

Determination of the Follow-up Receiver Noise-Temperature Contribution

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This article presents the derivations of both the exact and approximate equations for determining the follow-up receiver noise temperature from measured output powers when the first-stage low-noise amplifier (LNA) power supply is turned on and off. Although approximate equations have been used by DSN engineers since the early 1960s when the LNA was a maser, it is important to know whether the equation and experimental procedure are still valid when the LNA is a cryogenically cooled high-electron mobility transistor (HEMT) rather than a maser. In this article, it is shown that the approximate equations and experimental procedure being used for masers can also be used for HEMTs for obtaining an accurate value of the follow-up receiver temperature. However, if the first stage amplifier is not cryogenically cooled, but is at room temperature, it is shown that, if the approximate equation is used, a correction factor (also derived in this article) might have to be applied in order to get an accurate value. Since the derivations do not appear to be documented in any reports, the equations are rederived and presented in this article.

I. Introduction

From the early 1960s to the present, the method used to measure the follow-up noise-temperature contribution has been known as the on-off Y-factor measurement method. The approximate equation as given in [1] is

$$(T_f)_a = \frac{T_{oph}}{Y_{oo}} \quad (1)$$

where T_{oph} is the measured system temperature when the source that is connected to the maser input is an ambient load and Y_{oo} is the measured ratio of the output powers when the maser pump power supply is turned on and off when the source is an ambient load. The original derivations applied to systems wherein the front-end low-noise amplifier (LNA) was a maser with an effective input noise temperature of about 4.6 K and a gain of about 40 dB [2].

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It is currently of interest to determine whether the assumptions made in the derivation are still valid when the LNA is a high-emission mobility transistor (HEMT) where the effective input noise temperature at 32 GHz can be as high as 60 K and the gain is typically 25 dB.

The original notes on the derivations and assumptions used probably exist in laboratory notebooks or personal notebooks that are no longer retrievable. Therefore, this article serves to permanently document the steps in the derivations and point out what assumptions were made. It will be shown that the equations are still valid for systems wherein the LNA is a cryogenically cooled HEMT rather than a maser and also for room-temperature medium-noise-figure amplifiers. It will also be shown that when the LNA is turned off, the amplifier becomes a very lossy pad. Examples utilizing the equations for these three types of amplifiers will be presented.

II. Derivations

Figure 1 shows a basic DSN receiving system in an ambient load source configuration. When the source at port 1 is an ambient load, the operating system temperature at port 1 is

$$T_{oph} = T_h + T_e \quad (2)$$

and

$$T_e = T_{LNA} + T_f \quad (3)$$

where

T_h = the ambient load source temperature, K

T_e = the effective input noise temperature of the receiver, K

T_{LNA} = the effective input noise temperature of the LNA defined at port 1, K

T_f = follow-up receiver temperature defined at port 1, K

The ambient load temperature is $T_h = T_{pc} + 273.2$ K, where T_{pc} is the physical temperature of the load in deg C. The follow-up receiver temperature, T_f , at port 1 is related to the follow-up receiver temperature, T_{f2} , defined at port 2 by the relationship

$$T_f = \frac{T_{f2}}{G_1} \quad (4)$$

where G_1 is the gain of the LNA.

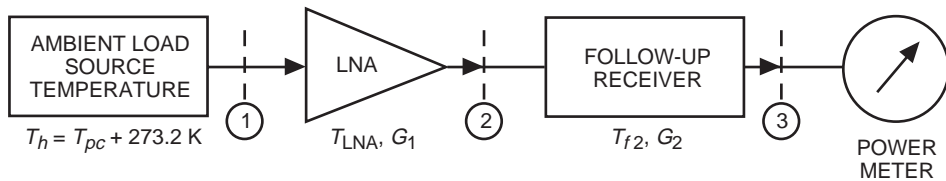


Fig. 1. The low-noise receiver system connected to an ambient load noise source.

Substitution of Eqs. (3) and (4) into Eq. (2) gives

$$\begin{aligned} T_{oph} &= T_h + T_{\text{LNA}} + T_f \\ &= T_h + T_{\text{LNA}} + \frac{T_{f2}}{G_1} \end{aligned} \quad (5)$$

When the bias supply (if the LNA is a HEMT) or the pump power supply (if the LNA is a maser) is turned off, instead of being an amplifier, the LNA becomes a lossy pad at a cryogenic physical temperature of T_{p1} . The system for this “off” configuration is shown in Fig. 2. The operating system temperature for Fig. 2 is

$$T_{oph}' = T_h + T'_{\text{LNA}} + \frac{T_{f2}}{G'_1} \quad (6)$$

where T'_{LNA} is the effective noise temperature of the LNA when it becomes a lossy pad. This noise temperature is expressed as [3]

$$T'_{\text{LNA}} = (L - 1)T_{p1} \quad (7)$$

where L is the pad loss factor > 1 , T_{p1} is the physical temperature of the pad, and

$$G'_1 = \frac{1}{L} \quad (8)$$

Substitutions into Eq. (6) give

$$T_{oph}' = T_h + (L - 1)T_{p1} + LT_{f2} \quad (9)$$

When on-off Y-factor measurements are made, the following output powers are measured on a power meter at the output of the follow-up receiver (see Figs. 1 and 2):

$$(P_{\text{out}})_{\text{on}} = kB [G_1 (T_h + T_{\text{LNA}}) + T_{f2}] G_2 \quad (10)$$

$$(P_{\text{out}})_{\text{off}} = kB [G'_1 (T_h + T'_{\text{LNA}}) + T_{f2}] G_2 \quad (11)$$

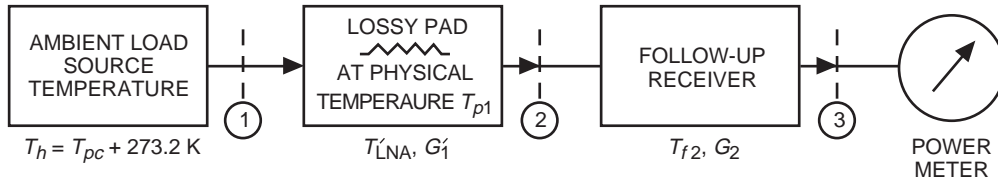


Fig. 2. The low-noise receiver system (connected to an ambient load noise source) when (1) the LNA bias supply is turned off if the LNA is a HEMT or (2) the pump power supply is turned off if the LNA is a maser.

where

k = Boltzmann's constant, 1.380×10^{-23} J/K

B = the receiving system noise bandwidth, Hz

G_2 = the gain of the follow-up receiver, defined between the input of the follow-up receiver and the input of the power meter

Defining the on-off Y-factor as the ratio of these two output powers, then

$$Y_{oo} = \frac{(P_{\text{out}})_{\text{on}}}{(P_{\text{out}})_{\text{off}}} \quad (12)$$

Substitution of Eqs. (10) and (11) gives

$$Y_{oo} = \frac{G_1 (T_h + T_{\text{LNA}}) + T_{f2}}{G'_1 (T_h + T'_{\text{LNA}}) + T_{f2}} \quad (13)$$

Dividing the numerator and denominator by G_1 results in

$$Y_{oo} = \frac{T_h + T_{\text{LNA}} + \frac{T_{f2}}{G_1}}{\frac{G'_1}{G_1} (T_h + T'_{\text{LNA}}) + \frac{T_{f2}}{G_1}} \quad (14)$$

We can further substitute $T_f = T_{f2}/G_1$ from Eq. (4) in the numerator to arrive at the expression

$$Y_{oo} = \frac{T_h + T_{\text{LNA}} + T_f}{\frac{G'_1}{G_1} (T_h + T'_{\text{LNA}}) + \frac{T_{f2}}{G_1}} = \frac{T_{oph}}{Den} \quad (15)$$

where, from substitutions of Eqs. (7) and (8),

$$Den = \frac{1}{G_1} [T_h L^{-1} + (1 - L^{-1}) T_{p1} + T_{f2}] \quad (16)$$

An error that is easy to make is to assume that

$$\left[Y_{oo} = \frac{T_{oph}}{T_{oph'}} \right]$$

However, comparison of Eq. (15) to Eqs. (5) and (9) shows that

$$Y_{oo} \neq \frac{T_{oph}}{T_{oph'}}$$

and to derive the correct expression for Y_{oo} , we must take the ratios of output powers that are measured [see Eq. (12)] rather than the ratio of system temperatures.

Manipulation of Eq. (15) and solving for T_f results in the expression

$$T_f = (T_f)_{\text{approx}} - C_f \quad (17)$$

where the new approximate formula is

$$(T_f)_{\text{approx}} = \frac{T_h + T_{\text{LNA}}}{Y_{oo} - 1} \quad (18)$$

and C_f is a correction factor expressed as

$$C_f = \left(\frac{Y_{oo}}{Y_{oo} - 1} \right) \frac{G'_1}{G_1} (T_h + T'_{\text{LNA}}) \quad (19)$$

Substitutions of Eqs. (7) and (8) give

$$C_f = \left(\frac{Y_{oo}}{Y_{oo} - 1} \right) \frac{1}{G_1} [T_h L^{-1} + (1 - L^{-1}) T_{p1}] \quad (20)$$

Equation (17) is the exact expression, where no assumptions were made. To obtain Eq. (1), make the assumption that $Y_{oo} \gg 1$, so that Eqs. (18) and (19) become

$$T_f \approx \frac{T_h + T_{\text{LNA}}}{Y_{oo}} - C'_f \quad (21)$$

where

$$C'_f = \frac{1}{G_1} [T_h L^{-1} + (1 - L^{-1}) T_{p1}] \quad (22)$$

If we further assume that the correction factor C'_f may be ignored and assume that $T_h + T_{\text{LNA}} \gg T_f$, then

$$T_{oph} = T_h + T_{\text{LNA}} + T_f \approx T_h + T_{\text{LNA}} \quad (23)$$

These assumptions used in Eq. (21) lead to

$$T_f \approx \frac{T_{oph}}{Y_{oo}} \quad (24)$$

which is the same as Eq. (1). Both T_{oph} and Y_{oo} are measured values. The derivation of Eq. (1) is now complete, and the assumptions made have been pointed out. It was recently discovered that, instead of using Eq. (24), DSS 13 currently uses the approximate formula

$$T_f \simeq \frac{T_{pc} + 273.2 + T_{\text{LNA}}}{Y_{oo} - 1} \quad (25)$$

which is the same as Eq. (18) and is slightly more accurate than Eq. (1).

III. Applications

A. Example 1: The LNA Is a Cryogenically Cooled HEMT

In the system shown in Fig. 1, when the LNA voltage is turned off, assume that the LNA becomes a pad with a loss of 40 dB at a physical temperature of $T_{p1} = 12$ K. These are values that closely fit the HEMT parameters of the monopulse feed receiving system currently at DSS 13.² The following is a summary of system parameters:

$$\begin{aligned} T_h &= 293.2 \text{ K} \\ T_{\text{LNA}} &= 51 \text{ K} \\ (G_1)_{\text{dB}} &= 28 \text{ dB or } G_1 = 631 \\ L_{\text{dB}} &= 40 \text{ dB or } L = 10^4 \\ T_{p1} &= 12 \text{ K} \end{aligned}$$

Assume that the follow-up receiver in Fig. 1 consists of a post-amplifier with an effective input noise temperature of $(T_e)_{PA} = 359$ K and a gain of $G_{PA} = 10^3$, followed by a downconverter with $(T_e)_{dc} = 1225$ K. Using the general equation for the effective noise temperature of amplifiers in cascade [3,4], and applying it to this case, we derive the equation

$$T_{f2} = (T_e)_{PA} + \frac{(T_e)_{dc}}{G_{PA}} \quad (26)$$

and calculate $T_{f2} = 360.2$ K at port 2. With this a priori knowledge of T_{f2} , we can calculate the value of Y_{oo} that would be measured. From Eq. (5), the numerator of Y_{oo} is

$$\begin{aligned} T_{oph} &= T_h + T_{\text{LNA}} + \frac{T_{f2}}{G_1} \\ &= 293.2 + 51 + \frac{360.2}{631} \\ &= 344.8 \text{ K} \end{aligned}$$

From Eq. (16), the denominator of Y_{oo} is

$$Den = \frac{1}{G_1} [T_h L^{-1} + (1 - L^{-1}) T_{p1} + T_{f2}]$$

so substitution of values gives

² J. Bowen, personal communication, Jet Propulsion Laboratory, Pasadena, California, August 5, 2000.

$$Den = \frac{1}{631} \left[\frac{293.2}{10,000} + \left(1 - \frac{1}{10,000} \right) 12 + 360.2 \right]$$

$$= 0.59$$

The Y_{oo} values that would be measured are [see Eq. (15)],

$$Y_{oo} = \frac{T_{oph}}{Den} = \frac{344.8}{0.59} = 584.4$$

and

$$(Y_{oo})_{dB} = 10 \log_{10} Y_{oo} = 27.67 \text{ dB}$$

Use of the approximate formula given by Eq. (1) yields

$$(T_f)_a = \frac{T_{oph}}{Y_{oo}} = \frac{344.8}{584.4} = 0.59 \text{ K}$$

as compared with the true value of

$$T_f = \frac{T_{f2}}{G_1} = \frac{360.2}{631} = 0.57 \text{ K}$$

where for purposes of this example T_{f2} was known a priori. The actual difference is 0.02 K, which may be considered to be small enough to ignore.

B. Example 2: The LNA Is a Maser

Assume that the LNA is a maser instead of a HEMT. For a maser having 40-dB gain, when the pump power supply is turned off, the maser becomes a pad of about 50-dB loss at a physical temperature of about 4.2 K.³ The basic system of Fig. 1 has the following given parameters:

$$T_h = 293.2 \text{ K}$$

$$T_{LNA} = 4.6 \text{ K}$$

$$(G_1)_{dB} = 40 \text{ dB or } G_1 = 10^4$$

$$L_{dB} = 50 \text{ dB or } L = 10^5$$

$$T_{p1} = 4.2 \text{ K}$$

As in Example 1, let $T_{f2} = 360.2 \text{ K}$ at port 2 be known a priori. We can now calculate the value of Y_{oo} that would be measured. From Eq. (5),

$$T_{oph} = T_h + T_{LNA} + \frac{T_{f2}}{G_1}$$

³ Ibid.

so that substitution of values gives

$$\begin{aligned} T_{oph} &= 293.2 + 4.6 + \frac{360.2}{10^4} \\ &= 297.84 \text{ K} \end{aligned}$$

From Eq. (15), the denominator of Y_{oo} is

$$Den = \frac{1}{G_1} [T_h L^{-1} + (1 - L^{-1}) T_{p1} + T_{f2}]$$

so substitution of values gives

$$\begin{aligned} Den &= \frac{1}{10^4} [(293.2) (10^{-5}) + (1 - 10^{-5}) 4.2 + 360.2] \\ &= 0.0364 \end{aligned}$$

Substitution of these values into Eq. (15) gives the Y_{oo} values that would be measured:

$$Y_{oo} = \frac{T_{oph}}{Den} = \frac{297.84}{0.0364} = 8182.4$$

and

$$(Y_{oo})_{dB} = 39.13 \text{ dB}$$

From the approximate formula given by Eq. (1), we obtain

$$(T_f)_a = \frac{T_{oph}}{Y_{oo}} = \frac{297.84}{8182.4} = 0.0364$$

as compared with the true value of

$$T_f = \frac{T_{f2}}{G_1} = \frac{360.2}{10^4} = 0.036$$

The difference is negligible.

C. Example 3: The First-Stage Amplifier Is a Room-Temperature Amplifier Instead of a Cooled LNA

Assume that in the system shown in Fig. 1 that $T_h = 293.2$ K, $T_{LNA} = 290$ K, and $G_1 = 10^3$, and, when the bias supply voltages are turned off, assume $L_{dB} = 30$ dB or $L = 10^3$ and $T_{p1} = 300$ K. Assume further that the follow-up receiver temperature at port 2, T_{f2} , is known a priori to be 1500 K. Then,

$$\begin{aligned}
T_{oph} &= T_h + T_{LNA} + \frac{T_{f2}}{G_1} \\
&= 293.2 + 290 + \frac{1500}{10^3} \\
&= 584.7 \text{ K}
\end{aligned}$$

The denominator of Y_{oo} is given in Eq. (16) as

$$Den = \frac{1}{G_1} [T_h L^{-1} + (1 - L^{-1}) T_{p1} + T_{f2}]$$

so substitution of values gives

$$\begin{aligned}
Den &= \frac{1}{10^3} [293.2 (10^{-3}) + (1 - 10^{-3}) 300 + 1500] \\
&= 1.8 \text{ K}
\end{aligned}$$

The values of Y_{oo} that would be measured are

$$Y_{oo} = \frac{T_{oph}}{Den} = \frac{584.7}{1.80} = 324.83$$

and

$$(Y_{oo})_{dB} = 25.12 \text{ dB}$$

Using the approximate formula given by Eq. (1), we obtain the approximate follow-up receiver-temperature value of

$$(T_f)_a = \frac{T_{oph}}{Y_{oo}} = \frac{584.7}{324.83} = 1.8 \text{ K}$$

as compared with the true value of

$$T_f = \frac{T_{f2}}{G_1} = \frac{1500}{10^3} = 1.5 \text{ K}$$

where T_{f2} was known a priori. The difference is 0.3 K.

The approximate correction factor given by Eq. (22) can be used with the approximate formula given by Eq. (1) to obtain

$$\begin{aligned}
C'_f &= \frac{1}{G_1} [T_h L^{-1} + (1 - L^{-1}) T_{p1}] \\
&= \frac{1}{10^3} [293.2 (10^{-3}) + (1 - 10^{-3}) 300] \\
&= 0.3 \text{ K}
\end{aligned}$$

So if we use this correction factor, we obtain

$$T_f \approx (T_f)_a - C'_f = 1.8 - 0.3 = 1.5 \text{ K}$$

which agrees with the true value for this example.

Although for this example the L_{dB} for a room-temperature amplifier was estimated to be 30 dB, the actual value can be measured for each amplifier or the value might be obtainable from the manufacturer. The purpose of Example 3 is to show that a non-negligible error in determining T_f could exist if the approximate formula given by Eq. (1) is used without a correction factor.

It is sometimes the case in field operations that T_f is measured with the Y-factor method but, several months later, the measured value is much different. Consider the case where both L_{dB} and $(G_1)_{\text{dB}}$ have changed over time. It can be seen from Eq. (4) that accurate knowledge of $(G_1)_{\text{dB}}$ is critical for determining T_f . On the other hand, if $L_{\text{dB}} > 20$ dB, L_{dB} does not have to be known to better than 3 dB. For simplicity, consider the following worst-case scenario. If L_{dB} becomes 10 dB less in any of the three examples, calculation results will not be significantly affected. However, if over a period of time $(G_1)_{\text{dB}}$ has become lower by 3 dB, the value of T_f that will be measured by the Y-factor on-off method will be higher by a factor of 2.

IV. Concluding Remarks

The derivation of the approximate formula currently being used by DSN engineers to determine the follow-up receiver temperature has been presented. It was shown that this simple equation is sufficiently accurate to use for systems wherein the LNA is either a maser or a HEMT. If the first-stage amplifier is a room-temperature amplifier, a correction factor might have to be used to get close to the true value of the follow-up receiver temperature.

Acknowledgment

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References

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