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Mutual Interference of Ranging and Telemetry

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Telemetry and ranging sidebands may interfere with each other. Theoretical models are proposed for assessing this danger. An interfering sideband may be specular or diffuse. If specular, it looks like a continuous-wave interferer, and a signal-to-noise ratio degradation is calculated as a function of the proximity of the interferer to the range tone frequency (for ranging), or subcarrier (for telemetry with a subcarrier), or carrier (for telemetry without a subcarrier). If diffuse, the interferer increases the effective noise floor. The theoretical model for telemetry degradation due to a specular interferer is verified by experiment.

I. Introduction

The telemetry and ranging signals from either the same spacecraft or different spacecraft may interfere with each other if their spectral components overlap in frequency. For example, when the telemetry modulation sidebands overlay a range tone, they will increase the effective noise floor of the range measurement. As a second example, the intermodulation products may fall near the telemetry subcarrier frequency with a detrimental effect on telemetry. This article outlines the theory needed to check for possible interference problems of this kind. The emphasis will be on telemetry, ranging, and intermodulation product sidebands from the same carrier, but much of the theory will also be applicable to the case of more than one spacecraft.

First, we define telemetry signal-to-noise ratio (SNR) degradation due to the presence of a continuous-wave (CW) interferer. Then we define ranging SNR degradation due to the presence of a CW interferer. These definitions can be used to estimate the effect of a specular interferer. Diffuse interferers are also of concern: they increase the effective noise floor. A few general comments are made about the potential for mutual interference of ranging and telemetry. Then we take a detailed look at one example signal, which is representative of a modulated downlink carrier from a satellite as tracked by a 26-m station.

II. Telemetry SNR Degradation Due to a CW Interferer

In this section, we calculate an SNR degradation that can be used as an approximate measure of the effect of a CW interferer on telemetry demodulation. First, we derive the SNR degradation for the case of a square-wave subcarrier and non-return to zero (NRZ) data. Later, we state the results for a sine-wave subcarrier with NRZ data, for NRZ data directly modulated onto the carrier, and for bi-ϕ

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(Manchester-coded) data directly modulated onto the carrier. Derivations for these latter results are not presented here because they are very similar to that for a square-wave subcarrier with NRZ data.

A. Square-Wave Subcarrier With NRZ Data

We consider a signal of the form

$$\sqrt{2P_D}d(t)\mathrm{S}(t)\cos(\omega_C t) + n(t) + \sqrt{2P_I}\sin\left[(\omega_C+\omega_I)t+\phi\right]$$

In general, there also would be a residual-carrier component to the signal; this component is not of interest here and so is not modeled. The signal has carrier angular frequency ω_C. The data, $d(t) = \pm 1$, are phase modulated onto a square-wave subcarrier, $\mathrm{S}(t) = \pm 1$, which, in turn, is phase modulated onto the carrier; the data sideband power is P_D. White Gaussian noise, $n(t)$, is present and has a one-sided power spectral density, N_0. The CW interferer has power P_I and an angular frequency differing from that of the carrier by ω_I. The excess phase, ϕ, in the argument of the CW interferer serves as a reminder that there is no relationship between the phases of this interferer and the signal.

A coherent demodulator is shown in Fig. 1. The local oscillator, $2\cos(\omega_C t)$, has the proper frequency and phase for phase-coherent demodulation. At the output of the carrier demodulator and the input to the subcarrier demodulator, there is signal plus noise plus interferer of the form

$$\sqrt{2P_D}d(t)\mathrm{S}(t) + n'(t) + \sqrt{2P_I}\sin(\omega_I t+\phi)$$

The baseband noise, $n'(t)$, has one-sided noise spectral density (within its passband) equal to $2N_0$; that is to say, the baseband noise, $n'(t)$, has a power spectral density (within its passband) that is twice that of $n(t)$, owing to the amplitude of 2 used in modeling the local oscillator. After multiplication by the local square wave, which is assumed to equal $\mathrm{S}(t)$, the input to the averaging filter is

$$\sqrt{2P_D}d(t) + n''(t) + \sqrt{2P_I}\mathrm{S}(t)\sin(\omega_I t+\phi)$$

The new baseband noise, $n''(t)$, which equals $n'(t)\times\mathrm{S}(t)$, is approximately white at low Fourier frequencies and has a one-sided noise spectral density there of approximately $2N_0$. This approximation is a good one whenever the bandwidth of the filter preceding the multiplication by $\mathrm{S}(t)$ is large compared with the subcarrier frequency, as it typically is for deep-space communications. The square wave $\mathrm{S}(t)$ can be written as a Fourier series:

$$\mathrm{S}(t) = \sum_{\substack{k=1\\ \mathrm{odd}}}^{\infty} \frac{4}{k\pi}\sin(k\omega_T t) \tag{1}$$

where ω_T is the telemetry subcarrier angular frequency.

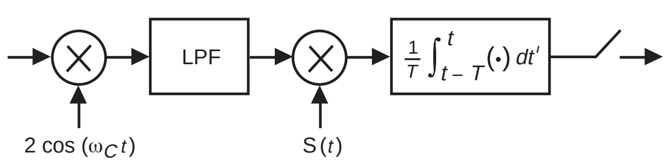

Fig. 1. A coherent demodulator for phase-shift-keyed telemetry with a square-wave subcarrier.

The interferer, $\sqrt{2P_I}\mathrm{S}(t)\sin(\omega_I t + \phi)$, at the input to the averaging filter has the one-sided power spectral density

$$S_I(f) = \sum_{\substack{m=-\infty \\ \text{odd}}}^{\infty} \frac{4P_I}{m^2\pi^2}\delta\left(f - \frac{1}{2\pi}|m\omega_T - \omega_I|\right) \tag{2}$$

where $\delta(\cdot)$ is the Dirac delta function. The impulse response, $h(t)$, of the averaging filter is shown in Fig. 2. The integration time, T, equals the symbol period of the data stream, $d(t)$. The corresponding transfer function is obtained by a Fourier transform:

$$H(j2\pi f) = \mathcal{F}\{h(t)\} \tag{3}$$

The squared magnitude of this transfer function evaluates to

$$|H(j2\pi f)|^2 = \mathrm{sinc}^2(fT) \tag{4}$$

where

$$\mathrm{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \tag{5}$$

The one-sided power spectral density of the interferer coming out of the averaging filter is

$$S_I(f) \times |H(j2\pi f)|^2$$

The one-side power spectral density of the noise coming out of the averaging filter is

$$2N_0 \times |H(j2\pi f)|^2$$

Finally, the output of the averaging filter is sampled at the symbol rate such that samples are taken at the symbol boundaries. This sampling, taken together with the averaging filter, constitutes a matched filter for the NRZ symbols of $d(t)$. The signal power in these demodulated samples is $2P_D$. The noise power is

$$\sigma_N^2 = \int_0^\infty 2N_0 \times |H(j2\pi f)|^2 \, df = \frac{N_0}{T} \tag{6}$$

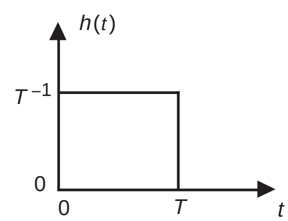

Fig. 2. Impulse response of the averaging filter.

The interferer power is

$$\sigma_I^2 = \int_0^\infty S_I(f) \times |H(j2\pi f)|^2 \, df = \sum_{\substack{m=-\infty \\ \text{odd}}}^{\infty} \frac{4P_I}{m^2\pi^2} \times \text{sinc}^2 \left[(m\omega_T - \omega_I) \frac{T}{2\pi} \right] \tag{7}$$

Equation (7) appears formidable; but if $\omega_T \gg 2\pi/T$, as it normally will, then the sum in that equation can be approximated by just its dominant term. To be precise, the dominant term equals

$$\max_{m \sim \text{odd}} \frac{4P_I}{m^2\pi^2} \text{sinc}^2 \left[(m\omega_T - \omega_I) \frac{T}{2\pi} \right]$$

The odd integer M is here used to denote the m corresponding to this maximum value; that is to say, M is the harmonic number of ω_T corresponding to the dominant term in the sum of Eq. (7). Therefore, the interferer power is normally well approximated by

$$\sigma_I^2 = \frac{4P_I}{M^2\pi^2} \times \text{sinc}^2 \left[(M\omega_T - \omega_I) \frac{T}{2\pi} \right] \tag{8}$$

The telemetry SNR degradation, η_T, due to the presence of the CW interferer is defined as

$$\eta_T = \frac{\sigma_N^2 + \sigma_I^2}{\sigma_N^2} = 1 + \frac{\sigma_I^2}{\sigma_N^2} \tag{9}$$

With this definition, $\eta_T \geq 1$. Combining Eqs. (6), (8), and (9) yields

$$\begin{aligned} \eta_T &= 1 + \frac{P_I T}{N_0} \times \frac{4}{M^2\pi^2} \times \text{sinc}^2 \left[(M\omega_T - \omega_I) \frac{T}{2\pi} \right] \\ &= 1 + \frac{P_I}{P_D} \times \frac{E_s}{N_0} \times \frac{4}{M^2\pi^2} \times \text{sinc}^2 \left[(M\omega_T - \omega_I) \frac{T}{2\pi} \right] \end{aligned} \tag{10}$$

where

$$\frac{E_s}{N_0} = \frac{P_D T}{N_0} \tag{11}$$

is the symbol-energy-to-noise spectral-density ratio.

An example of SNR degradation as calculated by Eq. (10) is shown in Fig. 3. In that figure, the CW interferer is closest to the upper fundamental harmonic of the subcarrier, so $M = 1$. It is assumed that E_s/N_0 is 0 dB. The abscissa is $|\omega_T - \omega_I| \times T/(2\pi)$. The ordinate is the SNR degradation in decibels. The three different curves correspond to the three values of P_I/P_D: +10, +3, and −3 dB.

For each curve in Fig. 3, E_s/N_0 and P_I/P_D are constant, so the variation in SNR degradation is due only to the passage of the interferer through the matched filter. Generally, we say that the attenuation of the interferer gets more severe and, therefore, the SNR degradation improves as the interferer becomes

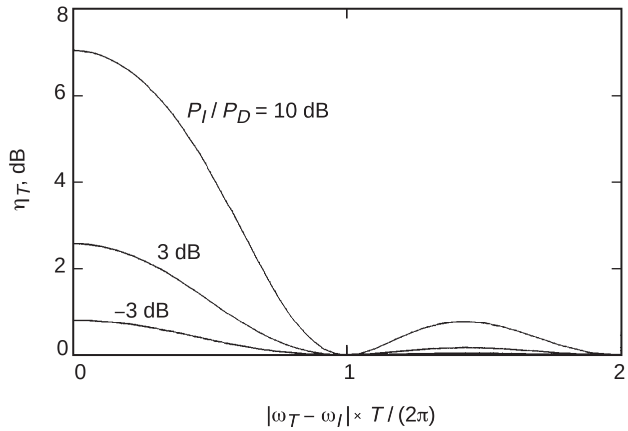

Fig. 3. η_T **versus** $|\omega_T - \omega_I| \times T/(2\pi)$**;** $M = 1$**,** $E_S/N_0 = 0$ **dB.**

further removed (in the frequency domain) from the fundamental of the telemetry subcarrier. However, this is not a monotonic function. There is, for example, a local minimum wherever the difference between the interferer and subcarrier frequencies is an integer multiple of the symbol rate. These minima arise because the integration of a sine wave over an integer number of cycles is exactly zero; so the attenuation of the interference by the matched filter is complete at those frequencies. What is not shown in Fig. 3, but which must be borne in mind, is that the SNR degradation will experience a local maximum wherever the interferer frequency equals an odd harmonic of the subcarrier frequency.

The SNR degradation as defined here is a useful, easy-to-calculate measure of the effect of a CW interferer on telemetry demodulation. However, it must be added that there are two important reasons why this SNR degradation is an incomplete and approximate accounting of the danger arising from a nearby CW interferer. First, SNR degradation is not the same as loss. A true loss calculation must account for the nonlinear relationship between probability of bit error and SNR. Nonetheless, since interference is additive (unlike radio loss), the approximation of loss by SNR degradation is believed to be a good one. In Subsection II.C, experimental evidence is given in support of this approximation. Second, if the CW interferer lies within about one carrier-loop bandwidth of $\omega_C/(2\pi)$ or within about one subcarrier-loop bandwidth of an odd harmonic of $\omega_T/(2\pi)$, the operation of the synchronization loops may be affected. Fortunately, these loop bandwidths are normally quite small, so this would be an uncommon occurrence.

B. Sine-Wave Subcarrier With NRZ Data

We now consider the case when NRZ data phase-shift key a sine-wave subcarrier and this subcarrier, in turn, phase modulates the carrier. This produces data-bearing sidebands displaced from the residual carrier by odd-integer multiples of the subcarrier frequency and specular sidebands displaced from the residual carrier by even-integer multiples of the subcarrier frequency. At the receiver, only the fundamental harmonics are used for telemetry detection. The fundamental harmonics are detected as shown in Fig. 4. With a sine-wave subcarrier, it would greatly complicate the receiver signal processing to also

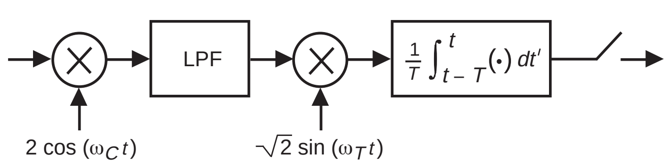

Fig. 4. A coherent demodulator for phase-shift-keyed telemetry with a sine-wave subcarrier.

harvest the higher-order odd harmonics. Since these higher-order odd harmonics are quite weak relative to the fundamental harmonics, there is no incentive to bother with them.

We consider a signal of the form

$$2\sqrt{P_D}d(t)\sin(\omega_T t)\cos(\omega_C t) + n(t) + \sqrt{2P_I}\sin\left[(\omega_C + \omega_I)t + \phi\right]$$

As mentioned above, there are also higher-order harmonics as well as a residual carrier present, but as these play no role in the analysis to follow, they are not included in our model. The P_D is the data power associated with just the fundamental sidebands of the subcarrier. The carrier angular frequency is ω_C, and the subcarrier angular frequency is ω_T. The data $d(t) = \pm 1$. White Gaussian noise, $n(t)$, is present and has a one-sided power spectral density, N_0. The CW interferer has power P_I and an angular frequency differing from that of the carrier by ω_I.

The output of the carrier demodulator is of the form

$$2\sqrt{P_D}d(t)\sin(\omega_T t) + n'(t) + \sqrt{2P_I}\sin(\omega_I t + \phi)$$

The baseband noise, $n'(t)$, has one-sided noise spectral density (within its passband) equal to $2N_0$. After multiplication by the local sine wave, the input to the averaging filter is

$$\sqrt{2P_D}d(t) + n''(t) + \sqrt{P_I}\cos\left[(\omega_T - \omega_I)t - \phi\right]$$

The new baseband noise, $n''(t)$, has a one-sided noise spectral density of $2N_0$. The interferer at the input to the averaging filter has the one-sided power spectral density

$$S_I(f) = \frac{P_I}{2}\delta\left(f - \frac{1}{2\pi}|\omega_T - \omega_I|\right) \tag{12}$$

Finally, at the sampled output of the averaging filter, the noise power is, as before, $\sigma_N^2 = N_0/T$. The interferer power is

$$\sigma_I^2 = \frac{P_I}{2}\mathrm{sinc}^2\left[(\omega_T - \omega_I)\frac{T}{2\pi}\right] \tag{13}$$

The SNR degradation for NRZ data with a sine-wave subcarrier is, therefore,

$$\begin{aligned}
\eta_T &= 1 + \frac{\sigma_I^2}{\sigma_N^2} \\
&= 1 + \frac{P_I T}{2N_0} \times \mathrm{sinc}^2\left[(\omega_T - \omega_I)\frac{T}{2\pi}\right] \\
&= 1 + \frac{P_I}{P_D} \times \frac{E_s}{N_0} \times \frac{1}{2} \times \mathrm{sinc}^2\left[(\omega_T - \omega_I)\frac{T}{2\pi}\right] \qquad (14)
\end{aligned}$$

C. NRZ Data Directly Modulated Onto the Carrier

In the case when NRZ data directly modulate the carrier, the SNR degradation is given by

$$\eta_T = 1 + \frac{P_I}{P_D} \times \frac{E_s}{N_0} \times \operatorname{sinc}^2\left(\frac{\omega_I}{2\pi}T\right) \tag{15}$$

where P_D is the total data sideband power. The derivation of Eq. (15) is similar to those of the preceding subsections and will not be given here.

A series of experiments was undertaken to measure the loss due to the presence of a CW interferer. A Block-V Receiver with carrier synchronization by Costas loop was used. The input to the receiver consisted of a suppressed-carrier signal with NRZ data directly modulated onto the carrier, a CW interferer, and additive white noise. The data were uncoded, the bit rate was 10^5 b/s, and $E_s/N_0 = E_b/N_0 = 6.8$ dB. The results are given in Table 1. Each row in this table corresponds to a different frequency offset, $\omega_I/(2\pi)$, of the CW interferer from the carrier frequency. The ratio of interferer power to signal power, P_I/P_D, was chosen to make the SNR degradation roughly the same for all frequency offsets. The third column of Table 1 shows the SNR degradation, in decibels, as predicted by Eq. (15). The fourth column of Table 1 shows the measured loss. The loss was obtained by measuring the bit-error rate (BER), associating it with an effective E_b/N_0 through the relationship

$$\text{BER} = \frac{1}{2}\operatorname{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right) \tag{16}$$

and then noting the difference (in decibels) between the available E_b/N_0 (6.8 dB) and this effective E_b/N_0. In Eq. (16), erfc($\cdot$) is the complementary error function. In comparing columns three and four of Table 1, it becomes clear that the SNR degradation, η_T, as calculated from Eq. (15) is a good approximation to the true loss. It must be mentioned that two measurements, not reported in Table 1, were made for which the loss was much worse than the computed η_T. These contrary data points may have been the result of something else happening within the receiver, such as a half-cycle slip in the Costas loop, which would cause an unduly large number of bit errors. We plan to investigate this further.

Table 1. Measured loss compared with predicted SNR degradation; $T = 10^{-5}$ s, E_s/N_0 = 6.8 dB.

$\omega_I/(2\pi)$, kHz	P_I/P_D, dB	Predicted SNR degradation, dB	Measured loss, dB
5	−10	1.69	1.48
10	−10	1.65	1.38
20	−9	1.84	1.63
50	−6	1.72	1.63
70	−1	1.80	1.73
80	+3	1.83	1.68
250	+8	1.73	1.73
350	+11	1.76	1.48
450	+13	1.70	1.68
550	+15	1.78	1.83

D. Bi-ϕ Data Directly Modulated Onto the Carrier

In the case when bi-ϕ (Manchester-coded) data directly modulate the carrier, the SNR degradation is given by

$$\eta_T = 1 + \frac{P_I}{P_D} \times \frac{E_s}{N_0} \times \frac{\sin^4\left(\dfrac{\omega_I T}{4}\right)}{\left(\dfrac{\omega_I T}{4}\right)^2} \tag{17}$$

where P_D is the total data sideband power.

III. Ranging SNR Degradation Due to a CW Interferer

For ranging also, we want to be able to calculate an SNR degradation due to a CW interferer. A simple calculation of this type is only an approximate measure of the effect of a CW interferer on ranging, but at least it is convenient. Justification of the equations given here is similar to that for the SNR degradation of telemetry by a CW interferer that is discussed in detail in Subsection II.A.

For square-wave ranging, SNR degradation η_R can be calculated from

$$\eta_R = 1 + \frac{P_I}{P_R} \times \frac{P_R T_R}{N_0} \times \sum_{\substack{m=-\infty \\ \text{odd}}}^{\infty} \frac{4}{m^2\pi^2} \times \text{sinc}^2\left[(m\omega_R - \omega_I)\frac{T_R}{2\pi}\right] \tag{18}$$

where ω_R is the ranging component angular frequency and T_R is the component integration time. The sum in Eq. (18) usually will be well approximated by its dominant term:

$$\max_{m\sim\text{odd}} \frac{4}{m^2\pi^2}\text{sinc}^2\left[(m\omega_R - \omega_I)\frac{T_R}{2\pi}\right]$$

Here the odd integer M will denote the m corresponding to the above maximum. The SNR degradation of Eq. (18) then becomes approximately

$$\eta_R = 1 + \frac{P_I}{P_R} \times \frac{P_R T_R}{N_0} \times \frac{4}{M^2\pi^2} \times \text{sinc}^2\left[(M\omega_R - \omega_I)\frac{T_R}{2\pi}\right] \tag{19}$$

If T_R is large, as it typically is for deep-space ranging, then a CW interferer would have to be quite close to an odd harmonic of a ranging component before it would pose a threat.

For sine-wave ranging, SNR degradation η_R can be calculated from

$$\eta_R = 1 + \frac{P_I}{P_R} \times \frac{P_R T_R}{N_0} \times \frac{1}{2} \times \text{sinc}^2\left[(\omega_R - \omega_I)\frac{T_R}{2\pi}\right] \tag{20}$$

IV. Mutual Interference of Telemetry and Ranging

We make a few general comments about the potential for mutual interference of telemetry and ranging for two different application areas: deep space and satellites. Following this section, the remainder of this article discusses a satellite example in some detail.

In deep-space communications, the telemetry subcarrier ordinarily is a square wave. The ranging signal has, in the recent past, consisted of sequential square waves. (That is beginning to change; pseudo-random range codes are now being introduced.) The highest-frequency (square-wave) ranging component usually is filtered to a sine wave by the bandpass filtering of the spacecraft transponder channel. Furthermore, some lower-frequency components are chopped by a higher-frequency component. Therefore, a complete analysis of mutual interference between ranging and telemetry would require us to consider all components of the sequential square-wave ranging scheme, some of which are filtered and some of which are chopped. Fortunately, a complete analysis is often unnecessary. It is often evident that the ranging sidebands can have no effect on telemetry (other than consuming a fraction of the available link power). The noise sidebands contributed to the downlink by the ranging channel are typically too diffuse to significantly affect the noise floor for telemetry. Furthermore, the signal-to-noise ratio in the ranging channel of a deep-space transponder typically is quite small, so that the ranging signal sidebands are small and pose no threat to telemetry. One might think that telemetry sidebands greatly interfere with the range measurement. This can happen. Specular components of telemetry and the intermodulation products must lie very close to an odd harmonic of a ranging component to have an effect on the range measurement, due to the long integration times for deep-space ranging. Since there is no coherent relationship between the telemetry subcarrier and the ranging signal and since the ranging component frequencies normally are well separated from the telemetry subcarrier, specular interference to the range measurement is seldom a problem. Diffuse telemetry sidebands are more likely to have an effect on ranging; but the effect is to increase the noise floor for the range measurement. Adjusting the modulation indices can counter this effect.

With (low-Earth) satellites, it is a different story. The communication range is much smaller, the ranging signal sidebands are bigger, and the range measurement integration times are much shorter. In general, the mutual interference of telemetry and ranging is a bigger issue for satellites. In the following section, we present example calculations for a satellite downlink with telemetry and ranging as it might be tracked by a 26-m station.

V. Example: Sine-Wave Subcarrier and Tone Ranging

The following example situation is modeled. The telemetry subcarrier is a sine wave. Tone ranging is used, and only the major range tone (MRT) is present. The uplink has been filtered so that no significant harmonics of the MRT appear on the uplink. The signal-to-noise ratio in the turnaround ranging channel of the transponder is large, so that only the MRT comes through this channel.

A downlink carrier modulated by a range tone and telemetry with a sine-wave subcarrier is modeled as

$$\sqrt{2P_T}\sin\left[\omega_C t + \alpha d(t)\sin(\omega_T t) + \sqrt{2}\beta\sin(\omega_R t)\right]$$

The total downlink power is P_T. The telemetry modulation index is α radians *peak*. The root-mean-square (rms) downlink carrier-phase deviation caused by the signal plus noise in the turnaround ranging channel is β radians *rms*. The angular frequencies of the carrier, telemetry subcarrier, and MRT are ω_C, ω_T, and ω_R, respectively. The telemetry data are $d(t) = \pm 1$; it is assumed throughout that $+1$ and -1 occur with equal probability and that each data bit is statistically independent of every other. We also assume that additive white Gaussian noise is present with one-sided power spectral density N_0.

For most of this section, we treat ω_T, ω_R, α, β, and T as unspecified parameters in order to keep our equations as generally applicable as possible. But later we do substitute the specific set of values shown in Table 2 for these parameters. In this way, we obtain numeric results for a typical parameter set.

Table 2. Parameters.

Parameter	Value
$\omega_T/(2\pi)$	256 kHz
$\omega_R/(2\pi)$	100 kHz
α	1.1 rad peak
β	0.28 rad rms
$1/T$	60 ksym/s

The expression for the modulated downlink carrier may be expanded with the help of some trigonometric identities and the Jacobi–Anger identities (see the Appendix). The different components then may be identified. The residual carrier is

$$\sqrt{2P_T}\mathrm{J}_0(\alpha)\mathrm{J}_0\big(\sqrt{2}\beta\big)\sin(\omega_C t)$$

The telemetry sidebands are

$$2\sqrt{2P_T}d(t)\mathrm{J}_0\big(\sqrt{2}\beta\big)\left[\sum_{m=1}^{\infty}\mathrm{J}_{2m-1}(\alpha)\sin\{(2m-1)\omega_T t\}\right]\cos(\omega_C t)$$

$$+\,2\sqrt{2P_T}\mathrm{J}_0\big(\sqrt{2}\beta\big)\left[\sum_{m=1}^{\infty}\mathrm{J}_{2m}(\alpha)\cos(2m\omega_T t)\right]\sin(\omega_C t)$$

It will be noticed that the odd harmonics of the subcarrier are diffuse [i.e., modulated by $d(t)$] and that the even harmonics of the subcarrier are specular. The ranging sidebands are

$$2\sqrt{2P_T}\mathrm{J}_0(\alpha)\left[\sum_{n=1}^{\infty}\mathrm{J}_{2n-1}\big(\sqrt{2}\beta\big)\sin\{(2n-1)\omega_R t\}\right]\cos(\omega_C t)$$

$$+\,2\sqrt{2P_T}\mathrm{J}_0(\alpha)\left[\sum_{n=1}^{\infty}\mathrm{J}_{2n}\big(\sqrt{2}\beta\big)\cos(2n\omega_R t)\right]\sin(\omega_C t)$$

There are specular intermodulation products:

$$4\sqrt{2P_T}\left[\sum_{m=1}^{\infty}\mathrm{J}_{2m}(\alpha)\cos(2m\omega_T t)\right]\left[\sum_{n=1}^{\infty}\mathrm{J}_{2n-1}\big(\sqrt{2}\beta\big)\sin\{(2n-1)\omega_R t\}\right]\cos(\omega_C t)$$

$$+\,4\sqrt{2P_T}\left[\sum_{m=1}^{\infty}\mathrm{J}_{2m}(\alpha)\cos(2m\omega_T t)\right]\left[\sum_{n=1}^{\infty}\mathrm{J}_{2n}\big(\sqrt{2}\beta\big)\cos(2n\omega_R t)\right]\sin(\omega_C t)$$

There are diffuse intermodulation products:

$$4\sqrt{2P_T}d(t)\left[\sum_{m=1}^{\infty}\mathrm{J}_{2m-1}(\alpha)\sin\{(2m-1)\omega_T t\}\right]\left[\sum_{n=1}^{\infty}\mathrm{J}_{2n}\big(\sqrt{2}\beta\big)\cos(2n\omega_R t)\right]\cos(\omega_C t)$$

$$-4\sqrt{2P_T}d(t)\left[\sum_{m=1}^{\infty}\mathrm{J}_{2m-1}(\alpha)\sin\{(2m-1)\omega_T t\}\right]\left[\sum_{n=1}^{\infty}\mathrm{J}_{2n-1}\big(\sqrt{2}\beta\big)\sin\{(2n-1)\omega_R t\}\right]\sin(\omega_C t)$$

Both the fundamental telemetry sidebands and the fundamental MRT sidebands contain the term $\cos(\omega_C t)$, and so both telemetry and ranging will be insensitive to all terms involving $\sin(\omega_C t)$ (i.e., the terms in carrier-phase quadrature).

A. Interference to the Fundamental Range Tone

The specular interferers to the fundamental harmonic of the MRT are all of the following description: intermodulation products involving even harmonics of the subcarrier and odd harmonics of the MRT. The baseband angular frequencies ω_I of the interferers are

$$\omega_I = |2m\omega_T - (2n-1)\omega_R|\,, \quad m = 1,2,3,\cdots, \quad n = 1,2,3,\cdots \tag{21}$$

(At *radio* frequency, the angular frequencies of the interferers are $\omega_C + \omega_I$.) The ratio of the power P_I in an interferer of this type to the power P_R of the fundamental harmonic of the MRT is

$$\frac{P_I}{P_R} = \frac{\mathrm{J}_{2m}^2(\alpha)\mathrm{J}_{2n-1}^2(\sqrt{2}\beta)}{\mathrm{J}_0^2(\alpha)\mathrm{J}_1^2(\sqrt{2}\beta)} \tag{22}$$

The ratio P_I/P_R as given by Eq. (22) may be used in Eq. (20) to determine η_R.

Odd harmonics of the subcarrier are potential diffuse interferers to the MRT. The baseband angular frequencies are

$$\omega_I = (2m-1)\omega_T, \quad m = 1,2,3,\cdots \tag{23}$$

The one-sided baseband power spectral density placed at the MRT frequency by the $2m-1$ harmonic of the subcarrier is

$$S_{2m-1,0} = 4P_T T\mathrm{J}_{2m-1}^2(\alpha)\mathrm{J}_0^2\big(\sqrt{2}\beta\big)\mathrm{sinc}^2\left[\frac{(\omega_I-\omega_R)T}{2\pi}\right] \tag{24}$$

where T is the symbol period.

The diffuse intermodulation products that pose a threat to the MRT (and to telemetry) have the baseband angular frequencies

$$\omega_I = |(2m-1)\omega_T - 2n\omega_R|\,, \quad m = 1,2,3,\cdots, \quad n = 1,2,3,\cdots \tag{25}$$

The one-sided baseband power spectral density placed at the MRT frequency by the diffuse intermodulation product involving the $2m-1$ harmonic of the subcarrier and the $2n$ harmonic of the MRT is

$$S_{2m-1,2n} = 4P_T T \mathrm{J}^2_{2m-1}(\alpha)\mathrm{J}^2_{2n}\left(\sqrt{2}\beta\right)\mathrm{sinc}^2\left[\frac{(\omega_I - \omega_R)T}{2\pi}\right] \tag{26}$$

The diffuse odd harmonics of the subcarrier and the diffuse intermodulation products cause an increase in the effective noise floor in the vicinity of the MRT frequency by a factor

$$\begin{aligned} \eta &= \frac{1}{N_0}\left[N_0 + \sum_{m=1}^{\infty}\sum_{n=0}^{\infty} S_{2m-1,2n}\right] \\ &= 1 + 4T\frac{P_T}{N_0}\sum_{m=1}^{\infty}\sum_{n=0}^{\infty} \mathrm{J}^2_{2m-1}(\alpha)\mathrm{J}^2_{2n}\left(\sqrt{2}\beta\right)\mathrm{sinc}^2\left[\frac{(\omega_I - \omega_R)T}{2\pi}\right] \end{aligned} \tag{27}$$

Equation (27) takes into account both odd harmonics of the subcarrier, whose effect is characterized by Eq. (24), and the diffuse intermodulation products involving odd harmonics of the subcarrier and even harmonics of the MRT, whose effect is characterized by Eq. (26).

A computer program was written to search for interferers that might cause trouble to ranging. The results of this search are discussed here for the example set of parameter values given in Table 2. No significant specular interferers close to the MRT frequency were found. The factor η by which the effective noise floor in the vicinity of the MRT frequency increases due to the presence of diffuse interferers is shown in Fig. 5. The reader may be alarmed by the large decibel increase in noise floor for values of P_T/N_0 greater than about 65 dB-Hz, but this should be put in proper perspective. For a P_T/N_0 of 65 dB-Hz with the modulation indices given, the E_s/N_0 is 13.3 dB. For any reasonably modern coding scheme, this represents far more power than needed for telemetry detection. Therefore, the right-hand side of Fig. 5 reflects a booming downlink, not a typical deep-space scenario.

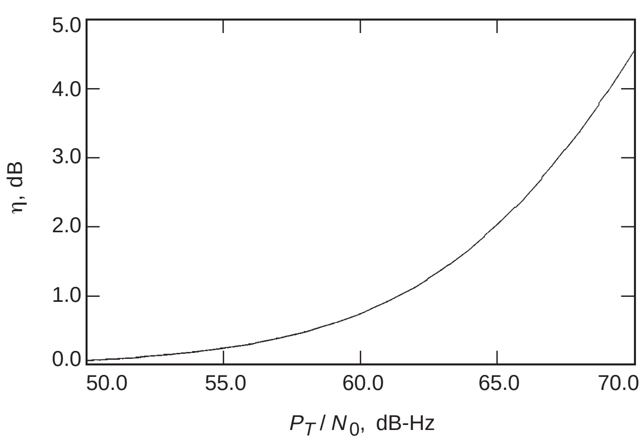

Fig. 5. The factor η as given by Eq. (27) for the parameters in Table 2.

B. Interference to Telemetry

Odd harmonics of the MRT can interfere with telemetry. The baseband angular frequencies are

$$\omega_I = (2n-1)\omega_R, \quad n = 1, 2, 3, \cdots \tag{28}$$

The ratio of the power P_I in this type of interferer to the power P_D in the telemetry fundamental sidebands is

$$\frac{P_I}{P_D} = \frac{\mathrm{J}_0^2(\alpha)\mathrm{J}_{2n-1}^2(\sqrt{2}\beta)}{\mathrm{J}_1^2(\alpha)\mathrm{J}_0^2(\sqrt{2}\beta)} \tag{29}$$

Intermodulation products involving even harmonics of the subcarrier and odd harmonics of the MRT are potential specular interferers to telemetry. The frequencies of this type of interferer are given in Eq. (21). The ratio of the power in this type of interferer to the power in the fundamental telemetry sidebands is

$$\frac{P_I}{P_D} = \frac{\mathrm{J}_{2m}^2(\alpha)\mathrm{J}_{2n-1}^2(\sqrt{2}\beta)}{\mathrm{J}_1^2(\alpha)\mathrm{J}_0^2(\sqrt{2}\beta)} \tag{30}$$

The ratio P_I/P_D of Eq. (29) or Eq. (30) can be substituted into Eq. (14) to determine η_T. If the difference between the subcarrier frequency and the frequency of a specular interferer is approximately less than the tracking bandwidth of the subcarrier synchronization loop, then the specular interferer could pose a threat to subcarrier synchronization. This is an unlikely scenario since the subcarrier tracking-loop bandwidth typically is quite small and there is not ordinarily a harmonic relationship between subcarrier and MRT.

Intermodulation products involving odd harmonics of the subcarrier and even harmonics of the MRT are potential diffuse interferers to telemetry. The frequencies of this type of interferer are given in Eq. (25). The ratio of the power in this type of interferer to the power in the fundamental telemetry sidebands is

$$\frac{P_I}{P_D} = \frac{\mathrm{J}_{2m-1}^2(\alpha)\mathrm{J}_{2n}^2(\sqrt{2}\beta)}{\mathrm{J}_1^2(\alpha)\mathrm{J}_0^2(\sqrt{2}\beta)} \tag{31}$$

A computer program was written to search for interferers that might cause trouble to telemetry. The results of this search are discussed here for the example set of parameter values given in Table 2. No significant diffuse interferers close to ω_T were found. The most significant specular interferer is the 100-kHz MRT fundamental. The ratio P_I/P_D for this specular interferer is -4.3 dB. The corresponding η_T, as calculated from Eq. (14), when $E_s/N_0 = 3$ dB is only 0.02 dB. Clearly, in the example considered here, there is no significant degradation of telemetry performance due to the presence of ranging and intermodulation sidebands.

VI. Conclusion

SNR degradation of telemetry due to the presence of a CW interferer is characterized for several telemetry schemes: NRZ data with a square-wave subcarrier, NRZ data with a sine-wave subcarrier, NRZ data directly modulated onto the carrier, and bi-ϕ data directly modulated onto the carrier. For the case of NRZ data directly modulated onto the carrier, we experimentally determined the loss due to the presence of a CW interferer and found it to agree very well with the theoretical characterization of

SNR degradation. Also, a model for SNR degradation of ranging due to the presence of a CW interferer is proposed here.

Mutual interference of telemetry and ranging is, in general, a complicated issue. An example is discussed in some detail in this article. Specular interferers are, in essence, CW interferers, and their contribution to SNR degradation is calculated as discussed in the first part of this article. Diffuse interferers increase the local effective noise floor. If ranging and telemetry sidebands are well separated and if no harmonic relationships exist between telemetry subcarrier and ranging tone frequencies, then there is often no significant mutual interference between telemetry and ranging.

Appendix
Jacobi–Anger Identities

The Jacobi–Anger identities are as follows:

$$\cos(y \sin x) = \mathrm{J}_0(y) + 2 \sum_{n=1}^{\infty} \mathrm{J}_{2n}(y) \cos(2nx) \tag{A-1}$$

$$\sin(y \sin x) = 2 \sum_{n=1}^{\infty} \mathrm{J}_{2n-1}(y) \sin[(2n-1)x] \tag{A-2}$$