Noise Temperature of a Lossy Flat-Plate Reflector for the Elliptically Polarized Wave Case

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This article presents the derivation of equations necessary to calculate noise temperature of a lossy flat-plate reflector. Reflector losses can be due to metallic surface resistivity and multilayer dielectric sheets, including thin layers of plating, paint, and primer on the reflector surface. The incident wave is elliptically polarized, which is general enough to include linear and circular polarizations as well. The derivations show that the noise temperature for the circularly polarized incident wave case is simply the average of those for perpendicular and parallel polarizations.

I. Introduction

Although equations for power in an incident and reflected elliptically polarized wave can be derived in a straightforward manner, the equations for the associated noise temperatures are not well known nor, to the authors' knowledge, can they be found in published literature. It is especially of interest to know what the relations are when expressed in terms of perpendicular and parallel polarizations and the corresponding reflection coefficients. The following presents the derivations of noise-temperature equations for three cases of interest.

II. Theory

A. Power Relationships

For the coordinate system geometry shown in Fig. 1, the fields for an incident elliptically polarized plane wave at the reflection point are [1,2]

$$\overline{E}_i = E_{xi}\hat{\mathbf{a}}_{xi} + E_{yi}\hat{\mathbf{a}}_{yi} \tag{1}$$

$$\overline{H}_i = H_{xi}\hat{\mathbf{a}}_{xi} + H_{yi}\hat{\mathbf{a}}_{yi} \tag{2}$$

where

$$E_{xi} = E_1 e^{j(\omega t - kz_i)} \tag{3}$$

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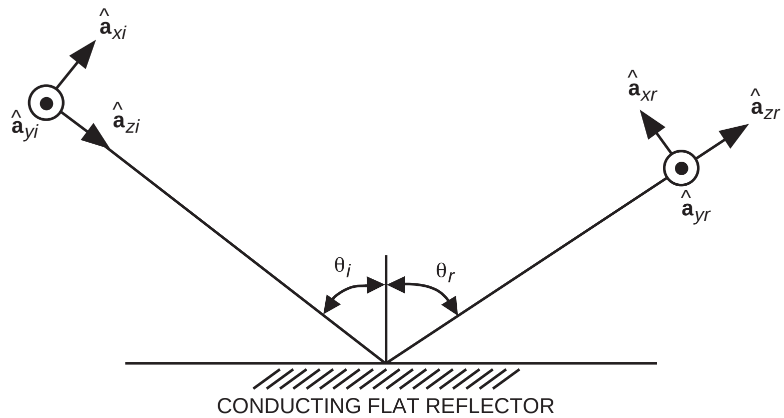

Fig. 1. The coordinate system for incident and reflected plane waves. The symbols with a boldface $\hat{\mathbf{a}}$ are unit vectors, and θ_i and θ_r are angles of incidence and reflection, respectively. The plane of incidence is the plane of this page.

$$E_{yi} = E_2 e^{j(\omega t - kz_i + \delta)} \tag{4}$$

$$H_{xi} = -\frac{E_{yi}}{\eta} \tag{5}$$

$$H_{yi} = \frac{E_{xi}}{\eta} \tag{6}$$

where ω is the angular frequency, t is time, η is the characteristic impedance of free space, k is the free-space wave number, and z_i is the distance from an arbitrarily chosen source point on the incident wave ray path to the reflection point on the reflector surface (Fig. 1). In Eqs. (3) and (4), it is important to note that E_1 and E_2 are scalar magnitudes and δ is the phase difference between E_{xi} and E_{yi}.

The Poynting vector [1] for the incident wave is expressed as

$$\overline{P}_i = \frac{1}{2}\,\mathrm{Re}\left(\overline{E}_i \times \overline{H}_i^*\right) \tag{7}$$

where $\times$, $*$, and Re denote the cross product, complex conjugate, and real part, respectively. Then assuming all of the incident power travels through an area A in the direction of the Poynting vector, the total incident wave power is

$$P_{Ti} = \int \left(\overline{P}_i \cdot \hat{\mathbf{a}}_{zi}\right) dA \tag{8}$$

where $\cdot$ denotes the dot product. Substitution of Eqs. (1) through (7) into Eq. (8) results in

$$P_{Ti} = \frac{1}{2\eta}\left(E_1^2 + E_2^2\right) A \tag{9}$$

The equations for the reflected wave are obtained by replacing the subscript i with r in all of the equations for the incident wave except for Eqs. (3) and (4). From Fig. 1, it can be seen that the expressions for E_{xr} and E_{yr} are

$$E_{xr} = \Gamma_{||} E_{xi} e^{-jkz_r} \tag{10}$$

$$E_{yr} = \Gamma_{\perp} E_{yi} e^{-jkz_r} \tag{11}$$

where

$\Gamma_{||} =$ the voltage reflection coefficient for parallel polarization at the reflection point and is a function of incidence angle θ_i (see Fig. 1)

$\Gamma_{\perp} =$ the voltage reflection coefficient for perpendicular polarization at the reflection point and is a function of incidence angle θ_i

$z_r =$ the distance from the reflection point on the reflector surface to an arbitrary observation point along the reflected ray path

Then following steps similar to those used to obtain Eq. (9), the total power for the reflected wave can be derived as

$$P_{Tr} = \frac{1}{2\eta} \left[\left|\Gamma_{||}\right|^2 E_1^2 + \left|\Gamma_{\perp}\right|^2 E_2^2 \right] A \tag{12}$$

It is assumed that the lossy conductor in Fig. 1 has sufficient thickness so that no power is transmitted out the bottom side. Then the dissipated power is

$$P_d = P_{Ti} - P_{Tr} \tag{13}$$

B. Noise-Temperature Relationships

For the geometry of Fig. 1, the noise temperature due to a lossy reflector is

$$T_n = \left(\frac{P_d}{P_{Ti}} \right) T_p \tag{14}$$

where T_p is the physical temperature of the reflector in units of K. For example, if the lossy conductor is at a physical temperature of 20 deg C, then $T_p = 293.16$ K. Use of Eqs. (9), (12), and (13) in Eq. (14) gives

$$T_n = \left(1 - \left|\Gamma_{ep}\right|^2 \right) T_p \tag{15}$$

where

$$\left|\Gamma_{ep}\right|^2 = \frac{\left|\Gamma_{||}\right|^2 E_1^2 + \left|\Gamma_{\perp}\right|^2 E_2^2}{E_1^2 + E_2^2} \tag{16}$$

Equation (15) is the elliptically polarized wave noise-temperature equation that is general enough to apply to linear and circular polarizations as well. In the following, the noise-temperature expressions for three different polarization cases are derived.

Case 1. If the incident wave is linearly polarized with the E-field perpendicular to the plane of incidence, then $E_1 = 0$ and Eq. (15) becomes

$$T_n = (T_n)_\perp = \left(1 - |\Gamma_\perp|^2\right) T_p \tag{17}$$

Case 2. If the incident wave is linearly polarized with the E-field parallel to the plane of incidence, then $E_2 = 0$ and Eq. (15) becomes

$$T_n = (T_n)_{||} = \left(1 - \left|\Gamma_{||}\right|^2\right) T_p \tag{18}$$

Case 3. If the incident wave is circularly polarized, then $E_1 = E_2$ and

$$T_n = (T_n)_{cp} = \left[1 - \frac{\left(\left|\Gamma_{||}\right|^2 + |\Gamma_\perp|^2\right)}{2}\right] T_p \tag{19}$$

Note then that $(T_n)_{cp}$ is also just the average of $(T_n)_\perp$ and $(T_n)_{||}$ or

$$(T_n)_{cp} = \frac{1}{2}\left[(T_n)_\perp + (T_n)_{||}\right] \tag{20}$$

The reader is reminded that, since the reflection coefficients are functions of incidence angle θ_i, the noise temperatures are also functions of θ_i as well as of polarization.

C. Excess Noise-Temperature Relationships

It is of interest to see what the relationship is for excess noise temperature as well. For painted reflector noise-temperature analyses [3], it is convenient to use the term excess noise temperature (ENT). It is defined in [3] as the total noise temperature of a painted reflector minus the noise temperature of the reflector (bare metal) without paint. Mathematically, it is expressed as

$$\Delta T_n = T_{n2} - T_{n1} = \left(1 - |\Gamma_2|^2\right) T_p - \left(1 - |\Gamma_1|^2\right) T_p \tag{21}$$

where Γ_1 and Γ_2 are the input voltage reflection as seen looking at the unpainted (bare conductor) and painted reflector surfaces, respectively, and are functions of incidence angle and polarization. These reflection coefficients can be obtained through the use of multilayer equations such as those given in [4].

Then from Eqs. (17) through (21) it follows that, for the perpendicular-, parallel-, and circular-polarization cases,

$$\begin{aligned} (\Delta T_n)_\perp &= (T_{n2})_\perp - (T_{n1})_\perp \\ &= \left(1 - |\Gamma_2|_\perp^2\right) T_p - \left(1 - |\Gamma_1|_\perp^2\right) T_p \end{aligned} \tag{22}$$

$$
\begin{aligned}
(\Delta T_n)_{||} &= (T_{n2})_{||} - (T_{n1})_{||} \\
&= \left(1 - |\Gamma_2|^2_{||}\right) T_p - \left(1 - |\Gamma_1|^2_{||}\right) T_p
\end{aligned} \tag{23}
$$

$$
(\Delta T_n)_{cp} = (T_{n2})_{cp} - (T_{n1})_{cp} \tag{24}
$$

Substitution of Eq. (20) into Eq. (24) gives

$$
\begin{aligned}
(\Delta T_n)_{cp} &= \frac{1}{2}\left[(T_{n2})_{\perp} + (T_{n2})_{||}\right] - \frac{1}{2}\left[(T_{n1})_{\perp} + (T_{n1})_{||}\right] \\
&= \frac{1}{2}\left\{\left[(T_{n2})_{\perp} - (T_{n1})_{\perp}\right] + \left[(T_{n2})_{||} - (T_{n1})_{||}\right]\right\}
\end{aligned} \tag{25}
$$

Substitution of Eqs. (22) and (23) into Eq. (25) gives

$$
(\Delta T_n)_{cp} = \frac{1}{2}\left[(\Delta T_n)_{\perp} + (\Delta T_n)_{||}\right] \tag{26}
$$

Equation (25) shows that the ENT for the circular-polarization case is simply the average of the ENTs of perpendicular and parallel polarizations. Although not shown mathematically, the ENTs are functions of incidence angle θ_i.

III. Concluding Remarks

In the article, noise-temperature equations were derived from power equations for the incident and reflected wave. The relationships between noise temperatures of the different polarized wave cases were not obvious to the authors until the equations were derived from basic theoretical considerations. Hence, this article serves to document the relationships and derivations. These noise-temperature formulas have proven to be useful for painted reflector studies [3] and will be useful for studies of plating [5] or rain [4,6] on reflector surfaces as well.

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