Analysis and Optimization of the Performance of a Convolutionally Encoded Deep-Space Link in the Presence of Spacecraft Oscillator Phase Noise

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In order to reduce the cost of deep-space missions, NASA is exploring the possibility of using new, cheaper technologies. Among these is the possibility of replacing ultra-stable oscillators (USOs) onboard the spacecraft with oscillators with measurable phase noise. In addition, it is proposed that these spacecraft use higher 32-GHz (Ka-band) radio frequencies in order to save mass. In this article, the performance of a convolutionally encoded deep-space link using non-USO-type oscillators onboard the spacecraft at Ka-band is analyzed. It is shown that the ground-receiver tracking-loop bandwidth settings need to be optimized and that, by selecting an oscillator with good phase-noise characteristics, the minimum required power onboard the spacecraft could be reduced by as much as 10 dB.

I. Introduction

Due to budget constraints, new NASA deep-space missions have been forced to use newer technologies and to do without items once considered essential on deep-space missions. One of the new technologies that is being considered is the use of higher radio frequencies, namely 32-GHz (Ka-band), and one of the items that these new missions have to do without is the expensive ultra-stable oscillator (USO). In this article, we analyze the combined effect of the use of higher frequencies (i.e., Ka-band) with non-USO-type oscillators on the telemetry performance of a residual-carrier modulation link employing convolutional codes for error correction. Furthermore, we will show how this analysis is used to optimize the performance of the channel by optimizing the receiver tracking-loop settings and why this optimization is necessary, especially at low data rates.

In Section II of this article, we present the theoretical model of a channel employing a non-USO-type oscillator and show how the tracking-loop bandwidth could be optimized based on this analysis. In Section III, we show why this optimization is necessary by demonstrating that a bandwidth-optimized loop tracks the signal significantly better than does a non-optimized loop. In Section IV, we present the results of such an optimization for the Deep Space Network's (DSN's) Block V Receiver (BVR) and NASA's (7,1/2) and (15,1/6) convolutional codes. The range of phase-noise levels for which this analysis is performed represents the Ka-band performance of the non-USO-type oscillators in use by the current

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[1] Communications Systems and Research Section.

set of NASA's deep-space missions. These results indicate that, while at high data rates the noise level of the oscillator has very little effect (less than 1 dB) on the total power required onboard the spacecraft to close the link, the choice of the oscillator could mean as much as 10 dB less/more power for closing the link at low data rates. In Section V, we present our conclusions.

II. Theoretical Approach

Most non-USO-type oscillators exhibit a one-sided power spectral density given by

$$S_\phi(\Delta f) = \frac{C_3}{\Delta f^3} \tag{1}$$

where Δf is the absolute frequency offset from the center frequency of the oscillator and C_3 is the oscillator spectral density at a 1-Hz offset. In residual-carrier-modulation transmission, the phase and the frequency of the carrier are tracked by a phase-locked loop (PLL). It is assumed that the tracking phase error, $\Delta\phi$, has approximately a Tikhonov probability density function [1–3] given by

$$P_{\Delta\phi}(\theta) = \begin{cases} \dfrac{\exp\left(\dfrac{\cos(\theta)}{\sigma^2_{\Delta\phi}}\right)}{2 \times \pi \times I_0(\sigma^{-2}_{\Delta\phi})} & -\pi \le \theta < \pi \\ 0 & \text{otherwise} \end{cases} \tag{2}$$

where $\sigma^2_{\Delta\phi}$ is the variance of the tracking phase error. Assuming that the PLL filter has a bandwidth of B_l, then $\sigma^2_{\Delta\phi}$ is given by[2]

$$\sigma^2_{\Delta\phi} = \frac{k \times C_3}{B_l^2} + \frac{B_l}{\dfrac{P_c}{N_0}} \tag{3}$$

where P_c/N_0 is the carrier-signal-to-noise ratio and k is the PLL filter's coefficient.

In order to improve the tracking performance of the PLL, we want to minimize $\sigma^2_{\Delta\phi}$ for a given P_c/N_0. Thus, we need to calculate the optimum value of B_l, $B_{l_{\text{opt}}}$, in Eq. (3). Doing so gives

$$B_{l_{\text{opt}}} = \sqrt[3]{2 \times k \times C_3 \times \frac{P_c}{N_0}} \tag{4}$$

Substituting Eq. (4) in Eq. (3), we obtain

$$\sigma^2_{\Delta\phi,\text{opt}}\left(\frac{P_c}{N_0}\right) = \frac{3 \times \sqrt[3]{k \times C_3}}{\left(\dfrac{2P_c}{N_0}\right)^{2/3}} \tag{5}$$

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[2] S. Shambayati, "Preliminary Results on Optimum Settings for BVR for Tracking of Voyager 1," JPL Interoffice Memorandum (internal document), Jet Propulsion Laboratory, Pasadena, California, 1995.

One uses $\sigma^2_{\Delta\phi}$ to calculate the radio loss of convolutional codes. There are three models for radio losses of such codes [1]: high rate, low rate, and medium rate.

The high-rate model is used when the data rate is much higher than the PLL bandwidth. The bit-error probability for the convolutional code in that case is given by

$$\mathrm{BER}_{(\mathrm{high})} = \int_{\theta=-\pi}^{\pi} f_{\mathrm{BER}}\left(\cos^2(\theta) \times \frac{E_b}{N_0}\right) P_{\Delta\phi}(\theta) d\theta \tag{6}$$

where f_{BER} is the ideal bit-error rate (BER) function for the convolutional code, $P_{\Delta\phi}(\theta)$ is the probability density function of the tracking error as given in Eq. (2), and E_b/N_0 is the input bit-signal-to-noise ratio.

The low-rate model applies when the data rate is much lower than the PLL bandwidth. In this case, it is assumed that the bit-error rate is equal to the value of f_{BER} at the average bit-signal-to-noise ratio. In other words,

$$\mathrm{BER}_{(\mathrm{low})} = f_{\mathrm{BER}}\left(\int_{\theta=-\pi}^{\pi} \cos^2(\theta) \times \frac{E_b}{N_0} P_{\Delta\phi}(\theta) d\theta\right) \tag{7}$$

The medium-rate model applies when the other two models do not. In this case, there is no straight analytical solution, and the medium-rate model is calculated by interpolation between the high-rate model and the low-rate model [1]:

$$\mathrm{BER}_{(\mathrm{med})} = a_c \times \mathrm{BER}_{(\mathrm{high})} + (1 - a_c) \times \mathrm{BER}_{(\mathrm{low})} \tag{8}$$

According to [1], a_c is given by

$$a_c = \frac{\delta_c}{4}\left\{1 - \frac{\delta_c}{8}\left[1 - \exp\left(\frac{-8}{\delta_c}\right)\right]\right\} \tag{9}$$

where

$$\delta_c = \frac{1}{B_l \times T} \tag{10}$$

and T is the period of time over which the value of the bit is determined. For convolutional codes, T is usually several times the constraint length of the code. In this article, it is assumed that T is five times the constraint length of the code. Thus, for (7,1/2) code, T is 35 bit times and, for (15,1/6) code, it is 75 bit times.

When phase noise is present, we use $B_{l_{\mathrm{opt}}}$ for B_l in order to minimize the bit-error rate; therefore, Eqs. (6) through (8) could all be viewed as functions of the bit duration: k, C_3, P_c/N_0, and E_b/N_0. Therefore, given a bit rate, R_b, and k, C_3, and P_c/N_0, and a required bit-error rate, p_b, Eqs. (6) through (10) could be used to calculate the required E_b/N_0. Therefore, we can define the required E_b/N_0, $(E_b/N_0)_r$, as a function of k, C_3, P_c/N_0, R_b, and p_b:

$$\left(\frac{E_b}{N_0}\right)_r = f_{\mathrm{SNR}}\left(k, C_3, \frac{P_c}{N_0}, R_b, p_b\right) \tag{11}$$

Note that, in general, there is no closed-form solution for f_{SNR}; however, using numerical methods, its value could be approximated.

Let $(E_b/N_0)_{p_b}$ be the value of E_b/N_0 for which the bit-error rate is p_b under perfect phase-tracking conditions (i.e., $\sigma^2_{\Delta\phi} = 0$). Then the radio loss, L_{radio}, is given by

$$L_{\mathrm{radio}} = \frac{\left(\frac{E_b}{N_0}\right)_r}{\left(\frac{E_b}{N_0}\right)_{p_b}} \tag{12}$$

where $(E_b/N_0)_r$ is given by Eq. (11).

Given $(E_b/N_0)_r$, P_c/N_0, and R_b, then the total power-to-noise ratio, P_t/N_0, is given by

$$\frac{P_t}{N_0} = \frac{P_c}{N_0} + R_b \times \left(\frac{E_b}{N_0}\right)_r \tag{13}$$

Since $(E_b/N_0)_r$ is a function of k, C_3, P_c/N_0, R_b, and p_b, then P_t/N_0 is also a function of these parameters. Therefore, given an oscillator (hence C_3), a ground receiver (hence k), and a bit-error rate, p_b, for any data rate R_b, we can minimize P_t/N_0 as a function of P_c/N_0. This is important since on deep-space missions the limiting factor in telecommunications is always total available power. Let $(P_c/N_0)_{\mathrm{opt}}$ be the value for which P_t/N_0 is minimized, and let $(P_t/N_0)_{\mathrm{min}}$ be the minimized value; then the optimum modulation index, m_{opt}, could be obtained by

$$m_{\mathrm{opt}} = \mathrm{Arccos}\left(\sqrt{\frac{\left(\frac{P_c}{N_0}\right)_{\mathrm{opt}}}{\left(\frac{P_t}{N_0}\right)_{\mathrm{min}}}}\right) \tag{14}$$

It should be noted that, in the literature [4], instead of C_3 another metric, L_1, is used, where

$$L_1 = 10 \times \log(0.5 \times C_3) \tag{15}$$

The units of L_1 are dBc/Hz. For the rest of this article, all the results will be identified in terms of L_1.

In the following section, we show the sensitivity of loop signal-to-noise ratio to the level phase noise. Then, we will use the equations in this section to calculate $(P_t/N_0)_{\mathrm{min}}$ and m_{opt} for different data rates and different levels of oscillator phase noise for (15,1/6) and (7,1/2) convolutional codes for a bit-error rate of 0.001 when the signal is demodulated using a BVR.

III. Sensitivity of PLL Loop Signal-to-Noise Ratio to Oscillator Phase Instability and the Necessity of Optimization of the Loop Bandwidth

One question that arises from the discussions presented in the previous section is whether or not it is necessary to optimize the PLL loop bandwidth. To answer this question, one has to first define a metric that indicates how accurately the PLL tracks the phase of the received signal. The PLL loop signal-to-noise ratio (LSNR) is such a metric. LSNR is defined as $\sigma_{\Delta\phi}^{-2}$. In this section, we will show that it is necessary to optimize the loop bandwidth by comparing the LSNR of an optimized receiver with the LSNR of a receiver with constant loop bandwidth for different values of loop bandwidth and different phase-noise levels. Three loop bandwidth values of 3 Hz, 10 Hz, and 30 Hz are chosen for this comparison. The phase-noise levels that are selected are $L_1 = -10.57$ dBc/Hz, $L_1 = -4.55$ dBc/Hz, and $L_1 = 7.49$ dBc/Hz. The PLL is assumed to have a second-order underdamped loop with $k = 8.7$ (this is the actual value used for the DSN's BVR receiver PLL [4]). The results are presented in Figs. 1 through 4.

Figure 1 represents the optimum loop bandwidth, $B_{l_{\rm opt}}$, as a function of P_c/N_0 for the values of L_1 under consideration. First of all, we note that values of $B_{l_{\rm opt}}$ cover a wide range. This indicates that the value of the loop bandwidth needs to be optimized for the specific link conditions. Second of all, we note that, as L_1 increases, the value of $B_{l_{\rm opt}}$ increases for the same value of P_c/N_0. This is due to the fact that the carrier energy is more spread out for larger values of L_1. Therefore, the loop bandwidth needs to be wider to capture the energy in the carrier for higher values of L_1.

Figures 2 through 4 show LSNR values for optimized loop bandwidths and bandwidths of 3 Hz, 10 Hz, and 30 Hz for $L_1 = -10.57$ dBc/Hz, $L_1 = -4.55$ dBc/Hz, and $L_1 = 7.49$ dBc/Hz, respectively. As we can see from these figures, the optimized bandwidth has a decidedly higher LSNR than does the fixed bandwidth. The only time that the fixed bandwidth LSNR is comparable to that of the optimized bandwidth is when the optimized bandwidth happens to be close in value to the fixed bandwidth. This indicates that, in the presence of significant phase noise on the spacecraft oscillator, it is necessary to optimize the ground-receiver loop bandwidth in order to minimize ground-receiver loop-tracking error.

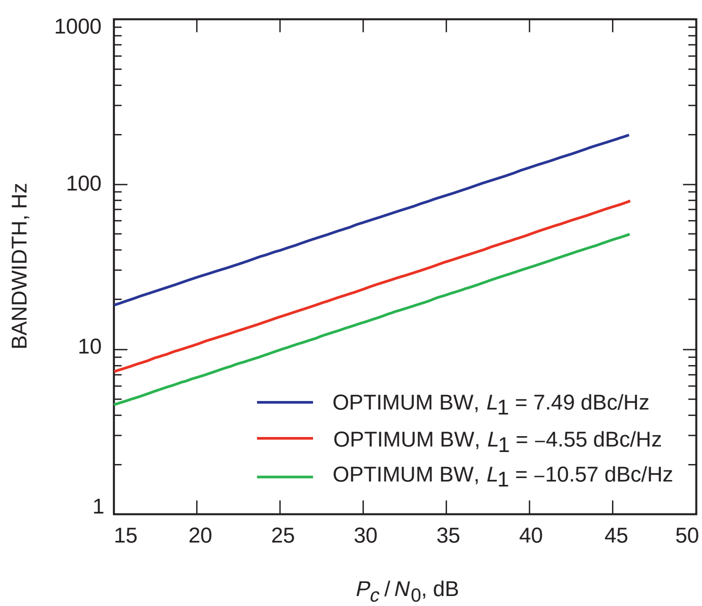

Fig. 1. Optimum loop bandwidth (BW) versus P_c / N_0.

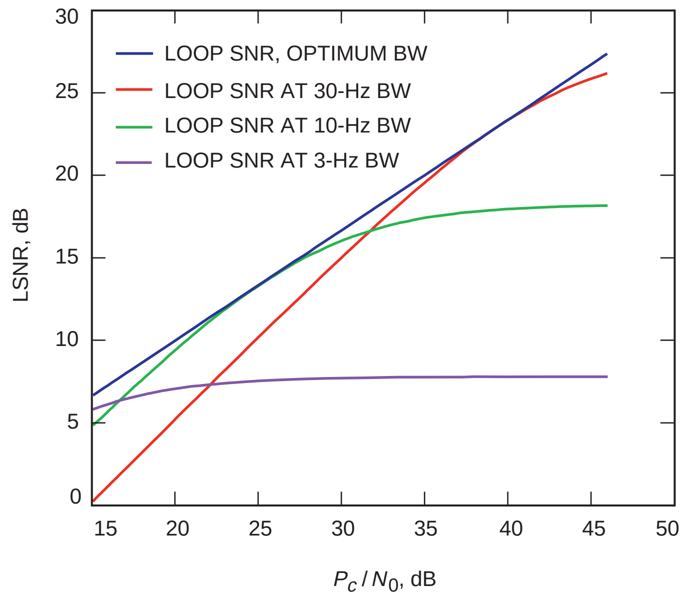

Fig. 2. LSNR versus P_c / N_0, for L_1 = –10.57 dBc.

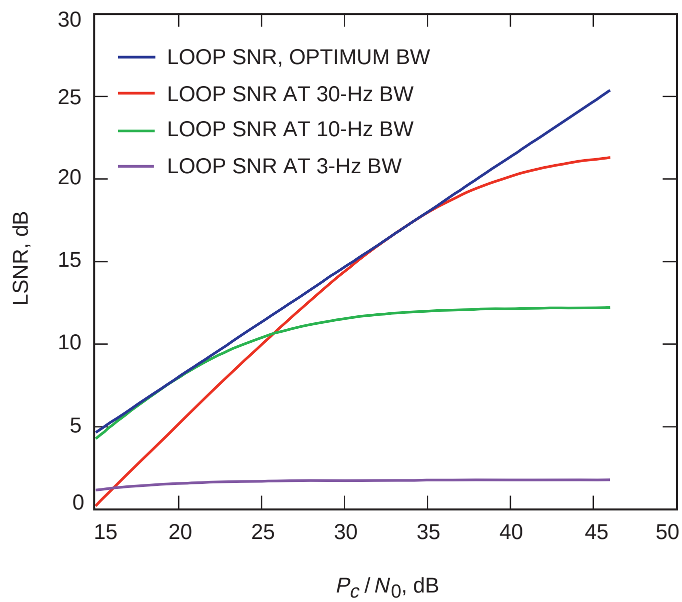

Fig. 3. LSNR versus P_c / N_0, for L_1 = –4.55 dBc/Hz.

IV. Noise Deep-Space Link Optimization with (15,1/6) and (7,1/2) Convolutional Codes

The BVR is used by the DSN to track residual-carrier and suppressed-carrier data from deep-space missions. The BVR uses a digital PLL to track the carrier of a residual-carrier signal. This PLL requires a minimum of 10-dB LSNR,[3] which corresponds to a 0.01 variance for the phase error. Furthermore, the BVR always suffers a minimum of 0.3-dB radio loss [4] in actual operations. Given these two constraints,

[3] P. W. Kinman, personal communication, Case Western Reserve University, Cleveland, Ohio, 1998.

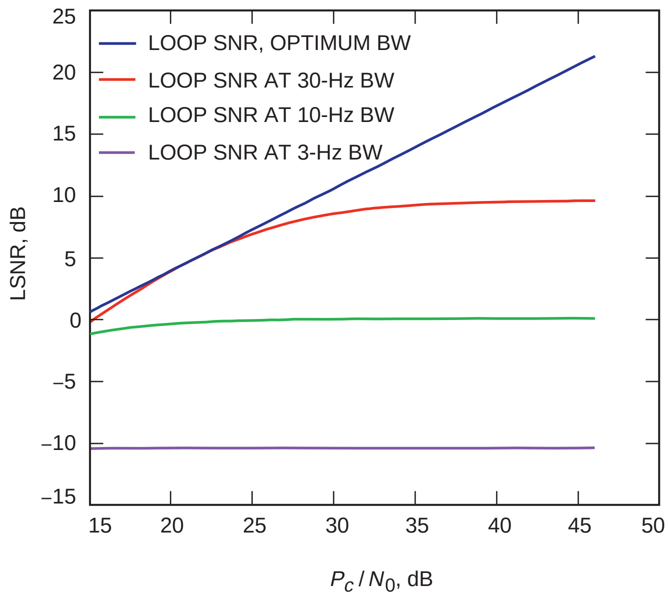

Fig. 4. LSNR versus P_c / N_0, for L_1 = 7.49 dBc.

we will use the approach outlined in the previous section to evaluate $(P_t/N_0)_{\rm min}$ for different data rates and different oscillator phase-noise characteristics.

Two kinds of convolutional codes are considered here: (7,1/2) and (15,1/6). The bit-error rate functions, $f_{\rm BER}$, for both of these codes has the form [1]

$$f_{\rm BER}\left(\frac{E_b}{N_0}\right) = \begin{cases} \exp\left(\alpha_0 - \alpha_1 \times \dfrac{E_b}{N_0}\right) & \dfrac{E_b}{N_0} > \dfrac{\ln(2)+\alpha_0}{\alpha_1} \\ 0.5 & \text{otherwise} \end{cases} \tag{16}$$

For (7,1/2) code, $\alpha_0 = 4.4514$ and $\alpha_1 = 5.7230$ [1]. For (15,1/6) code, $\alpha_0 = 9.807$ and $\alpha_1 = 14.064$.[4] For analysis purposes, we consider L_1 values between -10.57 dBc/Hz, which corresponds roughly to the type of auxiliary oscillator on the Deep Space 1 (DS1) spacecraft[5] operating at Ka-band, and 7.57 dBc/Hz, which corresponds roughly to the type of auxiliary oscillator on the Voyager spacecraft operating at Ka-band.[6] The data rates that are considered are between 10 b/s and 1 Mb/s, which covers the range of data rates typically used for deep-space missions. The bit-error rate at which the optimization is performed is 0.001 at the output of the convolutional decoder. This corresponds to the case when the convolutional code is concatenated with a (255,223) Reed–Solomon code and the output bit-error rate of the Reed–Solomon decoder is less than 10^{-6}. Note that this assumes an infinite interleaving depth for the Reed–Solomon code words. In practice, when the interleaving depth is finite, the bit-error rate performance of the Reed–Solomon code for higher data rates is much greater than 10^{-6}.

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[4] S. J. Dolinar, "Empirical Formula for the Performance of the Recommended (15,1/6) Convolutional Code," JPL Interoffice Memorandum (internal document), Jet Propulsion Laboratory, Pasadena, California, 1990.

[5] C. Chen, personal communication, Jet Propulsion Laboratory, Pasadena, California, 1998.

[6] P. W. Kinman, "Tracking the Voyager 1 Auxiliary Oscillator with Narrow Carrier Loop," Memorandum, Case Western Reserve University, Cleveland, Ohio, 1995.

Both $(P_t/N_0)_{\rm min}$ and $m_{\rm opt}$ for the two convolutional codes are represented in Figs. 5 through 8. While the $(P_t/N_0)_{\rm min}$ curves (Figs. 5 and 7) seem smooth, there are irregularities in the curves for $m_{\rm opt}$. This is due to the numerical techniques that were used to obtain $(P_t/N_0)_{\rm min}$ and $m_{\rm opt}$, and to the inaccuracy of the medium-rate model. While the numbers in these figures do not represent the true optimum values, they are accurate enough for engineering and design purposes. Thus, the following conclusions could be drawn from them:

(1) The more unstable an oscillator, the lower the modulation index should be for the receiver to be able to track the signal optimally. The reason for this is as follows: Since the instability of the oscillator means that the signal energy is spread out over a wider band, we need a larger loop bandwidth to track the signal. Therefore, by using a larger bandwidth, we allow an increase in the noise energy entering the tracking loop. This means that in order to achieve the desired LSNR we need to have more power in the carrier. The only way to achieve this is by decreasing the modulation index.

(2) To communicate at lower data rates, it is necessary to have a low-noise oscillator in order to lower $(P_t/N_0)_{\rm min}$. As we can see from Figs. 5 and 7, as the data rate increases, the values of $(P_t/N_0)_{\rm min}$ for different oscillator phase-noise levels converge. However, at lower data rates, the difference between $(P_t/N_0)_{\rm min}$ for a low-noise (more stable) oscillator and a noisy (less stable) oscillator could be as high as 10 dB. This is due to the fact that, at high data rates, the total energy received on the ground is dominated by the energy in the data. This means that, in order to minimize P_t/N_0, one has to minimize the energy required in the data and, thus, minimize the required bit signal-to-noise ratio, $(E_b/N_0)_r$ [see Eqs. (11) and (13)]. In order to minimize $(E_b/N_0)_r$, one has to increase the LSNR for the PLL and, thus, P_c/N_0 [see Eq. (5)]. Since the BVR has a minimum loss of 0.3 dB, for any noise level at high data rates, only enough energy is needed in the carrier to bring the radio loss in the PLL down to 0.3 dB. For (7,1/2) code, this is achieved at an LSNR of 14.4 dB, and, for (15,1/6) code, this is achieved at an LSNR of 15.3 dB. Selecting 16-dB LSNR as an upper bound for these values, we can see from Figs. 2 through 4 that this is achieved when P_c/N_0 is between 30 and 38 dB. At high data rates, typical values for $(P_t/N_0)_{\rm min}$ are higher than 45 dB (see Figs. 5 and 7). At this level of P_t/N_0, the difference between 30 dB and 38 dB in P_c/N_0 translates to a difference of only 0.7 dB in P_t/N_0. On the other hand, at low data rates, P_t/N_0 is dominated by P_c/N_0. A minimum of 10-dB LSNR is required for the PLL to track the phase. Therefore, the difference between $(P_t/N_0)_{\rm min}$ for a noisy oscillator and $(P_t/N_0)_{\rm min}$ for a more stable oscillator at low data rates is the difference between the value of P_c/N_0 that gives an LSNR of 10 dB for the noisy oscillator and the value of P_c/N_0 that gives an LSNR of 10 dB for the more stable oscillator. As we can see from Figs. 2 through 4, the noisiest oscillator considered here ($L_1 = 7.49$ dBc/Hz) requires roughly 10 dB more P_c/N_0 (30 dB to 20 dB) than the most stable oscillator considered ($L_1 = -10.57$ dBc/Hz). Therefore, the noisiest oscillator requires approximately 10 dB more P_t/N_0 in order to close the link than does the most stable oscillator under consideration. This shows that, by selecting an oscillator with good phase-noise characteristics, we can require less power from the spacecraft in order to close the link at low data rates.

Note that the results presented here are valid only for convolutional codes or concatenated codes with infinite interleaving. For block codes (including turbo codes) and concatenated codes with finite interleaving, this approach should be modified to take into account the fact that the decisions are made over blocks/frames. This means that the decision period is equal to the block/frame time (the duration of time it takes to transmit a single block/frame) and not some number of constraint lengths (in the case of this article, five times the constraint length) of the convolutional code. Therefore, in order to apply the model presented in this article to these types of codes, one has to obtain the frame-error rate function for these codes and replace T, the decision period, with T_f, the frame duration, in Eq. (10),

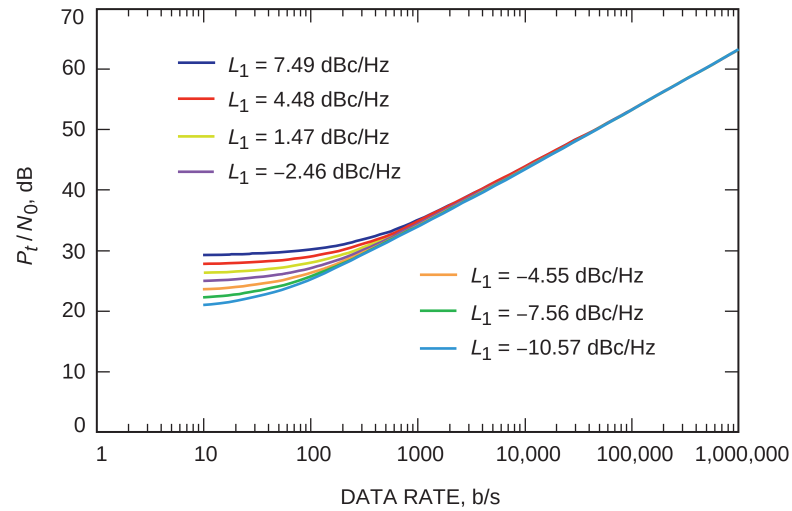

Fig. 5. Minimum P_t / N_0 versus data rate for different phase-noise levels, for (7,1/2) code and BER = 0.001.

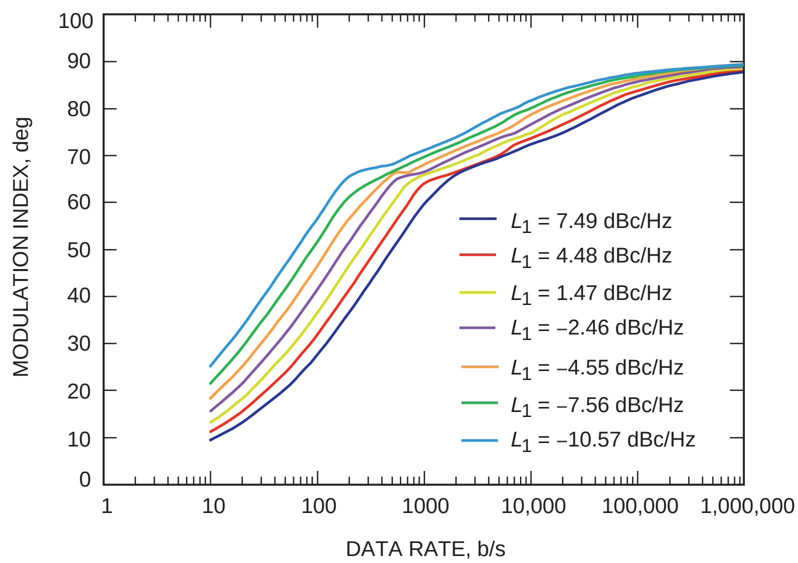

Fig. 6. Optimum modulation index versus data rate for different phase-noise levels, for (7,1/2) code and BER = 0.001.

and perform the analysis for required frame/block error rates. Furthermore, these results are applicable only to standard residual-carrier tracking of binary-phase-shift keyed (BPSK) signals. The results for other tracking methods, such as sideband-aided and data-aided tracking, are different from the results presented in this article, due to different loss characteristics exhibited by such schemes.

V. Conclusions

In this article, we have developed the methodology for evaluating the effect of spacecraft oscillator phase noise on telemetry performance of a convolutionally coded, residual-carrier-modulated, deep-space link. We have shown that, when the oscillator phase noise is significant, it is necessary to optimize the loop bandwidth of the ground receiver in order to minimize the required transmission power from

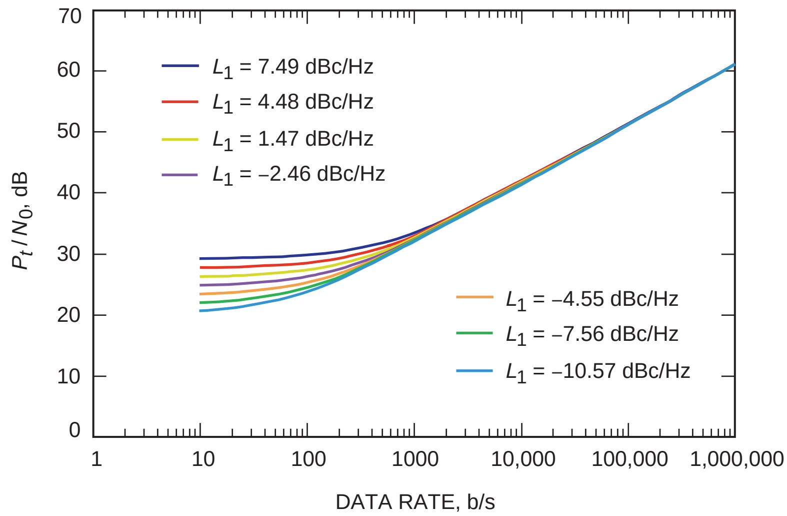

Fig. 7. Minimum P_t / N_0 versus data rate for different phase-noise levels, for (15,1/6) code and BER = 0.001.

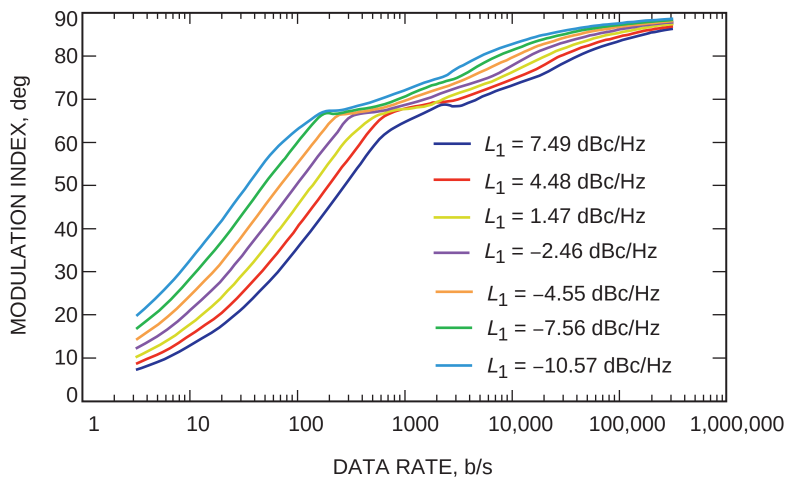

Fig. 8. Optimum modulation index versus data rate for different phase-noise levels, for (15,1/6) code and BER = 0.001.

the spacecraft. Furthermore, we have shown that, for the standard NASA convolutional codes—that is, (7,1/2) and (15,1/6)—selecting an oscillator with good phase-noise characteristics could reduce spacecraft transmission-power requirements significantly (by as much as 10 dB). This is due to the fact that, at low data rates, the minimum power requirement for the receiver to maintain lock is higher for oscillators with higher noise levels.

References

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[2] J. G. Proakis, *Digital Communications*, 2nd ed., New York: McGraw-Hill, 1989.

[3] W. C. Lindsey and M. K. Simon, *Telecommunication Systems Engineering*, Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1973.

[4] P. W. Kinman, *810-5: Deep Space Network/Flight Project Interface Handbook*, "TLM-21 DSN Telemetry System, Block-V Receiver," 810-5, Jet Propulsion Laboratory, Pasadena, California, 1996.