT_s}^{(k+1)T_s} e^{j\Delta\omega t}dt = \frac{1}{T_s}\int_0^{T_s} e^{j\Delta\omega(t+kT_s)}dt \\ A_k' &= A_{ck}' + jA_{sk}' = \frac{1}{T_s}\int_{(k+1/2)T_s}^{(k+3/2)T_s} e^{j\Delta\omega t}dt = \frac{1}{T_s}\int_0^{T_s} e^{j\Delta\omega(t+(k+1/2)T_s)}dt \end{aligned}\right\} \tag{8}$$

then we can rewrite Eq. (6) as

$$\left.\begin{aligned} z_s(t) &= T_s\sqrt{\frac{P_s}{2}}b_k - N_2' + T_s\sqrt{P_I}\ \mathrm{Re}\left\{A_k' e^{j\Delta\theta}\right\}, \quad \left(k+\frac{3}{2}\right)T_s \le t \le \left(k+\frac{5}{2}\right)T_s \\ z_c(t) &= T_s\sqrt{\frac{P_s}{2}}a_k + N_1 + T_s\sqrt{P_I}\ \mathrm{Im}\left\{A_k e^{j\Delta\theta}\right\}, \quad (k+1)T_s \le t \le (k+2)T_s \end{aligned}\right\} \tag{9}$$

Further note that

$$\left.\begin{aligned} |A_k'| &= \left|\frac{1}{T_s}\int_0^{T_s} e^{j\Delta\omega(t+(k+1/2)T_s)}dt\right| = \left|\frac{1}{T_s}\int_0^{T_s} e^{j\Delta\omega t}dt\right| = \left|\frac{\sin\frac{\Delta\omega T_s}{2}}{\frac{\Delta\omega T_s}{2}}\right| \\ |A_k| &= \left|\frac{1}{T_s}\int_0^{T_s} e^{j\Delta\omega(t+kT_s)}dt\right| = \left|\frac{1}{T_s}\int_0^{T_s} e^{j\Delta\omega t}dt\right| = \left|\frac{\sin\frac{\Delta\omega T_s}{2}}{\frac{\Delta\omega T_s}{2}}\right| \end{aligned}\right\} \tag{10}$$

which are independent of k. Also,

$$\left.\begin{aligned} \alpha_k' &\triangleq \arg A_k' = \tan^{-1}\frac{A_{sk}'}{A_{ck}'} \\ \alpha_k &\triangleq \arg A_k = \tan^{-1}\frac{A_{sk}}{A_{ck}} \end{aligned}\right\} \tag{11}$$

which, unlike $|A_k'|$ and $|A_k|$, are functions of the index of the bit interval, k. In particular, analogously to Eq. (15) of [1], we have

$$\left.\begin{aligned} \alpha_k' &= \left(k+\frac{1}{2}\right)\Delta\omega T_s + \alpha_0 \\ \alpha_k &= k\Delta\omega T_s + \alpha_0 \end{aligned}\right\} \tag{12}$$

where

$$\alpha_0 \triangleq \arg A_0 = \tan^{-1}\frac{\dfrac{1}{T_s}\int_0^{T_s}\sin\Delta\omega t dt}{\dfrac{1}{T_s}\int_0^{T_s}\cos\Delta\omega t dt} = \tan^{-1}\left(\frac{1-\cos\Delta\omega T_s}{\sin\Delta\omega T_s}\right) = \tan^{-1}\left(\frac{\dfrac{\sin^2(\eta_s/2)}{\eta_s/2}}{\dfrac{\sin\eta_s}{\eta_s}}\right) \tag{13}$$

and we have further introduced the shorthand notation for normalized frequency offset:

$$\eta_s \triangleq \Delta\omega T_s = 2\pi\Delta f T_s \tag{14}$$

For evaluation of average error probability, it is sufficient to consider the arbitrarily selected zeroth symbol intervals corresponding to $k=0$, in which case Eq. (9) becomes

$$\left.\begin{aligned} z_s(t) &= T_s\sqrt{\frac{P_s}{2}}b_k - N_2' + T_s\sqrt{P_I}\left|\frac{\sin\frac{\eta_s}{2}}{\frac{\eta_s}{2}}\right|\cos\left(\Delta\theta+\frac{\eta_s}{2}+\alpha_0\right), \quad \frac{3T_s}{2}\le t\le\frac{5T_s}{2} \\ z_c(t) &= T_s\sqrt{\frac{P_s}{2}}a_k + N_1 + T_s\sqrt{P_I}\left|\frac{\sin\frac{\eta_s}{2}}{\frac{\eta_s}{2}}\right|\sin\left(\Delta\theta+\alpha_0\right), \quad T_s\le t\le 2T_s \end{aligned}\right\} \tag{15}$$

where α_0 is defined in Eq. (13). Comparing these outputs to zero thresholds results in decisions on b_0 and a_0, respectively. Assuming $b_0 = 1$, the conditional probability of error for the decision on b_0 is given by

$$\begin{aligned} P_b(E|\Delta\theta)|_{b_0=1} &= \Pr\{z_s(t)<0|b_0=1\} \\ &= \frac{1}{2}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{2P_I}{P_s}}\left|\frac{\sin\frac{\eta_s}{2}}{\frac{\eta_s}{2}}\right|\cos\left(\Delta\theta+\frac{\eta_s}{2}+\alpha_0\right)\right]\right\} \end{aligned} \tag{16a}$$

Similarly, assuming $b_0 = -1$, the conditional probability of error is given by

$$\begin{aligned} P_b(E|\Delta\theta)|_{b_0=-1} &= \Pr\{z_s(t)\ge 0|b_0=-1\} \\ &= \frac{1}{2}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{2P_I}{P_s}}\left|\frac{\sin\frac{\eta_s}{2}}{\frac{\eta_s}{2}}\right|\cos\left(\Delta\theta+\frac{\eta_s}{2}+\alpha_0\right)\right]\right\} \end{aligned} \tag{16b}$$

where $R_d \triangleq P_sT_b/N_0 = P_sT_s/2N_0$ is the bit signal-to-noise ratio (SNR). Since the hypotheses $b_0 = -1$ and $b_0 = 1$ are equiprobable, then averaged over the I-channel data, the conditional bit-error probability (BEP) is

$$P_{bI}\left(E\left|\Delta\theta\right.\right)=\frac{1}{2}P_b\left(E\left|\Delta\theta\right.\right)\big|_{b_0=1}+\frac{1}{2}P_b\left(E\left|\Delta\theta\right.\right)\big|_{b_0=-1} \tag{17}$$

For the Q channel, a decision on a_0 would produce

$$P_{bQ}\left(E\left|\Delta\theta\right.\right)=\frac{1}{2}P_b\left(E\left|\Delta\theta\right.\right)\big|_{a_0=1}+\frac{1}{2}P_b\left(E\left|\Delta\theta\right.\right)\big|_{a_0=-1} \tag{18}$$

where

$$\begin{aligned} P_b\left(E\left|\Delta\theta\right.\right)\big|_{a_0=1} &= \Pr\left\{z_c\left(t\right)<0\left|a_0=1\right.\right\} \\ &= \frac{1}{2}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{2P_I}{P_s}}\left|\frac{\sin\frac{\eta_s}{2}}{\frac{\eta_s}{2}}\right|\sin\left(\Delta\theta+\alpha_0\right)\right]\right\} \end{aligned} \tag{19a}$$

and

$$\begin{aligned} P_b\left(E\left|\Delta\theta\right.\right)\big|_{a_0=-1} &= \Pr\left\{z_c\left(t\right)\geq 0\left|a_0=-1\right.\right\} \\ &= \frac{1}{2}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{2P_I}{P_s}}\left|\frac{\sin\frac{\eta_s}{2}}{\frac{\eta_s}{2}}\right|\sin\left(\Delta\theta+\alpha_0\right)\right]\right\} \end{aligned} \tag{19b}$$

Finally, the conditional (on $\Delta\theta$) probability of error is obtained from the average of Eqs. (17) and (18), i.e.,

$$\begin{aligned} P_b\left(E\left|\Delta\theta\right.\right) = {} & \frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{2P_I}{P_s}}\left|\frac{\sin\frac{\eta_s}{2}}{\frac{\eta_s}{2}}\right|\cos\left(\Delta\theta+\frac{\eta_s}{2}+\alpha_0\right)\right]\right\} \\ & +\frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{2P_I}{P_s}}\left|\frac{\sin\frac{\eta_s}{2}}{\frac{\eta_s}{2}}\right|\cos\left(\Delta\theta+\frac{\eta_s}{2}+\alpha_0\right)\right]\right\} \\ & +\frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{2P_I}{P_s}}\left|\frac{\sin\frac{\eta_s}{2}}{\frac{\eta_s}{2}}\right|\sin\left(\Delta\theta+\alpha_0\right)\right]\right\} \\ & +\frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{2P_I}{P_s}}\left|\frac{\sin\frac{\eta_s}{2}}{\frac{\eta_s}{2}}\right|\sin\left(\Delta\theta+\alpha_0\right)\right]\right\} \end{aligned} \tag{20}$$

Finally, assuming a uniform distribution on $\Delta\theta$, which is appropriate in the absence of any a priori information concerning the relative phase between the desired and interference signals, the average BEP is given by

$$P_b(E) = \frac{1}{2\pi}\int_{-\pi}^{\pi} P_b(E\,|\Delta\theta)\, d\Delta\theta \tag{21}$$

A simple special case (corresponding to the worst degradation due to interference) of the above results is worth considering. When the frequency separation between the desired signal and the interferer is equal to zero, i.e., the interferer tone is right on the carrier frequency, then $\eta_s = 0$ (which implies from Eq. (13) that $\alpha_0 = 0$), and Eq. (20) simplifies to

$$\begin{aligned} P_b(E\,|\Delta\theta) = {} & \frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{2P_I}{P_s}}\cos\Delta\theta\right]\right\} + \frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{2P_I}{P_s}}\cos\Delta\theta\right]\right\} \\ & + \frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{2P_I}{P_s}}\sin\Delta\theta\right]\right\} + \frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{2P_I}{P_s}}\sin\Delta\theta\right]\right\} \end{aligned} \tag{22}$$

If we now make the additional worst-case assumption that the interferer tone is also in phase with the desired signal, i.e., $\Delta\theta = 0$, then Eq. (22) simplifies still further to

$$P_b(E) = \frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{2P_I}{P_s}}\right]\right\} + \frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{2P_I}{P_s}}\right]\right\} + \frac{1}{4}\text{erfc}\left\{\sqrt{R_d}\right\} \tag{23}$$

Note that the presence of the third term in Eq. (23) implies that, for OQPSK, one-half of the time the performance is not degraded by the interferer.

By comparison, for BPSK under the same assumptions, we obtain from the results in [1]

$$P_b(E) = \frac{1}{4}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{P_I}{P_s}}\right]\right\} + \frac{1}{4}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{P_I}{P_s}}\right]\right\} \tag{24}$$

Note the absence of an interference-free term in Eq. (24) for BPSK. Also, since in OQPSK the desired signal power is split between the I and Q channels, then the effect of the interferer on the data decisions made on each of these channels is double that of BPSK and, thus, Eq. (23) will yield a worse performance than Eq. (24). In fact, for a large interference-to-desired-signal-power ratio, the dominant terms in Eqs. (23) and (24) are the second ones, which, under these idealized circumstances, reveal (ignoring the difference in the fraction preceding the erfc function) a 3-dB penalty of OQPSK relative to BPSK.

Figure 2 is a plot of average BEP as computed from Eqs. (23) and (24) for several values of P_I/P_s. The results clearly indicate the behavior described above. Figure 3 is the corresponding plot for the more relaxed condition when $\Delta\theta \neq 0$ and η_s may or may not be equal to zero. As an example of the behavior, a value of $P_I/P_s = 0.04$ was selected. For OQPSK, the results are computed from Eqs. (20) and (21) whereas, for BPSK, the results are computed from [1]

$$P_b\left(E\,|\Delta\theta\right) = \frac{1}{4}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{P_I}{P_s}}\left|\frac{\sin\dfrac{\eta_b}{2}}{\dfrac{\eta_b}{2}}\right|\cos\left(\Delta\theta+\alpha_{0b}\right)\right]\right\}$$

$$+\frac{1}{4}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{P_I}{P_s}}\left|\frac{\sin\dfrac{\eta_b}{2}}{\dfrac{\eta_b}{2}}\right|\cos\left(\Delta\theta+\alpha_{0b}\right)\right]\right\} \tag{25}$$

and Eq. (21). Here $\eta_b \triangleq 2\pi\Delta f T_b = \eta_s/2$ and, analogously to Eq. (13),

$$\alpha_{0b} = \tan^{-1}\left(\frac{1-\cos\Delta\omega T_b}{\sin\Delta\omega T_b}\right) = \tan^{-1}\left(\frac{1-\cos\eta_b}{\sin\eta_b}\right) \tag{26}$$

Here we see that, depending on the value of the frequency offset between the interferer and the desired signal, the performance of OQPSK can be either the same, better than, or worse than BPSK. Note that, for $\Delta f T_s = 1.0$, the performance of OQPSK is unaffected by the interferer whereas, for $\Delta f T_b = 1.0$, the same would be true for BPSK.

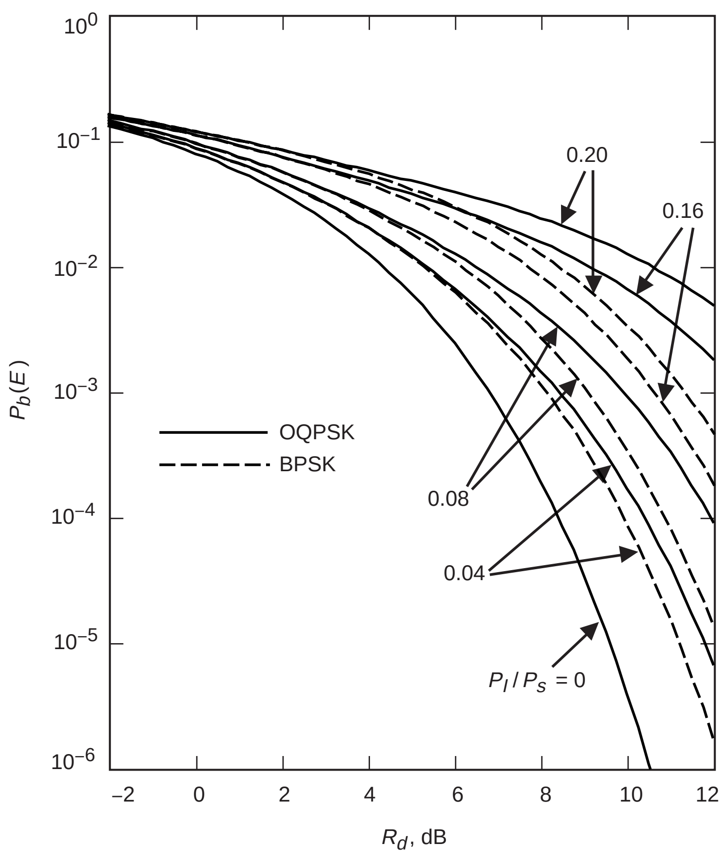

Fig. 2. The bit-error probability performance of OQPSK and BPSK in the presence of narrowband (tone) interference with the interferer perfectly aligned in phase and frequency with the desired signal.

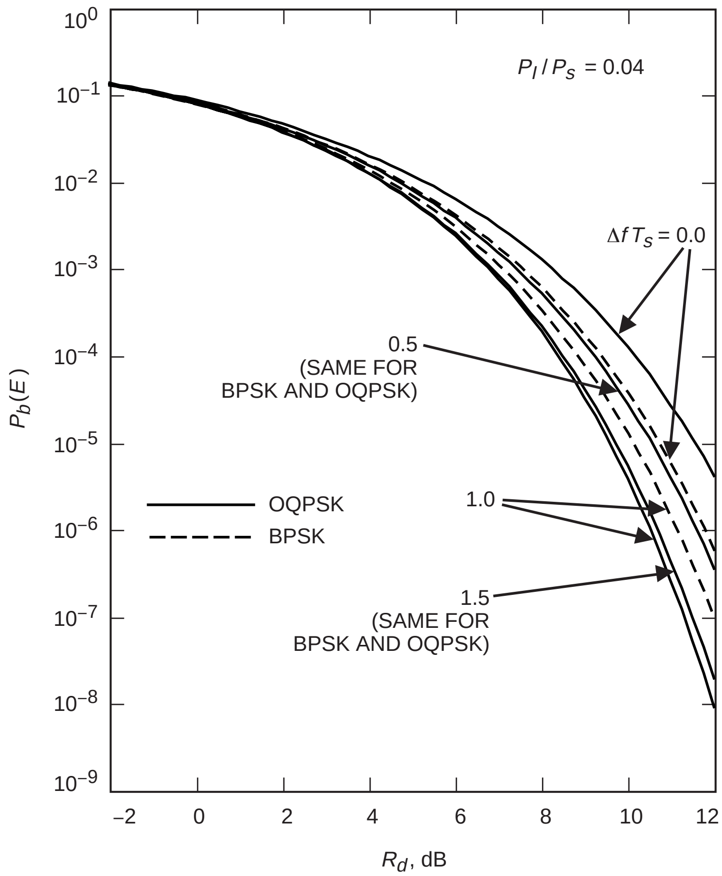

Fig. 3. The bit-error probability performance of OQPSK and BPSK in the presence of narrowband (tone) interference with the interferer at arbitrary (random) phase and fixed frequency offset with respect to the desired signal.

III. Performance in the Presence of Wideband Interference

For the wideband interferer case, the received signal is again given by Eq. (4), where now

$$s_I(t) = \sqrt{P_I}\left[m_{Ic}(t)\cos\left((\omega_c + \Delta\omega)t + \theta_I\right) + m_{Is}(t)\sin\left((\omega_c + \Delta\omega)t + \theta_I\right)\right] \tag{27}$$

with

$$\left.\begin{aligned} m_{Ic}(t) &= \sum_{n=-\infty}^{\infty} a'_n p\left(t - nT_s - \tau\right) \\ m_{Is}(t) &= \sum_{n=-\infty}^{\infty} b'_n p\left(t - nT_s - \frac{T_s}{2} - \tau\right) \end{aligned}\right\} \tag{28}$$

denoting the interference modulation, which is independent of the desired signal modulation and whose data rate is assumed to be equal to that of the desired signal. In Eq. (28), $\{a'_n\}$ and $\{b'_n\}$ are i.i.d. sequences taking on equiprobable values ± 1, $p(t)$ is still a unit amplitude rectangular pulse of duration T_s, and now τ denotes the time asynchronism of the interference with respect to the desired signal which, in the absence of any a priori information, is assumed to be uniformly distributed over a T_s-s interval. Analogously to Eq. (5), the I and Q demodulator baseband outputs become

$$\left.\begin{aligned}
\varepsilon_c(t) &= r(t)\sqrt{2}\cos(\omega_c t+\theta_s)\\
&= \sqrt{\frac{P_s}{2}}m_{sc}(t)+N_c(t)+\sqrt{\frac{P_I}{2}}m_{Ic}(t)\cos(\Delta\omega t+\Delta\theta)+\sqrt{\frac{P_I}{2}}m_{Is}(t)\sin(\Delta\omega t+\Delta\theta)\\
\varepsilon_s(t) &= r(t)\sqrt{2}\sin(\omega_c t+\theta_s)\\
&= \sqrt{\frac{P_s}{2}}m_{ss}(t)-N_s(t)-\sqrt{\frac{P_I}{2}}m_{Ic}(t)\sin(\Delta\omega t+\Delta\theta)+\sqrt{\frac{P_I}{2}}m_{Is}(t)\cos(\Delta\omega t+\Delta\theta)
\end{aligned}\right\} \tag{29}$$

After passing these signals through the I and Q integrate-and-dumps (I&Ds), the sample-and-hold values for the kth bit interval are given by

$$\left.\begin{aligned}
z_s(t) &= \int_{(k+1/2)T_s}^{(k+3/2)T_s}\varepsilon_s(t)\,dt = T_s\sqrt{\frac{P_s}{2}}b_k - N'_2 + T_s\sqrt{\frac{P_I}{2}}\left[\mathrm{Re}\left\{B'_k e^{j\Delta\theta}\right\}-\mathrm{Im}\left\{A'_k e^{j\Delta\theta}\right\}\right],\\
&\qquad\left(k+\frac{3}{2}\right)T_s \le t \le \left(k+\frac{5}{2}\right)T_s\\
z_c(t) &= \int_{kT_s}^{(k+1)T_s}\varepsilon_c(t)\,dt = T_s\sqrt{\frac{P_s}{2}}a_k + N_1 + T_s\sqrt{\frac{P_I}{2}}\left[\mathrm{Re}\left\{A_k e^{j\Delta\theta}\right\}+\mathrm{Im}\left\{B_k e^{j\Delta\theta}\right\}\right],\\
&\qquad(k+1)T_s \le t \le (k+2)T_s
\end{aligned}\right\} \tag{30}$$

where now

$$\left.\begin{aligned}
A'_k &\triangleq \frac{1}{T_s}\int_{(k+1/2)T_s}^{(k+3/2)T_s}\sum_{n=-\infty}^{\infty} a'_n p\,(t-nT_s-\tau)e^{j\Delta\omega t}dt \\
B'_k &\triangleq \frac{1}{T_s}\int_{(k+1/2)T_s}^{(k+3/2)T_s}\sum_{n=-\infty}^{\infty} b'_n p\left(t-nT_s-\frac{T_s}{2}-\tau\right)e^{j\Delta\omega t}dt \\
A_k &\triangleq \frac{1}{T_s}\int_{kT_s}^{(k+1)T_s}\sum_{n=-\infty}^{\infty} a'_n p\,(t-nT_s-\tau)e^{j\Delta\omega t}dt \\
B_k &\triangleq \frac{1}{T_s}\int_{kT_s}^{(k+1)T_s}\sum_{n=-\infty}^{\infty} b'_n p\left(t-nT_s-\frac{T_s}{2}-\tau\right)e^{j\Delta\omega t}dt
\end{aligned}\right\} \quad (31)$$

and as before N_1 and N'_2 are independent zero-mean Gaussian random variables with variance $\sigma^2_{N_1} = \sigma^2_{N'_2} = N_0T_s/2$. Combining Eqs. (30) and (31) and recognizing as before that, for evaluation of average error probability, it is sufficient to consider the arbitrarily selected zeroth symbol intervals corresponding to $k=0$, we obtain

$$\left.\begin{aligned}
z_s(t) &= T_s\sqrt{\frac{P_s}{2}}b_0 - N'_2 + T_s\sqrt{\frac{P_I}{2}}\left[\mathrm{Re}\left\{B'_0e^{j\Delta\theta}\right\} - \mathrm{Im}\left\{A'_0e^{j\Delta\theta}\right\}\right], \quad \frac{3T_s}{2}\le t\le\frac{5T_s}{2} \\
z_c(t) &= T_s\sqrt{\frac{P_s}{2}}a_0 + N_1 + T_s\sqrt{\frac{P_I}{2}}\left[\mathrm{Re}\left\{A_0e^{j\Delta\theta}\right\} + \mathrm{Im}\left\{B_0e^{j\Delta\theta}\right\}\right], \quad T_s\le t\le 2T_s
\end{aligned}\right\} \quad (32)$$

where, for $0\le\tau\le T_s/2$,

$$\left.\begin{aligned}
A'_0 &= \frac{1}{T_s}\int_{T_s/2}^{3T_s/2}\left[a'_0p\,(t-\tau)+a'_1p\,(t-T_s-\tau)\right]e^{j\Delta\omega t}dt \triangleq A'_{c0}+jA'_{s0} \\
B'_0 &= \frac{1}{T_s}\int_{T_s/2}^{3T_s/2}\left[b'_{-1}p\left(t+\frac{T_s}{2}-\tau\right)+b'_0p\left(t-\frac{T_s}{2}-\tau\right)\right]e^{j\Delta\omega t}dt \triangleq B'_{c0}+jB'_{s0} \\
A_0 &= \frac{1}{T_s}\int_{0}^{T_s}\left[a'_{-1}p\,(t+T_s-\tau)+a'_0p\,(t-\tau)\right]e^{j\Delta\omega t}dt \triangleq A_{c0}+jA_{s0} \\
B_0 &= \frac{1}{T_s}\int_{0}^{T_s}\left[b'_{-1}p\left(t+\frac{T_s}{2}-\tau\right)+b'_0p\left(t-\frac{T_s}{2}-\tau\right)\right]e^{j\Delta\omega t}dt \triangleq B_{c0}+jB_{s0}
\end{aligned}\right\} \quad (33)$$

Letting $\varepsilon_s \triangleq \tau/T_s$ denote the normalized time-synchronization error between the desired and interference signals, then the coefficients in Eq. (33) can be evaluated as follows:

$$\left.\begin{aligned}
A'_{c0} &= a'_0\left[\frac{\sin\left(\eta_s\left(1+\varepsilon_s\right)\right)-\sin\left(\dfrac{\eta_s}{2}\right)}{\eta_s}\right]+a'_1\left[\frac{\sin\left(\dfrac{3\eta_s}{2}\right)-\sin\left(\eta_s\left(1+\varepsilon_s\right)\right)}{\eta_s}\right]\\
A'_{s0} &= -a'_0\left[\frac{\cos\left(\eta_s\left(1+\varepsilon_s\right)\right)-\cos\left(\dfrac{\eta_s}{2}\right)}{\eta_s}\right]-a'_1\left[\frac{\cos\left(\dfrac{3\eta_s}{2}\right)-\cos\left(\eta_s\left(1+\varepsilon_s\right)\right)}{\eta_s}\right]\\
B'_{c0} &= b'_{-1}\left[\frac{\sin\left(\dfrac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right)-\sin\left(\dfrac{\eta_s}{2}\right)}{\eta_s}\right]+b'_0\left[\frac{\sin\left(\dfrac{3\eta_s}{2}\right)-\sin\left(\dfrac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right)}{\eta_s}\right]\\
B'_{s0} &= -b'_{-1}\left[\frac{\cos\left(\dfrac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right)-\cos\left(\dfrac{\eta_s}{2}\right)}{\eta_s}\right]-b'_0\left[\frac{\cos\left(\dfrac{3\eta_s}{2}\right)-\cos\left(\dfrac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right)}{\eta_s}\right]\\
A_{c0} &= a'_{-1}\frac{\sin\eta_s\varepsilon_s}{\eta_s}+a'_0\left[\frac{\sin\eta_s-\sin\eta_s\varepsilon_s}{\eta_s}\right]\\
A_{s0} &= -a'_{-1}\left[\frac{\cos\eta_s\varepsilon_s-1}{\eta_s}\right]-a'_0\left[\frac{\cos\eta_s-\cos\eta_s\varepsilon_s}{\eta_s}\right]\\
B_{c0} &= b'_{-1}\frac{\sin\left(\dfrac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right)}{\eta_s}+b'_0\left[\frac{\sin\eta_s-\sin\left(\dfrac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right)}{\eta_s}\right]\\
B_{s0} &= -b'_{-1}\left[\frac{\cos\left(\dfrac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right)-1}{\eta_s}\right]-b'_0\left[\frac{\cos\eta_s-\cos\left(\dfrac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right)}{\eta_s}\right]
\end{aligned}\right\}\tag{34}$$

Comparing $z_s\left(t\right)$ and $z_c\left(t\right)$ of Eq. (32) with zero thresholds results in decisions on b_0 and a_0, respectively. Assuming $b_0=1$, the conditional probability of error for the decision on b_0 is given by

$$\begin{aligned}
P_b\left(E\left|\Delta\theta,\varepsilon_s\right.\right)\Big|_{b_0=1} &= \Pr\left\{z_s\left(t\right)<0\left|b_0=1\right.\right\}\\
&= \frac{1}{2}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{P_I}{P_s}}\left(\left|B'_0\right|\cos\left(\Delta\theta+\beta'_0\right)-\left|A'_0\right|\sin\left(\Delta\theta+\alpha'_0\right)\right)\right]\right\}
\end{aligned}\tag{35a}$$

where $\beta'_0\triangleq\arg B'_0$ and $\alpha'_0\triangleq\arg A'_0$. Similarly, assuming $b_0=-1$, the conditional probability of error is given by

$$P_b\left(E\left|\Delta\theta,\varepsilon_s\right.\right)\big|_{b_0=-1} = \Pr\left\{z_s(t)\geq 0\left|b_0=-1\right.\right\}$$

$$= \frac{1}{2}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{P_I}{P_s}}\left(\left|B_0'\right|\cos\left(\Delta\theta+\beta_0'\right)-\left|A_0'\right|\sin\left(\Delta\theta+\alpha_0'\right)\right)\right]\right\} \quad (35b)$$

Since the hypotheses $b_0=-1$ and $b_0=1$ are equiprobable, then averaged over the I-channel data, the conditional BEP $P_{bI}\left(E\left|\Delta\theta,\varepsilon_s\right.\right)$ is analogous to Eq. (17):

$$P_{bI}\left(E\left|\Delta\theta,\varepsilon_s\right.\right) = \frac{1}{2}P_b\left(E\left|\Delta\theta,\varepsilon_s\right.\right)\big|_{b_0=1} + \frac{1}{2}P_b\left(E\left|\Delta\theta,\varepsilon_s\right.\right)\big|_{b_0=-1} \quad (36)$$

For the Q channel, a decision on a_0 would produce

$$P_{bQ}\left(E\left|\Delta\theta,\varepsilon_s\right.\right) = \frac{1}{2}P_b\left(E\left|\Delta\theta,\varepsilon_s\right.\right)\big|_{a_0=1} + \frac{1}{2}P_b\left(E\left|\Delta\theta,\varepsilon_s\right.\right)\big|_{a_0=-1} \quad (37)$$

where

$$P_b\left(E\left|\Delta\theta,\varepsilon_s\right.\right)\big|_{a_0=1} = \Pr\left\{z_c(t)<0\left|a_0=1\right.\right\}$$

$$= \frac{1}{2}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{P_I}{P_s}}\left(\left|A_0\right|\cos\left(\Delta\theta+\alpha_0\right)+\left|B_0\right|\sin\left(\Delta\theta+\beta_0\right)\right)\right]\right\} \quad (38a)$$

and

$$P_b\left(E\left|\Delta\theta,\varepsilon_s\right.\right)\big|_{a_0=-1} = \Pr\left\{z_c(t)\geq 0\left|a_0=-1\right.\right\}$$

$$= \frac{1}{2}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{P_I}{P_s}}\left(\left|A_0\right|\cos\left(\Delta\theta+\alpha_0\right)+\left|B_0\right|\sin\left(\Delta\theta+\beta_0\right)\right)\right]\right\} \quad (38b)$$

where $\beta_0 \triangleq \arg B_0$ and $\alpha_0 \triangleq \arg A_0$. Finally, the conditional (on $\Delta\theta$ and ε_s) BEP is obtained from the average of Eqs. (36) and (37), namely,

$$\begin{aligned}P_b\left(E\left|\Delta\theta,\varepsilon_s\right.\right) = &\frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{P_I}{P_s}}\left(\left|B_0'\right|\cos\left(\Delta\theta+\beta_0'\right)-\left|A_0'\right|\sin\left(\Delta\theta+\alpha_0'\right)\right)\right]\right\}\\ &+\frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{P_I}{P_s}}\left(\left|B_0'\right|\cos\left(\Delta\theta+\beta_0'\right)-\left|A_0'\right|\sin\left(\Delta\theta+\alpha_0'\right)\right)\right]\right\}\\ &+\frac{1}{8}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{P_I}{P_s}}\left(\left|A_0\right|\cos\left(\Delta\theta+\alpha_0\right)+\left|B_0\right|\sin\left(\Delta\theta+\beta_0\right)\right)\right]\right\}\end{aligned}$$

$$+\frac{1}{8}\mathrm{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{P_I}{P_s}}\left(|A_0|\cos\left(\Delta\theta+\alpha_0\right)+|B_0|\sin\left(\Delta\theta+\beta_0\right)\right)\right]\right\} \tag{39}$$

To compute the average BEP, we must average the conditional BEP obtained as above over the uniform distributions on $\Delta\theta$ and ε_s in the intervals $0 \le |\Delta\theta| \le \pi$ and $0 \le |\varepsilon_s| \le 1/2$, respectively, i.e.,

$$P_b\left(E\left|b'_{-1}, b'_0, a'_{-1}, a'_0, a'_1\right.\right) = \frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{-1/2}^{1/2} P_b\left(E\left|\Delta\theta, \varepsilon_s\right.\right) d\varepsilon_s d\Delta\theta \tag{40}$$

In addition, since the coefficients A_0, B_0, A'_0, and B'_0 depend through Eq. (34) on the random interference data bits $b'_{-1}, b'_0, a'_{-1}, a'_0$, and a'_1, we must further average over these equiprobable ± 1 random variables to get the final desired result, namely,

$$P_b\left(E\right) = \overline{P_b\left(E\left|b'_{-1}, b'_0, a'_{-1}, a'_0, a'_1\right.\right)}^{b'_{-1}, b'_0, a'_{-1}, a'_0, a'_1} \tag{41}$$

To simplify matters a bit in the evaluation of Eq. (41), we write the squared magnitudes and phases of these coefficients in closed form as follows:

$$\left.\begin{aligned}
|A'_0|^2 &= \left(\frac{\sin\left(\eta_s\left(\frac{1}{2}+\varepsilon_s\right)/2\right)}{\eta_s/2}\right)^2 + \left(\frac{\sin\left(\eta_s\left(\frac{1}{2}-\varepsilon_s\right)/2\right)}{\eta_s/2}\right)^2 \\
&\quad + a'_0 a'_1\left[\left(\frac{\sin\left(\eta_s\left(\frac{3}{2}-\varepsilon_s\right)/2\right)}{\eta_s/2}\right)\left(\frac{\sin\left(\eta_s\left(\frac{1}{2}+\varepsilon_s\right)/2\right)}{\eta_s/2}\right) - \left(\frac{\sin\left(\eta_s\left(\frac{1}{2}+\varepsilon_s\right)/2\right)}{\eta_s/2}\right)^2\right] \\
\alpha'_0 &= \tan^{-1}\frac{\cos\left(\frac{\eta_s}{2}\right)-\cos\left(\eta_s\left(1+\varepsilon_s\right)\right)+a'_0 a'_1\left[\cos\left(\eta_s\left(1+\varepsilon_s\right)\right)-\cos\left(\frac{3\eta_s}{2}\right)\right]}{\sin\left(\eta_s\left(1+\varepsilon_s\right)\right)-\sin\left(\frac{\eta_s}{2}\right)+a'_0 a'_1\left[\sin\left(\frac{3\eta_s}{2}\right)-\sin\left(\eta_s\left(1+\varepsilon_s\right)\right)\right]} \\
&\quad + \pi\left(\frac{1-\mathrm{sgn}\ a'_0}{2}\right)
\end{aligned}\right\} \tag{42a}$$

$$
\left.
\begin{aligned}
\left|B_0'\right|^2 =\ & \left(\frac{\sin\left(\frac{\eta_s\varepsilon_s}{2}\right)}{\frac{\eta_s}{2}}\right)^2 + \left(\frac{\sin\left(\frac{\eta_s\left(1-\varepsilon_s\right)}{2}\right)}{\frac{\eta_s}{2}}\right)^2 \\
& + b_{-1}'b_0'\left[\left(\frac{\sin\left(\frac{\eta_s\left(2-\varepsilon_s\right)}{2}\right)}{\frac{\eta_s}{2}}\right)\left(\frac{\sin\left(\frac{\eta_s\varepsilon_s}{2}\right)}{\frac{\eta_s}{2}}\right) - \left(\frac{\sin\left(\frac{\eta_s\varepsilon_s}{2}\right)}{\frac{\eta_s}{2}}\right)^2\right] \\
\beta_0' =\ & \tan^{-1}\frac{\cos\left(\frac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right) - \cos\left(\frac{3\eta_s}{2}\right) + b_{-1}'b_0'\left[\cos\left(\frac{\eta_s}{2}\right) - \cos\left(\frac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right)\right]}{\sin\left(\frac{3\eta_s}{2}\right) - \sin\left(\frac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right) + b_{-1}'b_0'\left[\sin\left(\frac{\eta_s\left(1+2\varepsilon_s\right)}{2}\right) - \sin\left(\frac{\eta_s}{2}\right)\right]} \\
& + \pi\left(\frac{1-\operatorname{sgn}\, b_0'}{2}\right)
\end{aligned}
\right\} \tag{42b}
$$

$$
\left.
\begin{aligned}
\left|A_0\right|^2 =\ & \left(\frac{\sin\left(\frac{\eta_s\varepsilon_s}{2}\right)}{\frac{\eta_s}{2}}\right)^2 + \left(\frac{\sin\left(\frac{\eta_s\left(1-\varepsilon_s\right)}{2}\right)}{\frac{\eta_s}{2}}\right)^2 \\
& + a_{-1}'a_0'\left[\left(\frac{\sin\left(\frac{\eta_s\left(1-\varepsilon_s\right)}{2}\right)}{\frac{\eta_s}{2}}\right)\left(\frac{\sin\left(\frac{\eta_s\left(1+\varepsilon_s\right)}{2}\right)}{\frac{\eta_s}{2}}\right) - \left(\frac{\sin\left(\frac{\eta_s\left(1-\varepsilon_s\right)}{2}\right)}{\frac{\eta_s}{2}}\right)^2\right] \\
\alpha_0 =\ & \tan^{-1}\frac{\cos\eta_s\varepsilon_s - \cos\eta_s + a_{-1}'a_0'\left[1-\cos\eta_s\varepsilon_s\right]}{\sin\eta_s - \sin\eta_s\varepsilon_s + a_{-1}'a_0'\sin\eta_s\varepsilon_s} + \pi\left(\frac{1-\operatorname{sgn}\, a_0'}{2}\right)
\end{aligned}
\right\} \tag{42c}
$$

$$
\left.
\begin{aligned}
|B_0|^2 &= \left(\frac{\sin\left(\dfrac{\eta_s\,(1/2+\varepsilon_s)}{2}\right)}{\dfrac{\eta_s}{2}}\right)^2 + \left(\frac{\sin\left(\dfrac{\eta_s\,(1/2-\varepsilon_s)}{2}\right)}{\dfrac{\eta_s}{2}}\right)^2 \\
&\quad + b'_{-1}b'_0\left[\left(\frac{\sin\left(\dfrac{\eta_s\,(3/2+\varepsilon_s)}{2}\right)}{\dfrac{\eta_s}{2}}\right)\left(\frac{\sin\left(\dfrac{\eta_s\,(1/2-\varepsilon_s)}{2}\right)}{\dfrac{\eta_s}{2}}\right) - \left(\frac{\sin\left(\dfrac{\eta_s\,(1/2-\varepsilon_s)}{2}\right)}{\dfrac{\eta_s}{2}}\right)^2\right] \\
\beta_0 &= \tan^{-1}\frac{\cos\left(\dfrac{\eta_s\,(1+2\varepsilon_s)}{2}\right) - \cos\eta_s + b'_{-1}b'_0\left[1-\cos\left(\dfrac{\eta_s\,(1+2\varepsilon_s)}{2}\right)\right]}{\sin\eta_s - \sin\left(\dfrac{\eta_s\,(1+2\varepsilon_s)}{2}\right) + b'_{-1}b'_0\sin\left(\dfrac{\eta_s\,(1+2\varepsilon_s)}{2}\right)} \\
&\quad + \pi\left(\frac{1-\operatorname{sgn} b'_0}{2}\right)
\end{aligned}
\right\} \tag{42d}
$$

As for the tone-interference case, we can again consider a simple special case of the above results wherein the frequency separation between the desired signal and the interferer is equal to zero, i.e., the interfering signal is right on the carrier frequency. Since for this case it is clear from Eq. (33) that A_0, B_0, A'_0, and B'_0 are all real, i.e., $\alpha_0, \beta_0, \alpha'_0$, and β'_0 all equal zero, then Eq. (39) simplifies to

$$
\begin{aligned}
P_b\left(E\,|\Delta\theta,\varepsilon_s\right) &= \frac{1}{8}\operatorname{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{P_I}{P_s}}\left(|B'_0|\cos\Delta\theta - |A'_0|\sin\Delta\theta\right)\right]\right\} \\
&\quad + \frac{1}{8}\operatorname{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{P_I}{P_s}}\left(|B'_0|\cos\Delta\theta - |A'_0|\sin\Delta\theta\right)\right]\right\} \\
&\quad + \frac{1}{8}\operatorname{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{P_I}{P_s}}\left(|A_0|\cos\Delta\theta + |B_0|\sin\Delta\theta\right)\right]\right\} \\
&\quad + \frac{1}{8}\operatorname{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{P_I}{P_s}}\left(|A_0|\cos\Delta\theta + |B_0|\sin\Delta\theta\right)\right]\right\}
\end{aligned} \tag{43}
$$

where taking the limit of the amplitude coefficients in Eq. (42) when $\eta_s = 0$ gives

$$\left.\begin{aligned}|A_0'|^2 &= \frac{1}{2}\left(1+a_0'a_1'\right)+2\varepsilon_s^2\left(1-a_0'a_1'\right)\\ |B_0'|^2 &= 1-2\varepsilon_s\left(1-\varepsilon_s\right)\left(1-b_0'b_1'\right)\\ |A_0|^2 &= 1-2\varepsilon_s\left(1-\varepsilon_s\right)\left(1-a_0'a_1'\right)\\ |B_0|^2 &= \frac{1}{2}\left(1+b_0'b_1'\right)+2\varepsilon_s^2\left(1-b_0'b_1'\right)\end{aligned}\right\} \tag{44}$$

Thus, from Eqs. (41) and (40), the average BEP is now

$$P_b\left(E\right)=\overline{\frac{1}{2\pi}\int_{-\pi}^{\pi}\int_{-1/2}^{1/2}P_b\left(E\left|\Delta\theta,\varepsilon_s\right.\right)d\varepsilon_s d\Delta\theta}^{\,b'_{-1}b'_0,a'_{-1}a'_0} \tag{45}$$

Note that only four terms (the possible combinations of $a_0'a_1'$ and $b_0'b_1'$) are required in the statistical average of Eq. (45) whereas thirty-two terms were required in Eq. (41). If now we furthermore make the additional simplifying assumptions that the interfering signal is in phase and time synchronization with the desired signal, i.e., $\Delta\theta=\varepsilon_s=0$, then Eq. (45), together with Eqs. (43) and (44), further simplifies to

$$P_b\left(E\right)=\frac{1}{4}\text{erfc}\left\{\sqrt{R_d}\left[1+\sqrt{\frac{P_I}{P_s}}\right]\right\}+\frac{1}{4}\text{erfc}\left\{\sqrt{R_d}\left[1-\sqrt{\frac{P_I}{P_s}}\right]\right\} \tag{46}$$

which from [1] would also be the limiting result for BPSK with a wideband BPSK interferer. Thus, the worst-case wideband OQPSK interferer in an OQPSK system produces the same degradation as the worst-case wideband BPSK interferer in a BPSK system. Furthermore, noticing that Eqs. (46) and (24) are identical, then by comparison with Eq. (23), we conclude that OQPSK is more sensitive to worst-case tone interference than it is to worst-case wideband OQPSK interference provided that in both cases the interference is centered at or near the carrier frequency.

Figure 4 is the analogous plot to Fig. 3 for the case of a wideband QPSK interferer. For OQPSK, the results are computed from Eq. (41) together with Eqs. (39), (40), and (42a) through (42d), whereas the BPSK results are computed from [1, Eq. (67)]. We observe here that the relative performances of the two modulation schemes depend on the value of $\Delta f T_s$, i.e., for $\Delta f T_s = 0$, OQPSK is worse whereas, for $\Delta f T_s \geq 0.5$, BPSK is worse. We remind the reader that these curves are plotted assuming that the phase and relative timing of the interfering and desired signals are random with respect to one another whereas the frequency separation between the two (relative to the symbol rate) is held fixed along a given curve.

IV. Conclusion

Depending on the nature of the interference, i.e., narrowband (tone) or wideband (modulated tone) and the specifications on its phase and frequency offsets relative to the desired signal, the performance of coherent offset QPSK in the presence of this interference can be better, the same, or worse than that of coherent BPSK. Thus, in comparing the two modulation/demodulation schemes in the presence of interference, one should be very specific as to the terms of the comparison.

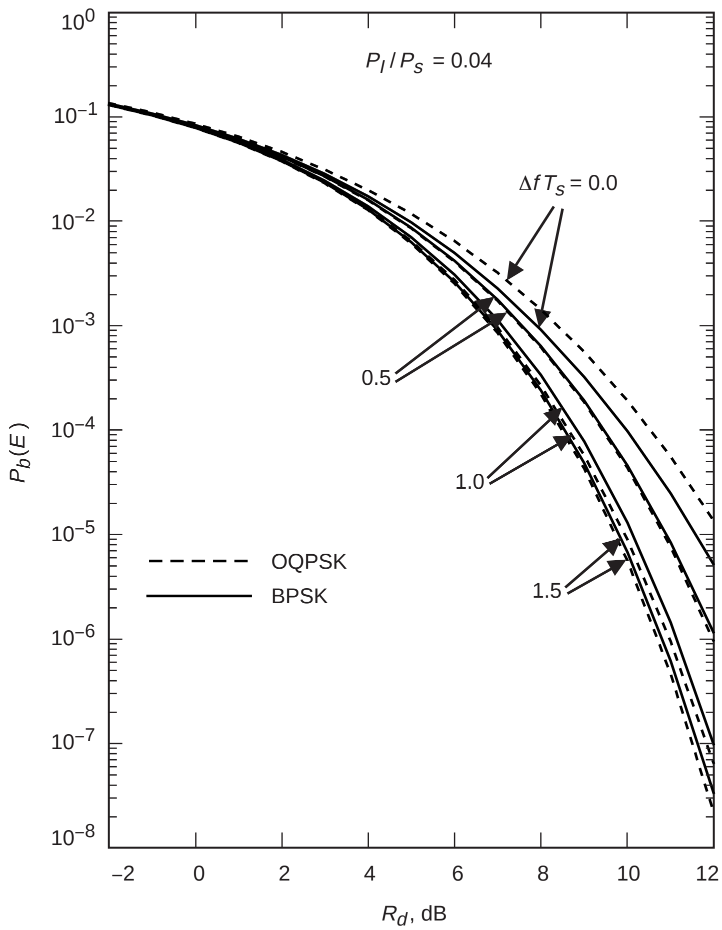

Fig. 4. The bit-error probability performance of OQPSK and BPSK in the presence of wideband interference with the interferer at arbitrary (random) phase and timing and fixed frequency offset with respect to the desired signal.

References

[1] M. K. Simon, "Performance of Coherent Binary Phase-Shift Keying (BPSK) with Costas-Loop Tracking in the Presence of Interference," *The Telecommunications and Mission Operations Progress Report 42-139, July–September 1999*, Jet Propulsion Laboratory, Pasadena, California, pp. 1–24, November 15, 1999. http://tmo.jpl.nasa.gov/tmo/progress_report/42-139/139A.pdf