Antenna System Noise-Temperature Calibration Mismatch Errors Revisited

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This article presents tutorial discussions of available and delivered system noise temperatures as well as antenna efficiency and how mismatch errors affect the expected values. Derivation of the mismatch error equations begins with fundamental considerations, and subsequent steps are purposely presented in detail. Mismatch errors are shown to be functions of the voltage reflection coefficients of the ambient load, antenna, and receiver. The errors can also be functions of the receiver correlation coefficient. Plots are presented showing how each of these coefficients affects deviations of the true operating-system noise-temperature values from the assumed matched-case values for a typical DSN receiving system at 8.45 GHz (X-band).

I. Introduction

The current method of calibrating antenna operating-system noise temperature for DSN antennas is the ambient-load method developed by Stelzried [1] and utilized by Otoshi [2] for calibrating the new beam-waveguide antenna systems. Although many elements comprise the antenna system, such as the sky, reflectors, horn, transmission lines, diplexer, and filter, these elements will be cascaded and shown as one element called the antenna input termination. In a similar manner, the receiver, which typically consists of a low-noise amplifier (LNA), post-amplifier, cables, filters, and downconverters, will also be cascaded and shown as a single element called the receiver. The two elements comprise the antenna receiving system as shown in Fig. 1. The system was simplified in this manner to facilitate mismatch error analyses. For mismatch error analysis, it is important that the antenna and receiver reflection coefficients be known.

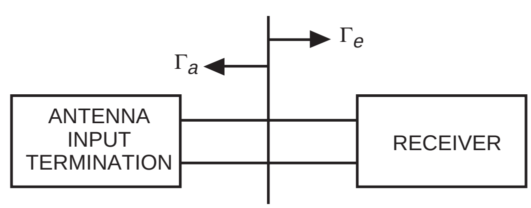

Fig. 1. The basic antenna receiving system.

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Very often only the magnitudes of the reflection coefficients are measured and the phases of the reflection coefficients are ignored. This lack of knowledge of the phases can cause a large range of uncertainties in determining the true value of the antenna operating-system noise temperature. The deviations of the actual system noise temperature from the expected values are referred to as mismatch errors. Expected values are based upon the assumption that all reflection coefficients in the system have zero magnitudes or that the voltage standing-wave ratios (VSWRs) are all equal to one. The expected values will be referred to in this article as the "assumed matched-case" values.

The system for calibrating DSN antenna systems is a total power radiometer and a thermal noise standard, which is an ambient load whose physical temperature is accurately measured. In [1], Stelzried presents mismatch errors associated with calibration of the antenna operating-system noise temperature, T_{op}, using this ambient load. His mismatch error analysis is based on Otoshi's mismatch error equations presented in [3]. To simplify the analysis, Stelzried omitted a term referred to as the correlated noise contribution. Recently, because of the need for the best possible calibrations of system operating noise temperatures for upcoming planetary encounters and gravitational wave experiments, a request was made of this author to document derivations of the mismatch error equations, including the correlated noise term, and to express the errors in terms of VSWRs.

In Section II of this article, a review will be made of IEEE definitions of noise temperatures and a discussion of mismatch factor, which is the factor that allows conversions between available and delivered noise temperatures. Section III presents mismatch errors associated with measurements of T_{op} using the ambient-load method. Section IV presents mismatch error equations for measurements of antenna efficiency in terms of measured delivered radio noise source temperatures. Mismatch error equations are presented for both delivered and available antenna operating-system noise temperatures in terms of reflection coefficients as well as VSWRs. Section V gives plotted examples of the effects of mismatch errors for delivered and available system noise temperatures of the newly designed DSN 8.45-GHz (X-band) receiving system whose LNA is a high-electron-mobility transistor (HEMT) having an effective input noise-temperature contribution of only 5 K. Section VI presents a brief summary and concluding remarks.

II. Review

A. IEEE Definitions

For a review of the fundamentals and the definitions of noise temperatures, the reader is referred to an excellent 1967 article by Miller et al. of the former National Bureau of Standards [4]. Also, it has been stated that, for narrow noise bandwidths, scattering parameters can be used for mismatch error analyses of noise temperatures in the same way they are used for continuous waves (CWs) [5].

For a simple case, the available power in watts from a resistor at a uniform temperature T in kelvins is

$$P = kTB \tag{1}$$

where k = Boltzmann's constant and B is the noise bandwidth in Hz. This resistor is a noise source and, when connected to the receiver, it becomes an "input termination" by an IEEE Standards Committee definition [6]. For a DSN low-noise system, the input termination can be the ambient load, a cryogenic load, or the antenna. The noise of a receiver was given the name the "effective input noise temperature" by the IEEE Standards Committee in 1962 [6]. For a DSN low-noise receiving system, the first amplifier is usually an LNA such as a maser or an HEMT, and the remaining components are lumped into a single element referred to as the "follow-up receiver." The LNA and follow-up receiver can be represented as simply the "receiver," as shown in Fig. 1. The reference plane is the junction of the input termination and the receiver. For this basic system consisting of an input termination and a receiver, the operating-system noise temperature as defined by the IEEE [6] is expressed as

$$T_{op} = T_i + T_e \tag{2}$$

where

$T_i =$ *available* noise temperature of the input termination

$T_e =$ effective input noise temperature of the receiver

What is not clear in the IEEE definition is whether T_e is the available or the delivered effective input noise temperature. It can be argued that all measurable noise temperatures are delivered noise temperatures and the most that can be delivered is the available noise temperature. Figure 2 shows equivalent receivers (consistent with the figure given by Miller et al. [4]), and it shows that T_e has the same source reflection coefficient as the input termination that is connected to the receiver. Then, if T_i is the *available* noise temperature of the input termination, it follows that T_e must also be the *available* effective input noise temperature of the receiver.

Therefore, we may write

$$(T_{op})_a = (T_i)_a + (T_e)_a \tag{3}$$

where the outer subscript "a" is used (in this article) to denote "available" noise temperature. An outer subscript "d" will be used in this article to denote "delivered" noise temperature, or the noise

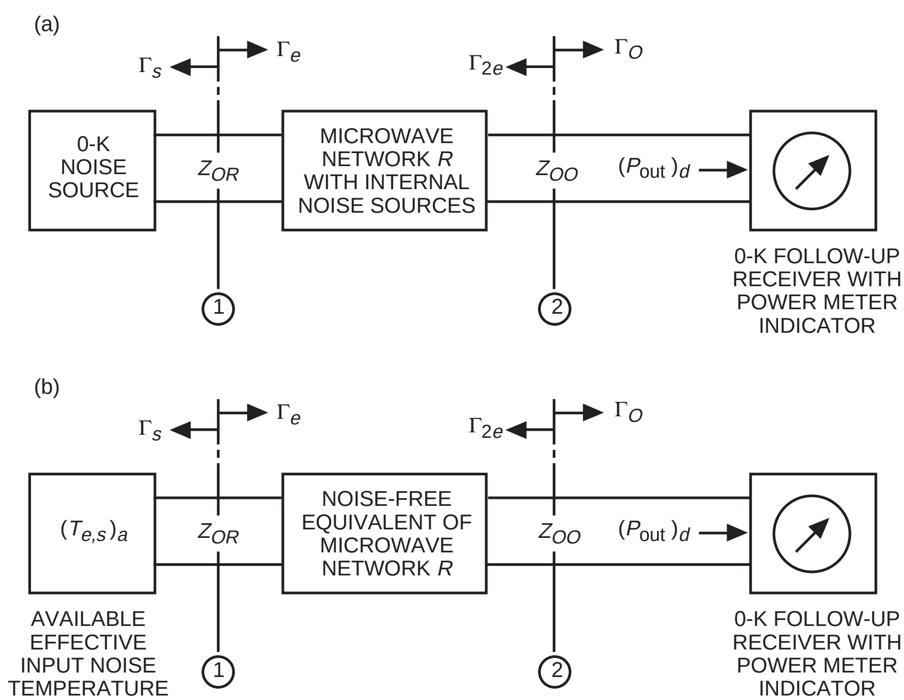

Fig. 2. Block diagram of the concept of effective input noise temperature: (a) actual receiver system and (b) equivalent receiver system. Note the dependence on Γ_S, which is the voltage reflection coefficient for an arbitrary input termination.

temperature that is actually delivered to the receiver. For example, the symbol $(T_{op})_d$ denotes delivered operating-system noise temperature.

Generally in the literature on noise temperatures, no outer subscript to denote available or delivered noise temperature is used because the matched case is assumed where all reflection coefficients are zero. If the receiver and generator (or source) reflection coefficients are zero, then the available and delivered noise temperatures are the same and there is no need to distinguish between available and delivered cases. However, when analyzing mismatch errors, it becomes necessary to make a distinction between the system noise temperature that is available and the system noise temperature that actually gets delivered to the receiver.

Available operating-system noise temperature, $(T_{op})_a$, can be defined as the maximum system noise temperature that can be delivered to the receiver. This maximum value is achieved only for two conditions: (1) the matched case where the source and receiver reflection coefficients are zero and (2) the mismatched case where source and receiver reflection coefficients are the complex conjugate of each other. The term complex conjugate as used here means that the magnitudes of the reflection coefficients are equal to each other, and the phase angle of the source reflection coefficient is the negative of the phase angle of the receiver reflection coefficient. These definitions will become clearer in the next section, where mismatch factor will be discussed. It will be shown that the author's definitions for the mismatched case are consistent with IEEE definitions [4].

Mismatch errors for this article are limited to mismatch errors for a system that consists of only an input termination and a receiver. If one wishes to determine the effects of mismatch on the noise temperatures of individual components of the input termination or of the receiver, one needs to refer to component mismatch error equations given in [3].

B. Mismatch Factor

Mismatch factor is a term that is often used in conjunction with discussions of delivered and available powers. Consider the basic system consisting of a generator (or source) and a load, as shown in Fig. 3. The source could be the antenna, and the load could be the receiver. For the configuration of Fig. 3, the mismatch factor for the generator–load interface is defined to be the ratio of power delivered to the load to the power available from the generator. The maximum power that the generator can deliver to the load is called available power. Instead of the Greek symbol α used previously by Otoshi in [3], the symbol M will be used. The first subscript on M will denote the generator, and the second subscript will denote the load. For example, the term M_{GL} denotes the mismatch factor for generator G and load L and is expressed mathematically [7] as

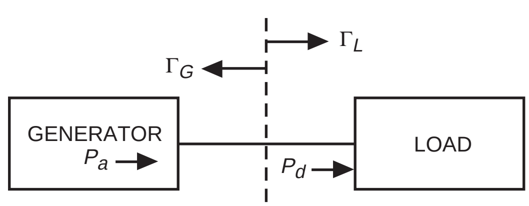

Fig. 3. The basic system, consisting of a generator and a load. P_a is the power *available* from the generator, and P_d is the power *delivered* to the load.

$$M_{GL} = \frac{P_d}{P_a} = \frac{\left(1 - |\Gamma_G|^2\right)\left(1 - |\Gamma_L|^2\right)}{|1 - \Gamma_G \Gamma_L|^2} \tag{4}$$

where Γ_G and Γ_L are the voltage reflection coefficients of the generator and load, respectively. Expansion of the denominator in terms of magnitudes and phase angles results in the expression

$$|1 - \Gamma_G \Gamma_L|^2 = 1 - 2\,|\Gamma_G \Gamma_L| \cos\theta + |\Gamma_G \Gamma_L|^2 \tag{5}$$

where

$$\theta = \psi_G + \psi_L$$

and ψ_G and ψ_L are the phase angles of reflection coefficients Γ_G and Γ_L, respectively. The vertical parallel || bars denote the magnitude of the complex quantities inside the parallel bars. With the advent of portable network analyzers that measure S-parameters, the complex reflection coefficients can be measured in the field, and the mismatch factor M_{GL} can then be calculated from Eqs. (4) and (5) to good accuracy. If M_{GL} is known, there is no need to perform further mismatch error analyses. However, without a network analyzer, one usually determines only VSWR or magnitudes of the reflection coefficients and not the phase angles of the reflection coefficients. Then the mismatch error analyses consist of solving for the worst-case limits of M_{GL} as follows. Differentiation of Eq. (5) with respect to θ and setting the resulting expression to zero will show that the maximums and minimums of Eq. (5) occur when

$$\theta = 0, 2\pi, 4\pi, \cdots, 2m\pi$$

where $m = 0, 1, 2, \cdots$ for minimums, and

$$\theta = \pi, 3\pi, 5\pi, \cdots, (2n-1)\pi$$

where $n = 1, 2, 3, \cdots$ for maximums. Substitution of these θ values back into Eq. (5) gives

$$|1 - \Gamma_G \Gamma_L|^2_{\min} = (1 - |\Gamma_G \Gamma_L|)^2 \tag{6}$$

and

$$|1 - \Gamma_G \Gamma_L|^2_{\max} = (1 + |\Gamma_G \Gamma_L|)^2 \tag{7}$$

Further substitution of these values into Eq. (4) gives

$$(M_{GL})_{\max} = \frac{\left(1 - |\Gamma_G|^2\right)\left(1 - |\Gamma_L|^2\right)}{(1 - |\Gamma_G \Gamma_L|)^2} \tag{8}$$

and

$$(M_{GL})_{\min} = \frac{\left(1 - |\Gamma_G|^2\right)\left(1 - |\Gamma_L|^2\right)}{(1 + |\Gamma_G \Gamma_L|)^2} \tag{9}$$

These are the local maximums and minimums. The global maximum of Eq. (8) can be found by letting magnitudes vary as well as the phases. The global maximum is found to be 1.0 and occurs only when the generator and receiver reflection coefficients are the complex conjugate of each other (i.e., $|\Gamma_G| = |\Gamma_L|$ and $\psi_G = -\psi_L$ or when $|\Gamma_G| = |\Gamma_L| = 0$). In this article, the local maximum and minimum will be utilized. For special cases, the local maximum can also be the global maximum.

Suppose it is desired that Eqs. (8) and (9) be expressed as VSWR instead of magnitudes of voltage reflection coefficients. Let S_G and S_L represent the VSWRs corresponding to Γ_G and Γ_L, respectively. From use of the general conversion formula of

$$|\Gamma_x| = \frac{S_x - 1}{S_x + 1} \tag{10}$$

the following equivalent relationships are obtained in terms of VSWR:

$$(M_{GL})_{\max} = \frac{4 S_G S_L}{(S_G + S_L)^2} \tag{11}$$

and

$$(M_{GL})_{\min} = \frac{4 S_G S_L}{(S_G S_L + 1)^2} \tag{12}$$

A global maximum mismatch factor equal to 1 occurs when $S_G = S_L$.

The mismatch factor is a useful fundamental relationship that appears in many analyses of mismatch errors. As will be shown later in the article, the mismatch factor will appear in the analyses of mismatch errors associated with measurements of antenna operating-system noise temperature and antenna efficiency.

C. Delivered T_{op}

Adopting the same nomenclature system as was used by Stelzried in [1], let the letters p, a, and e denote parameters of the ambient load, antenna, and receiver, respectively. Then for the antenna input termination case, Eq. (4) is rewritten as

$$M_{ae} = \frac{\left(1 - |\Gamma_a|^2\right)\left(1 - |\Gamma_e|^2\right)}{|1 - \Gamma_a \Gamma_e|^2} \tag{13}$$

and Eqs. (8), (9), (11), and (12) become

$$(M_{ae})_{\max} = \frac{\left(1 - |\Gamma_a|^2\right)\left(1 - |\Gamma_e|^2\right)}{(1 - |\Gamma_a \Gamma_e|)^2} = \frac{4 S_a S_e}{(S_a + S_e)^2} \tag{14}$$

and

$$(M_{ae})_{\min} = \frac{\left(1 - |\Gamma_a|^2\right)\left(1 - |\Gamma_e|^2\right)}{\left(1 + |\Gamma_a \Gamma_e|\right)^2} = \frac{4 S_a S_e}{\left(S_a S_e + 1\right)^2} \tag{15}$$

For the case where the input termination is the ambient load, replace all letter "a" subscripts in Eqs. (13) through (15) with the letter "p."

When the input termination is the antenna, the delivered operating-system noise temperature can be expressed as

$$(T_{op,a})_d = (T_a)_d + (T_{e,a})_d \tag{16}$$

where $T_{op,a}$ and $T_{e,a}$ are the operating-system and effective input noise temperatures, respectively. An additional subscript "a" on T_{op} and T_e is used to show their dependence on the reflection coefficient of the input termination, which in this case is Γ_a. The outer subscript "d" is used to denote delivered.

If instead the input termination is the ambient load, the delivered operating-system noise temperature would be expressed as

$$(T_{op,p})_d = (T_p)_d + (T_{e,p})_d \tag{17}$$

where the p after the comma in $T_{op,p}$ and $T_{e,p}$ denotes dependence on the ambient-load reflection coefficient Γ_p. Delivered operating-system noise temperature is what actually gets delivered to the receiver at the input port of the receiver.

D. Available T_{op}

To obtain available noise temperature from delivered noise temperature, one simply divides the delivered noise temperature by the mismatch factor applicable to the input port. For example, the available antenna operating-system noise temperature is obtained by dividing Eq. (16) by the mismatch factor M_{ae}:

$$(T_{op,a})_a = \frac{(T_{op,a})_d}{M_{ae}} \tag{18}$$

If one wishes to get the expression for the corresponding available effective input noise temperature, divide the delivered effective input noise-temperature term in Eq. (16) by the mismatch factor M_{ae} to obtain

$$(T_{e,a})_a = \frac{(T_{e,a})_d}{M_{ae}} \tag{19}$$

and the input termination available noise temperature is obtained from

$$(T_a)_a = \frac{(T_a)_d}{M_{ae}} \tag{20}$$

If the input termination is the ambient load rather than the antenna, replace a with p inside the parentheses and replace M_{ae} with M_{pe} in Eqs. (18) through (20). If instead the available noise temperature is known, then to obtain the delivered noise temperatures from available ones, simply multiply the available noise temperatures by the appropriate mismatch factors, which are M_{ae} and M_{pe} for the antenna and ambient-load configurations, respectively.

As discussed earlier, if the phases of Γ_a and Γ_e are not known and only the magnitudes are known, then only the limits or the mismatch factor M_{ae} can be solved for. For the above antenna case, $(M_{ae})_{\max}$ and $(M_{ae})_{\min}$ are given in Eqs. (14) and (15). To solve for the limits of error for the ambient-load input termination case, one need only substitute the symbol p for the symbol a in $(M_{ae})_{\max}$ and $(M_{ae})_{\min}$ to get $(M_{pe})_{\max}$ and $(M_{pe})_{\min}$.

III. Antenna System Temperature Measurements

A. Description of System Calibration Method Using an Ambient Load

The ambient-load method for measurement of antenna system noise temperature as originally developed by Stelzried assumes that the receiver is a total power radiometer and the output power is read on a power meter. The principle of operation of this calibration method is that two power meter readings are required to establish a linear power meter reading versus an operating-system noise-temperature curve [2]. The first required point of the curve corresponds to the power meter reading obtained when the power meter is zeroed by means of a remotely controlled coaxial switch that is terminated by a coaxial termination. This first point is an effectively zero system operating noise temperature on the calibration curve. The second required point on the calibration curve is the power meter reading that is observed when the waveguide switch is operated in the "ambient-load" path position. These two readings enable deriving a linear calibration curve of power meter reading versus system noise temperature [2]. Then the waveguide switch is switched to the antenna position. From the power meter reading, the antenna operating noise temperature is calculated from the linear relationship, assuming that mismatch effects do not cause deviation from the assumed linear relationship.

B. Neglect Effects of Mismatch

Although some of the symbols used here have been previously defined, they are included in the definitions of symbols provided in Table 1.

Values for the output powers (delivered to the output) are the same for either the "assumed matched" or "mismatched" case when the input termination connected to the receiver is the ambient load or the antenna. These measured powers are converted to a ratio usually called the Y-factor and expressed as

$$Y = \frac{(P_{op,p})_d}{(P_{op,a})_d} \tag{21}$$

and where $(P_{op,p})_d$ and $(P_{op,a})_d$ are the power meter readings observed when the input terminations connected to the receiver are the ambient load and antenna, respectively. In the assumed matched case (when mismatch effects are neglected), the following relationship is assumed:

$$Y = \frac{(P_{op,p})_d}{(P_{op,a})_d} = \frac{T_{op,p'}}{T_{op,a'}} \tag{22}$$

Table 1. Nomenclature.

Notation	Definition
Γ_p	Voltage reflection coefficient of the ambient load.
Γ_e	Voltage reflection coefficient of the receiver, consisting of the LNA and follow-up receiver.
Γ_a	Voltage reflection coefficient of the antenna/feed terminated in cold sky.
T_p	Noise temperature of the ambient load, K.
$T_{e'}$	Effective input noise temperature of the receiver for the assumed matched case, K.
T_r	Noise temperature that is generated by the receiver and radiates toward the input termination connected to the receiver, K.
T_{eo}	Internal receiver noise temperature, K. For more details, see [3].
$T_{a'}$	Antenna/feed noise temperature defined at the receiver input for the assumed matched case, K.
$(T_{op,a'})$	Antenna operating-system noise temperature for the assumed matched case.
$(T_{op,p'})$	Ambient-load operating-system noise temperature for the assumed matched case.
$(T_{op,a})_d, (T_{op,a})_a$	Delivered and available antenna operating-system noise temperatures, respectively, under mismatched conditions.
$(T_{op,p})_d, (T_{op,p})_a$	Delivered and available operating-system noise temperatures, respectively, when the input termination is the ambient load under mismatched conditions.
γ_p	Real part of the complex correlation coefficient that expresses the degree of correlation between T_{eo} and T_r. In general, the value of γ_p is not known but has the limits of $+1$ and -1. The p subscript denotes the ambient-load case.
M_{pe}, M_{ae}	Mismatch factors when the input terminations are the ambient load and the antenna, respectively.
S_p, S_e, S_a	The VSWRs of the ambient load, receiver, and antenna, respectively.
T_{LNA}	Noise temperature of the LNA, K.
T_{FU}	Follow-up receiver noise temperature, K.

Manipulation of this equation gives the following expression needed to determine the antenna operating-system noise temperature for the assumed matched case:

$$T_{op,a'} = \frac{T_{op,p'}}{Y} = T'_a + T'_e \tag{23}$$

where the primes denote the assumed matched-case values. It is assumed that

$$T_{op,p'} = T_{pc} + 273.16 + T_{e'} = T_p + T_{e'} \tag{24}$$

where T_{pc} is the ambient-load physical temperature in deg C and $T_{e'}$ is the effective input noise temperature of the receiver for the assumed matched case. The ambient-load physical temperature T_{pc} is known from readings of a thermometer embedded in the ambient-load housing. Real-time accurate information of the ambient load's physical temperature is monitored, digitized, and sent to the computer.

For a low-noise system composed of an LNA and a follow-up receiver,

$$T_{e'} = T_{\text{LNA}} + T_{FU} \tag{25}$$

For this particular ambient-load calibration technique, it is required that T_{LNA}, the noise temperature of the LNA, be known accurately from prior calibrations in the laboratory. It is assumed that calibrations of T_{LNA} were performed with matched ambient and cryogenic loads. After the LNA is installed, the follow-up noise-temperature contribution T_{FU} is measured in the field by the Y on–off method described in [8]. Then $T_{e'}$ is calculated from Eq. (25). It can be seen from Eq. (24) that all values are now known for calculating $T_{op,p'}$. If Y is measured in the field, then from use of Eq. (23), the assumed matched-case antenna system noise temperature can be calculated. In the DSN system, Y is measured automatically by measuring the receiver output powers while connecting the receiver first to the ambient load and then to the antenna by means of a computer-controlled switch [see Eq. (22)]. Then the station computer calculates the ambient-load system noise temperature from Eq. (24) in real time, and $T_{op,a'}$ is calculated from Eq. (23). A complete error analysis was performed by Stelzried [1] of this calibration method, but he did not account for all of the errors due to mismatch. A correlation term (to be discussed later) was thought to be small and, therefore, was purposely omitted. A separate error analysis was also performed by Otoshi [2], but, for simplicity, he did not include any of the errors due to mismatch or correlation. As will be seen in this article, the mismatch equations are complex and cumbersome.

C. Accounting for the Effects of Mismatch

1. Delivered Antenna System Noise Temperature. If mismatch effects are accounted for, one still begins with the measured Y-factor equation given in Eq. (21) but, for the mismatched case, instead of the assumed noise-temperature relationships of Eq. (22), the following correct relationship is used:

$$Y = \frac{(P_{op,p})_d}{(P_{op,a})_d} = \frac{(T_{op,p})_d}{(T_{op,a})_d} \tag{26}$$

Manipulation of Eq. (26) yields the following correct relationship for the delivered antenna operating-system noise temperature:

$$(T_{op,a})_d = \frac{(T_{op,p})_d}{Y} \tag{27}$$

where

$$\begin{aligned}(T_{op,p})_d &= (273.16 + T_{pc})M_{pe} + (T_{e,p})_d \\ &= T_p M_{pe} + (T_{e,p})_d \end{aligned} \tag{28}$$

Note again that Y is a measured power ratio and is the same whether it is for the mismatched or assumed matched case. The main effect of mismatch on the value of $(T_{op,a})_d$ is from the mismatch factor M_{pe} in Eq. (28), where

$$M_{pe} = \frac{\left(1 - |\Gamma_p|^2\right)\left(1 - |\Gamma_e|^2\right)}{|1 - \Gamma_p \Gamma_e|^2} \tag{29}$$

and the expression for the delivered effective input noise temperature $(T_{e,p})_d$ for the ambient-load case was derived by Otoshi [3] to be

$$(T_{e,p})_d = \left(1 - |\Gamma_e|^2\right) T_{eo} + \frac{|\Gamma_p|^2 \left(1 - |\Gamma_e|^2\right)^2}{|1 - \Gamma_p \Gamma_e|^2} T_r + \frac{2\gamma_p |\Gamma_p| \left(1 - |\Gamma_e|^2\right)}{|1 - \Gamma_p \Gamma_e|} \sqrt{\left(1 - |\Gamma_e|^2\right) T_{eo} T_r} \quad (30)$$

where T_r is the noise temperature (generated by the receiver), which radiates towards the input termination connected to the receiver and gets reflected back towards the receiver. This reflected receiver term combines with the internal receiver term T_{eo} in some correlated manner that is accounted for by the γ_p factor in the last term of Eq. (30). Other symbols are defined in Table 1. The expression for delivered effective input noise temperature when the input termination is the antenna can be derived by simply replacing the subscript p with the subscript a in Eq. (30). However, as will be seen later, the expression for $(T_{e,a})_d$ is not needed for deriving $(T_{op,a})_a$ when using the ambient-load method.

Substitution of Eq. (30) into Eq. (28) and then the subsequent substitution of Eq. (28) into Eq. (27) gives

$$(T_{op,a})_d =$$

$$\frac{1}{Y}\left[M_{pe} T_p + \left(1 - |\Gamma_e|^2\right) T_{eo} + \frac{|\Gamma_p|^2 \left(1 - |\Gamma_e|^2\right)^2}{|1 - \Gamma_p \Gamma_e|^2} T_r + \frac{2\gamma_p |\Gamma_p| \left(1 - |\Gamma_e|^2\right)}{|1 - \Gamma_p \Gamma_e|} \sqrt{\left(1 - |\Gamma_e|^2\right) T_{eo} T_r}\right] \quad (31)$$

The mismatch error for delivered antenna system noise temperature will be defined as[2]

$$(\in_{mm})_d = T_{op,a'} - (T_{op,a})_d \quad (32)$$

where $T_{op,a'}$ was given in Eq. (23) as

$$T_{op,a'} = \frac{1}{Y}(T_{op,p'}) = \frac{1}{Y}(T_p + T_{e'})$$

Substitution of Eqs. (23) and (27) into Eq. (32) gives

$$(\in_{mm})_d = \frac{1}{Y}\left[(T_{op,p'}) - (T_{op,p})_d\right] \quad (33)$$

$$(\in_{mm})_d = T_{op,a'} - \frac{(T_{op,p})_d}{Y} \quad (34)$$

For this error analysis, the value to use for Y is given by Eq. (22) and is shown below as

$$Y = \frac{T_p + T_{e'}}{T_{op,a'}} = \frac{T_{op,p'}}{T_{op,a'}}$$

where $T_{e'}$ is assumed to be known and T_p and $T_{op,a'}$ are measured.

[2] In this article, the convention wherein the error term is the negative of the correction term will be used.

The reader is reminded that Y is a constant even though it would seem that if return losses (reflection coefficients) are changed, then the value of Y should also change. It becomes less confusing if the expression of Y is eliminated. If the equation of Y given above is substituted into Eq. (34), then

$$(\in_{mm})_d = T_{op,a'} \left[1 - \frac{(T_{op,p})_d}{T_{op,p'}}\right] \tag{35}$$

where $(T_{op,p})_d$ is given by Eq. (28), and the terms $T_{op,a'}$ and $T_{op,p'}$ are constants because $T_{op,a'} = T_{a'} + T_{e'}$ and $T_{op,p'} = T_p + T_{e'}$, and $T_{a'}$, $T_{e'}$, and T_p are known values.

Now the factor Y is eliminated entirely and, from Eqs. (28) through (30), it can be seen that only by changing the ambient-load and receiver reflection coefficients and the correlation coefficient can one effect a change in the value of $(T_{op,p})_d$. The reflection coefficient of the antenna does not appear. It can also be seen from Eqs. (28) through (30) that, if the reflection coefficients of the ambient load and receiver are reduced, $(T_{op,p})_d$ approaches $T_{op,p'}$, which will make the mismatch error for $(T_{op,a})_d$, as given by Eq. (35), go towards zero.

For the purpose of calculating mismatch errors, the form of Eq. (33) containing the multiplying factor $1/Y$ will be used because this was the form previously used by Stelzried [1] and Otoshi [3]. For more clarity, Eq. (35) might have been used instead. Substitution of Eqs. (23) and (28) into Eq. (33) and collection of terms gives the error equation

$$(\in_{mm})_d = \in_1 + \in_2 + \in_3 + \in_4 \tag{36}$$

where

$$\in_1 = \frac{1}{Y}\left[1 - \frac{\left(1 - |\Gamma_p|^2\right)\left(1 - |\Gamma_e|^2\right)}{|1 - \Gamma_p \Gamma_e|^2}\right] T_p \tag{37}$$

It can be shown (see Eq. (44) of [3]) that if T_{eo} is calibrated through the use of matched ambient and cryogenic loads, then $T_{eo} = T_{e'}$. Then substitution of $T_{e'}$ for T_{eo} gives

$$\in_2 = \frac{1}{Y} |\Gamma_e|^2 T_{e'} \tag{38}$$

$$\in_3 = -\frac{1}{Y} \frac{|\Gamma_p|^2 \left(1 - |\Gamma_e|^2\right)^2}{|1 - \Gamma_p \Gamma_e|^2} T_r \tag{39}$$

and

$$\in_4 = -\frac{2}{Y} \frac{\gamma_p |\Gamma_p|}{|1 - \Gamma_p \Gamma_e|} \left(1 - |\Gamma_e|^2\right) \sqrt{\left(1 - |\Gamma_e|^2\right) T_r T_{e'}} \tag{40}$$

The expression for Y is given in both Eqs. (22) and (26). However, the simpler expression for Y that will be used is Eq. (22), given as

$$Y = \frac{T_{op,p'}}{T_{op,a'}} = \frac{T_p + T_{e'}}{T_{op,a'}}$$

Note that in Eqs. (37) through (40) $\in_1$ is the term that involves T_p, $\in_2$ involves $T_{e'}$, $\in_3$ involves T_r, and $\in_4$ involves the correlation factor. When only magnitudes of reflection coefficients are known (and not phases), the mismatch errors will vary between maximum and minimum values.

For the denominator for the above expressions, let

$$D = |1 - \Gamma_p \Gamma_e| \tag{41}$$

Then, if $D = \max$,

$$|1 - \Gamma_p \Gamma_e| = 1 + |\Gamma_p \Gamma_e|$$

and if $D = \min$,

$$|1 - \Gamma_p \Gamma_e| = 1 - |\Gamma_p \Gamma_e|$$

Table 2 shows two possible cases for upper and lower bounds for delivered antenna system noise temperatures. These bounds were derived in terms of reflection coefficients from substitution of $D = \min$ or $D = \max$ into Eqs. (37), (39), and (40). The bounds were converted to VSWRs through the use of Eq. (10). It is required that known values be substituted into the expressions in Table 2. Then the MAX and MIN of all values in Cases 1 and 2 are reported as the worst-case mismatch error values. MAX and MIN are functions that are available in worksheet programs such as Excel.

2. Available Antenna System Noise Temperature. As was discussed earlier, the expression for available system noise temperature is obtained by dividing the delivered system noise temperature by the mismatch factor at the input port as follows:

$$(T_{op,a})_a = \frac{1}{M_{ae}} (T_{op,a})_d \tag{42}$$

Substitution of the expressions of M_{ae} from Eq. (13) and $(T_{op,a})_d$ from Eq. (31) into Eq. (42) gives

$$\begin{aligned}(T_{op,a})_a = \frac{1}{Y} \Bigg\{ &\left| \frac{1 - \Gamma_a \Gamma_e}{1 - \Gamma_p \Gamma_e} \right|^2 \frac{\left(1 - |\Gamma_p|^2\right)}{\left(1 - |\Gamma_a|^2\right)} T_p + \frac{|1 - \Gamma_a \Gamma_e|^2}{\left(1 - |\Gamma_a|^2\right)} T_{eo} \\ &+ |\Gamma_p|^2 \left| \frac{1 - \Gamma_a \Gamma_e}{1 - \Gamma_p \Gamma_e} \right|^2 \frac{\left(1 - |\Gamma_e|^2\right)}{\left(1 - |\Gamma_a|^2\right)} T_r + \frac{2\gamma_p |\Gamma_p|}{|1 - \Gamma_p \Gamma_e|} \frac{|1 - \Gamma_a \Gamma_e|^2}{\left(1 - |\Gamma_a|^2\right)} \sqrt{\left(1 - |\Gamma_e|^2\right) T_{eo} T_r} \Bigg\}\end{aligned} \tag{43}$$

where $T_{eo} = T_{e'}$.

Table 2. Mismatch error upper and lower bounds for delivered antenna system noise temperature.

	$(\in_{mm})_d = \in_1 + \in_2 + \in_3 + \in_4$ $Y = (T_p + T_{e'})/T_{op,a'}$	
Term	Reflection coefficient	VSWR
	Case 1, $D = \min$	
$\in_1$	$\frac{1}{Y}\left[1 - \frac{\left(1-\lvert\Gamma_e\rvert^2\right)\left(1-\lvert\Gamma_p\rvert^2\right)}{\left(1-\lvert\Gamma_p\Gamma_e\rvert\right)^2}\right]T_p$	$\frac{1}{Y}\left[1-\frac{4S_eS_p}{(S_p+S_e)^2}\right]T_p$
$\in_2$	$\frac{1}{Y}\lvert\Gamma_e\rvert^2 T_{e'}$	$\frac{1}{Y}\left(\frac{S_e-1}{S_e+1}\right)^2 T_{e'}$
$\in_3$	$-\frac{1}{Y}\frac{\lvert\Gamma_p\rvert^2\left(1-\lvert\Gamma_e\rvert^2\right)^2}{\left(1-\lvert\Gamma_p\Gamma_e\rvert\right)^2}T_r$	$-4\frac{1}{Y}\left(\frac{S_p-1}{S_p+S_e}\right)^2\left(\frac{S_e}{S_e+1}\right)^2 T_r$
$\in_4$	$\frac{-2\gamma_p\lvert\Gamma_p\rvert\left(1-\lvert\Gamma_e\rvert^2\right)}{Y\left(1-\lvert\Gamma_p\Gamma_e\rvert\right)}\sqrt{\left(1-\lvert\Gamma_e\rvert^2\right)T_rT_{e'}}$	$-8\gamma_p\frac{1}{Y}\left(\frac{S_p-1}{S_p+S_e}\right)\frac{S_e}{(S_e+1)^2}\sqrt{S_eT_rT_{e'}}$
	Case 2, $D = \max$	
$\in_1$	$\frac{1}{Y}\left[1 - \frac{\left(1-\lvert\Gamma_e\rvert^2\right)\left(1-\lvert\Gamma_p\rvert^2\right)}{\left(1+\lvert\Gamma_p\Gamma_e\rvert\right)^2}\right]T_p$	$\frac{1}{Y}\left[1-\frac{4S_eS_p}{(S_pS_e+1)^2}\right]T_p$
$\in_2$	$\frac{1}{Y}\lvert\Gamma_e\rvert^2 T_{e'}$	$\frac{1}{Y}\left(\frac{S_e-1}{S_e+1}\right)^2 T_{e'}$
$\in_3$	$\frac{-\lvert\Gamma_p\rvert^2\left(1-\lvert\Gamma_e\rvert^2\right)^2}{Y\left(1+\lvert\Gamma_p\Gamma_e\rvert\right)^2}T_r$	$-4\frac{1}{Y}\left(\frac{S_p-1}{S_pS_e+1}\right)^2\left(\frac{S_e}{S_e+1}\right)^2 T_r$
$\in_4$	$\frac{-2\gamma_p\lvert\Gamma_p\rvert\left(1-\lvert\Gamma_e\rvert^2\right)}{Y\left(1+\lvert\Gamma_p\Gamma_e\rvert\right)}\sqrt{\left(1-\lvert\Gamma_e\rvert^2\right)T_rT_{e'}}$	$-8\gamma_p\frac{1}{Y}\left(\frac{S_p-1}{S_pS_e+1}\right)\frac{S_e}{(S_e+1)^2}\sqrt{S_eT_rT_{e'}}$

Mismatch error for the available antenna system noise-temperature case is defined as

$$(\in_{mm})_a = T_{op,a'} - (T_{op,a})_a \tag{44}$$

where $T_{op,a'}$ is given by Eq. (23) and $(T_{op,a})_a$ is given in Eq. (43).

It is interesting to note that an alternate expression of $(\in_{mm})_a$ can be derived as follows. Substitution of Eq. (27) into Eq. (42) gives

$$(T_{op,a})_a = \frac{1}{Y}\,\frac{(T_{op,p})_d}{M_{ae}} \tag{45}$$

Then substitution of Eqs. (23) and (45) into Eq. (44) gives

$$(\in_{mm})_a = \frac{1}{Y}\left[(T_{op,p'}) - \frac{(T_{op,p})_d}{M_{ae}}\right] \tag{46}$$

Substitution of Eq. (22), which was given as

$$Y = \frac{T_p + T_{e'}}{T_{op,a'}} = \frac{T_{op,p'}}{T_{op,a'}}$$

into Eq. (46) gives

$$(\in_{mm})_a = T_{op,a'}\left[1 - \frac{(T_{op,p})_d}{(T_{op,p'})M_{ae}}\right] \tag{47}$$

Note that Eq. (47) for the available case is the same as Eq. (35) for the delivered case except for the presence of mismatch factor M_{ae}. Further examination of Eq. (47) reveals that even if the ambient load and receiver reflection coefficients are reduced to zero so that $(T_{op,p})_d$ becomes equal to $T_{op,p'}$, the error for the available case will not go to zero unless the antenna reflection coefficient is also reduced to zero, causing M_{ae} to have a value of 1.

Even though Eq. (47) can be used directly, the form containing the $1/Y$ factor will be used instead to enable direct comparisons to earlier derivations. Substitution of Eq. (43) and the previously derived expression [see Eq. (23)] of

$$T_{op,a'} = \frac{T_{op,p'}}{Y} = \frac{T_p + T_{e'}}{Y}$$

into Eq. (44) and collection of terms gives

$$(\in_{mm})_a = \in_5 + \in_6 + \in_7 + \in_8 \tag{48}$$

where $(\in_{mm})_a$ is the mismatch error for available antenna system noise temperature and

$$\in_5 = \frac{1}{Y}\left[1 - \left|\frac{1-\Gamma_a\Gamma_e}{1-\Gamma_p\Gamma_e}\right|^2 \frac{\left(1-|\Gamma_p|^2\right)}{\left(1-|\Gamma_a|^2\right)}\right] T_p \tag{49}$$

$$\in_6 = \frac{1}{Y}\left[1 - \frac{|1-\Gamma_a\Gamma_e|^2}{\left(1-|\Gamma_a|^2\right)}\right] T_{e'} \tag{50}$$

$$\in_7 = -\frac{1}{Y}|\Gamma_p|^2 \left|\frac{1-\Gamma_a\Gamma_e}{1-\Gamma_p\Gamma_e}\right|^2 \frac{\left(1-|\Gamma_e|^2\right)}{\left(1-|\Gamma_a|^2\right)} T_r \tag{51}$$

$$\in_8 = -\frac{2}{Y}\gamma_p \frac{|\Gamma_p|}{|1-\Gamma_p\Gamma_e|} \frac{|1-\Gamma_a\Gamma_e|^2}{\left(1-|\Gamma_a|^2\right)} \sqrt{\left(1-|\Gamma_e|^2\right) T_r T_{e'}} \tag{52}$$

Note that $\in_5$ is the term that involves T_p, $\in_6$ involves $T_{e'}$, $\in_7$ involves T_r, and $\in_8$ involves the correlation coefficient γ_p. All $\in$ terms have a multiplying factor of $1/Y$ in front to make the equations of the same form previously used by Stelzried [1]. The form of error equation is useful because it makes it possible to see the individual noise source contributions. As stated previously, when only magnitudes of reflection coefficients are known (and not phases), the mismatch errors will vary between maximum and minimum values.

Let

$$N = |1 - \Gamma_a \Gamma_e| \tag{53}$$

and the expression for D was given in Eq. (41) as

$$D = |1 - \Gamma_p \Gamma_e|$$

If the phases of reflection coefficients Γ_a, Γ_e, and Γ_p are not known, then only the worst-case mismatch errors can be determined for four different cases when N and D are maximums or minimums, as follows:

$$\begin{aligned}
\text{If } N = \text{max}, &\quad |1 - \Gamma_a \Gamma_e| = 1 + |\Gamma_a \Gamma_e| \\
\text{If } N = \text{min}, &\quad |1 - \Gamma_a \Gamma_e| = 1 - |\Gamma_a \Gamma_e| \\
\text{If } D = \text{max}, &\quad |1 - \Gamma_p \Gamma_e| = 1 + |\Gamma_p \Gamma_e| \\
\text{If } D = \text{min}, &\quad |1 - \Gamma_p \Gamma_e| = 1 - |\Gamma_p \Gamma_e|
\end{aligned}$$

Substitution of these bounds into Eqs. (49) through (52) results in four possible cases for upper and lower bounds for available antenna system noise temperature (see Table 3). These bounds were first derived in terms of reflection coefficients and then converted to VSWRs through the use of Eq. (10). The values in Table 3 are computed, and then the MAX and MIN of all values of Cases 1 through 4 are reported as the worst-case maximum and minimum mismatch error values, respectively.

The equations given by Stelzried in [1] are for *available* system noise temperature, but they did not include terms involving T_r. His mismatch equations were given in terms of VSWR. After substitution of values, he chose the largest error value found in four cases similar to those in Table 3 as the worst-case error. He treats the mismatch errors as random probable errors and therefore used a 1/5 multiplying factor for converting the worst-case (3-sigma) errors into probable errors. In this article, the worst-case mismatch errors are not assumed to be random errors, but instead are assumed to be bias errors. Hence, the multiplying factor used in this article is 1 instead of 1/5.

IV. Antenna Efficiency Measurements

In the previous section, it was shown how mismatches affected deviations of delivered and available operating-system noise-temperature values from the assumed matched-case values. It will now be shown how mismatches similarly cause deviations of delivered and available antenna efficiencies from assumed matched-case values.

A. Delivered Antenna Efficiency

The measurement of antenna efficiency involves pointing the antenna "on" and "off" the peak of a radio source and measuring $(T_{op,a})_d$ at each position. The "on" measurement is made at the elevation

Table 3. Mismatch error upper and lower bounds for available antenna system noise temperature.

$$(\in_{mm})_a = \in_5 + \in_6 + \in_7 + \in_8$$
$$Y = (T_p + T_{e'})/T_{op,a'}$$

Term	Reflection coefficient	VSWR
Case 1, $N = \max, D = \max$		
$\in_5$	$\frac{1}{Y}\left[1-\frac{\left(1-\lvert\Gamma_p\rvert^2\right)}{\left(1-\lvert\Gamma_a\rvert^2\right)}\left(\frac{1+\lvert\Gamma_a\Gamma_e\rvert}{1+\lvert\Gamma_p\Gamma_e\rvert}\right)^2\right]T_p$	$\frac{1}{Y}\left[1-\left(\frac{S_p}{S_a}\right)\left(\frac{S_aS_e+1}{S_pS_e+1}\right)^2\right]T_p$
$\in_6$	$\frac{1}{Y}\left[1-\frac{(1+\lvert\Gamma_a\Gamma_e\rvert)^2}{\left(1-\lvert\Gamma_a\rvert^2\right)}\right]T_{e'}$	$\frac{1}{Y}\left[1-\frac{1}{S_a}\left(\frac{S_aS_e+1}{S_e+1}\right)^2\right]T_{e'}$
$\in_7$	$-\frac{1}{Y}\left(\frac{1+\lvert\Gamma_a\Gamma_e\rvert}{1+\lvert\Gamma_p\Gamma_e\rvert}\right)^2\frac{\lvert\Gamma_p\rvert^2\left(1-\lvert\Gamma_e\rvert^2\right)}{\left(1-\lvert\Gamma_a\rvert^2\right)}T_r$	$-\frac{1}{Y}\left[\frac{S_e}{S_a}\left(\frac{S_aS_e+1}{S_e+1}\right)^2\left(\frac{S_p-1}{S_pS_e+1}\right)^2\right]T_r$
$\in_8$	$-2\gamma_p\frac{1}{Y}\frac{(1+\lvert\Gamma_a\Gamma_e\rvert)^2\lvert\Gamma_p\rvert}{(1+\lvert\Gamma_p\Gamma_e\rvert)\left(1-\lvert\Gamma_a\rvert^2\right)}\sqrt{\left(1-\lvert\Gamma_e\rvert^2\right)T_rT_{e'}}$	$-2\gamma_p\frac{1}{Y}\left[\frac{1}{S_a}\left(\frac{S_aS_e+1}{S_e+1}\right)^2\left(\frac{S_p-1}{S_pS_e+1}\right)\right]\sqrt{S_eT_rT_{e'}}$
Case 2, $N = \min, D = \max$		
$\in_5$	$\frac{1}{Y}\left[1-\frac{\left(1-\lvert\Gamma_p\rvert^2\right)}{\left(1-\lvert\Gamma_a\rvert^2\right)}\left(\frac{1-\lvert\Gamma_a\Gamma_e\rvert}{1+\lvert\Gamma_p\Gamma_e\rvert}\right)^2\right]T_p$	$\frac{1}{Y}\left[1-\left(\frac{S_p}{S_a}\right)\left(\frac{S_a+S_e}{S_pS_e+1}\right)^2\right]T_p$
$\in_6$	$\frac{1}{Y}\left[1-\frac{(1-\lvert\Gamma_a\Gamma_e\rvert)^2}{\left(1-\lvert\Gamma_a\rvert^2\right)}\right]T_{e'}$	$\frac{1}{Y}\left[1-\frac{1}{S_a}\left(\frac{S_a+S_e}{S_e+1}\right)^2\right]T_{e'}$
$\in_7$	$-\frac{1}{Y}\left(\frac{1-\lvert\Gamma_a\Gamma_e\rvert}{1+\lvert\Gamma_p\Gamma_e\rvert}\right)^2\frac{\lvert\Gamma_p\rvert^2\left(1-\lvert\Gamma_e\rvert^2\right)}{\left(1-\lvert\Gamma_a\rvert^2\right)}T_r$	$-\frac{1}{Y}\left[\frac{S_e}{S_a}\left(\frac{S_a+S_e}{S_e+1}\right)^2\left(\frac{S_p-1}{S_pS_e+1}\right)^2\right]T_r$
$\in_8$	$-2\gamma_p\frac{1}{Y}\frac{(1-\lvert\Gamma_a\Gamma_e\rvert)^2\lvert\Gamma_p\rvert}{(1+\lvert\Gamma_p\Gamma_e\rvert)\left(1-\lvert\Gamma_a\rvert^2\right)}\sqrt{\left(1-\lvert\Gamma_e\rvert^2\right)T_rT_{e'}}$	$-2\gamma_p\frac{1}{Y}\left[\frac{1}{S_a}\left(\frac{S_a+S_e}{S_e+1}\right)^2\left(\frac{S_p-1}{S_pS_e+1}\right)\right]\sqrt{S_eT_rT_{e'}}$
Case 3, $N = \max, D = \min$		
$\in_5$	$\frac{1}{Y}\left[1-\frac{\left(1-\lvert\Gamma_p\rvert^2\right)}{\left(1-\lvert\Gamma_a\rvert^2\right)}\left(\frac{1+\lvert\Gamma_a\Gamma_e\rvert}{1-\lvert\Gamma_p\Gamma_e\rvert}\right)^2\right]T_p$	$\frac{1}{Y}\left[1-\left(\frac{S_p}{S_a}\right)\left(\frac{S_aS_e+1}{S_p+S_e}\right)^2\right]T_p$
$\in_6$	$\frac{1}{Y}\left[1-\frac{(1+\lvert\Gamma_a\Gamma_e\rvert)^2}{\left(1-\lvert\Gamma_a\rvert^2\right)}\right]T_{e'}$	$\frac{1}{Y}\left[1-\frac{1}{S_a}\left(\frac{S_aS_e+1}{S_e+1}\right)^2\right]T_{e'}$
$\in_7$	$-\frac{1}{Y}\left(\frac{1+\lvert\Gamma_a\Gamma_e\rvert}{1-\lvert\Gamma_p\Gamma_e\rvert}\right)^2\frac{\lvert\Gamma_p\rvert^2\left(1-\lvert\Gamma_e\rvert^2\right)}{\left(1-\lvert\Gamma_a\rvert^2\right)}T_r$	$-\frac{1}{Y}\left[\frac{S_e}{S_a}\left(\frac{S_aS_e+1}{S_e+1}\right)^2\left(\frac{S_p-1}{S_p+S_e}\right)^2\right]T_r$
$\in_8$	$-2\gamma_p\frac{1}{Y}\frac{(1+\lvert\Gamma_a\Gamma_e\rvert)^2}{(1-\lvert\Gamma_p\Gamma_e\rvert)}\frac{\lvert\Gamma_p\rvert}{\left(1-\lvert\Gamma_a\rvert^2\right)}\sqrt{\left(1-\lvert\Gamma_e\rvert^2\right)T_rT_{e'}}$	$-2\gamma_p\frac{1}{Y}\left[\frac{1}{S_a}\left(\frac{S_aS_e+1}{S_e+1}\right)^2\left(\frac{S_p-1}{S_p+S_e}\right)\right]\sqrt{S_eT_rT_{e'}}$

Table 3 (contd).

	$(\in_{mm})_a = \in_5 + \in_6 + \in_7 + \in_8$ $Y = (T_p + T_{e'})/T_{op,a'}$	
Term	Reflection coefficient	VSWR
	Case 4, $N = \min, D = \min$	
$\in_5$	$\frac{1}{Y}\left[1 - \frac{\left(1-\lvert\Gamma_p\rvert^2\right)}{\left(1-\lvert\Gamma_a\rvert^2\right)}\left(\frac{1-\lvert\Gamma_a\Gamma_e\rvert}{1-\lvert\Gamma_p\Gamma_e\rvert}\right)^2\right]T_p$	$\frac{1}{Y}\left[1-\left(\frac{S_p}{S_a}\right)\left(\frac{S_a+S_e}{S_p+S_e}\right)^2\right]T_p$
$\in_6$	$\frac{1}{Y}\left[1-\frac{(1-\lvert\Gamma_a\Gamma_e\rvert)^2}{\left(1-\lvert\Gamma_a\rvert^2\right)}\right]T_{e'}$	$\frac{1}{Y}\left[1-\frac{1}{S_a}\left(\frac{S_a+S_e}{S_e+1}\right)^2\right]T_{e'}$
$\in_7$	$-\frac{1}{Y}\left(\frac{1-\lvert\Gamma_a\Gamma_e\rvert}{1-\lvert\Gamma_p\Gamma_e\rvert}\right)^2\frac{\lvert\Gamma_p\rvert^2\left(1-\lvert\Gamma_e\rvert^2\right)}{\left(1-\lvert\Gamma_a\rvert^2\right)}T_r$	$-\frac{1}{Y}\left[\frac{S_e}{S_a}\left(\frac{S_a+S_e}{S_e+1}\right)^2\left(\frac{S_p-1}{S_p+S_e}\right)^2\right]T_r$
$\in_8$	$-2\gamma_p\frac{1}{Y}\frac{(1-\lvert\Gamma_a\Gamma_e\rvert)^2}{(1-\lvert\Gamma_p\Gamma_e\rvert)}\frac{\lvert\Gamma_p\rvert}{\left(1-\lvert\Gamma_a\rvert^2\right)}\sqrt{\left(1-\lvert\Gamma_e\rvert^2\right)T_rT_{e'}}$	$-2\gamma_p\frac{1}{Y}\left[\frac{1}{S_a}\left(\frac{S_a+S_e}{S_e+1}\right)^2\left(\frac{S_p-1}{S_p+S_e}\right)\right]\sqrt{S_eT_rT_{e'}}$

angle corresponding to the peak of the Gaussian-shaped radio source noise-temperature curve, and the "off" measurement is usually made at an elevation angle at least 5 half-power beam widths away from the peak of a point radio source. A more sophisticated procedure is to measure $(T_{op,a})_d$ at 5 points of the Gaussian-shaped radio source curve and then to perform a least-squares curve fit to the measured T_{op} values to obtain improved on-source and off-source values. For this mismatch error analysis, it is permissible to express either procedure mathematically as

$$(T_s)_{\text{meas}} = \left[(T_{op,a})_d\right]_{\text{on}} - \left[(T_{op,a})_d\right]_{\text{off}} \tag{54}$$

Although not shown explicitly, all of the above T_{op} values are functions of antenna elevation angle. From Eq. (26), we may write

$$Y_{\text{on}} = \frac{(T_{op,p})_d}{\left[(T_{op,a})_d\right]_{\text{on}}}$$

and

$$Y_{\text{off}} = \frac{(T_{op,p})_d}{\left[(T_{op,a})_d\right]_{\text{off}}}$$

Manipulation and substitution of these two equations into Eq. (54) give

$$(T_s)_{\text{meas}} = \left(\frac{1}{Y_{\text{on}}} - \frac{1}{Y_{\text{off}}}\right) \times (T_{op,p})_d \tag{55}$$

In terms of the assumed matched-case values,

$$Y_{\text{on}} = \frac{T_p + T_{e'}}{T_{a'} + T_{s'} + T_{e'}} \tag{56}$$

and

$$Y_{\text{off}} = \frac{T_p + T_{e'}}{T_{a'} + T_{e'}} \tag{57}$$

where $T_{s'}$ is the assumed matched-case value of the radio source noise temperature at elevation angle ψ, and $T_{a'}$ is the antenna temperature with no radio source present at elevation angle ψ.

Then,

$$\left(\frac{1}{Y_{\text{on}}} - \frac{1}{Y_{\text{off}}}\right) = \frac{T_{s'}}{T_p + T_{e'}} \tag{58}$$

and substitution of Eq. (58) into Eq. (55) gives

$$(T_s)_{\text{meas}} = \frac{T_{s'}}{T_p + T_{e'}}(T_{op,p})_d = T_{s'}\frac{(T_{op,p})_d}{T_{op,p'}} \tag{59}$$

Note that $T_{op,p'}$ and $(T_{op,p})_d$ are, respectively, the delivered operating-system noise temperatures for the assumed match and actual mismatched cases when the input termination is the ambient load. The value of $T_{op,p'}$ is a constant while, as may be seen from Eq. (28), that of $(T_{op,p})_d$ is a variable as a function of the ambient-load and receiver reflection coefficients. The mismatch error on the measurement of delivered source noise temperature is defined as

$$\in_{mm}[(T_s)_d] = T_{s'} - (T_s)_{\text{meas}} = T_{s'}\left[1 - \frac{(T_{op,p})_d}{T_{op,p'}}\right] \tag{60}$$

Division of Eq. (60) by Eq. (35) leads to

$$\in_{mm}[(T_s)_d] = \left(\frac{T_{s'}}{T_{op,a'}}\right)(\in_{mm})_d \tag{61}$$

where $(\in_{mm})_d$ is the mismatch error for delivered antenna operating noise temperature.

It can be seen that all of the maximum and minimum expressions already derived for $(\in_{mm})_d$ can be used. It is only necessary to multiply those expressions in Table 2 by the ratio $(T_{s'}/T_{op,a'})$.

The measured antenna efficiency is calculated from

$$\eta = \frac{(T_s)_{\text{meas}}}{T_{100}} \tag{62}$$

where T_{100} is the radio source noise temperature that would be measured if the antenna were perfect (i.e., it had no mismatches and no resistive losses) and, therefore, would have an antenna efficiency of 100 percent [9]. The value of T_{100} for some radio sources at 8.42 GHz and 32 GHz may be found in [9].

Substitution of Eq. (59) into Eq. (62) gives the expression for delivered antenna efficiency η_d as shown below:

$$\eta_d = \frac{T_{s'}}{T_{100}} \frac{(T_{op,p})_d}{T_{op',p}} = \eta' \frac{(T_{op,p})_d}{Top',p} \tag{63}$$

where η' is the antenna efficiency for the assumed matched case. The mismatch error for delivered antenna efficiency is

$$\in_{mm} (\eta_d) = \eta' - \eta_d = \eta' \left[1 - \frac{(T_{op,p})_d}{T_{op',p}} \right] \tag{64}$$

Division of Eq. (64) by Eq. (35) leads to

$$\in_{mm} (\eta_d) = \left(\frac{\eta'}{T_{op,a'}} \right) (\in_{mm})_d \tag{65}$$

Again, it can be seen that the mismatch errors for delivered antenna efficiency can be obtained by simply multiplying the worst-case errors given in Table 2 by the ratio $\eta'/T_{op,a'}$.

B. Available Antenna Efficiency

First the expression for available radio source noise temperature is obtained by dividing the expression for delivered (or measured) radio source noise temperature by the mismatch factor M_{ae} as follows:

$$(T_s)_a = \frac{(T_s)_{\text{meas}}}{M_{ae}} \tag{66}$$

Substitution of Eq. (59) into Eq. (66) gives

$$(T_s)_a = \frac{T_{s'}}{M_{ae}} \frac{(T_{op,p})_d}{T_{op,p'}} \tag{67}$$

Following the procedure of Eqs. (60) and (61), it can be shown that

$$\in_{mm} \left[(T_s)_a \right] = \frac{T_{s'}}{T_{op,a'}} (\in_{mm})_a \tag{68}$$

and values of maximum and minimum $(\in_{mm})_a$ are given in Table 3.

Available antenna efficiency is obtained by dividing the available source noise temperature by T_{100}, resulting in the expression

$$\eta_a = \frac{T_{s'}}{T_{100}} \left[\frac{1}{M_{ae}} \frac{(T_{op,p})_d}{T_{op,p'}} \right]$$

$$= \eta' \left[\frac{1}{M_{ae}} \frac{T_{op,p})_d}{T_{op,p'}} \right] \tag{69}$$

Mismatch error for available antenna efficiency is defined as

$$\in_{mm}(\eta_a) = \eta' - \eta_a \tag{70}$$

Substitution of Eq. (69) into Eq. (70) gives

$$\in_{mm}(\eta_a) = \eta' \left[1 - \frac{1}{M_{ae}} \frac{(T_{op,p})_d}{T_{op,p'}}\right] \tag{71}$$

Division of Eq. (71) by Eq. (47) leads to

$$\in_{mm}(\eta_a) = \frac{\eta'}{T_{op,a'}} (\in_{mm})_a \tag{72}$$

where $(\in_{mm})_a$ is the mismatch error for available antenna operating noise temperature derived in Section III. Now it can be seen the mismatch error for *available* antenna efficiency is simply the mismatch error for *available* antenna operating noise temperature multiplied by the ratio $(\eta'/T_{op,a'})$. The expressions for maximum and minimum $(\in_{mm})_a$ given in Table 3 are applicable.

V. Applications

A. Sample-Case Input Parameters

As was previously discussed, for delivered T_{op}, the mismatch errors are caused by the mismatch between the ambient load and LNA only. For available T_{op}, the errors are not only caused by mismatch between ambient load and the LNA receiving system, but also between the antenna and the LNA.

For a new DSN X-band feed system, maximum and minimum delivered and available $T_{op,a}$ values will be shown as functions of the return losses of (1) the antenna only, (2) the ambient load only, (3) the LNA only, and (4) the ambient load and LNA. While these return losses are varied, the other nominal return losses are fixed.

The nominal values for parameters of the new X-band (8.45-GHz) feed system are as follows:

$$\begin{aligned}
T_{op,a'} &= 13.7 \text{ K (zenith value)} \\
T_{\text{LNA}} &= 4.9 \text{ K} \\
T_{FU} &= 0.1 \text{ K} \\
T_{e'} &= T_{\text{LNA}} + T_{FU} = 5 \text{ K} \\
T_p &= 295 \text{ K} \\
T_r &= 6 \text{ K} \\
T_{a'} &= T_{op,a'} - T_{e'} = 8.7 \text{ K (zenith value)}
\end{aligned}$$

It should be stated that the correlation coefficient for this receiver is zero, because there is a cooled isolator in front of the LNA for improved matching purposes.[3] This isolator is cooled to a 6-K physical

[3] J. Fernandez, personal communication, Communications Ground Systems Section, Jet Propulsion Laboratory, Pasadena, California, November 2001.

temperature and hence T_r is assumed to be equal to 6 K. The nominal return losses of this feed system and calibration system are as follows:

$$\text{return loss ambient load} = -35 \text{ dB}$$
$$\text{return loss LNA} = -27 \text{ dB}$$
$$\text{return loss of feed horn} = -20 \text{ dB}$$

The return loss of the receiver is equal to the return loss of the LNA only because it will be assumed that any reflections from the follow-up receiver will be absorbed by the LNA (S_{12} approximately equal to 0). Return loss in dB is calculated from the relationship

$$RL_x = 20 \times \log_{10} |\Gamma_x| \tag{73}$$

Mismatch errors will be plotted as a function of return loss in dB rather than reflection coefficient magnitude or VSWR because return loss is the network analyzer output quantity that is most convenient to plot. Conversion of return loss in dB to magnitude of reflection coefficient can be made by the use of

$$|\Gamma_x| = 10^{RL_x/20} \tag{74}$$

and conversion of $|\Gamma_x|$ to VSWR can be made by the use of Eq. (10). For quick reference purposes, plots of RL_x versus $|\Gamma_x|$ and RL_x versus VSWR are given in the Appendix.

B. Sample-Case Antenna Operating-System Noise Temperature

Two methods for displaying the effects of mismatch are to (1) plot mismatch error limits as a function of return losses using the formulas in Tables 2 and 3 or (2) plot the maximum and minimum $T_{op,a}$ values as a function of return losses and let the spread between the maximum and minimum values represent the total uncertainties due to mismatch errors. For the plots presented in this article, Method 2 will be used. Each figure has two parts, (a) and (b). Part (a) is for the case where the correlation coefficient (CC) is equal to zero, and Part (b) is for the case where $CC = \pm 1$ and only the deviations from the $CC = 0$ case are plotted. As the values of all return losses (RL) become increasingly more negative dB, the value of the delivered or available antenna T_{op} converge towards the assumed matched-case antenna T_{op} value of 13.7 K. This characteristic will become clear in the plots to be presented. For convenience, if not specified, the symbol T_{op} refers to antenna system noise temperature. In all of the following plots, the MAX and MIN values were found using Excel worksheets and then plotted for each new return loss value.

First, it is of interest to calculate mismatch factors for the sample case. Figure 4 shows a plot of the mismatch factors M_{ae} at the antenna–LNA interface. The LNA return loss is held constant, but the antenna return loss is varied from -10 dB to -40 dB. It can be seen that the mismatch factor approaches unity and the uncertainty (represented by the spread between the maximum and minimum curve) gets smaller as the antenna is tuned towards the -40-dB return loss.

1. Delivered Antenna System Noise-Temperature Plots. Figure 5(a) shows the limits of *delivered* antenna T_{op} as a function of ambient-load return losses for the case of correlation coefficient $CC = 0$. The LNA return loss is held constant at its nominal value of -27 dB while the ambient-load return loss is allowed to vary. As the ambient-load return loss goes from -10 dB towards -40 dB, the *delivered* antenna T_{op} approaches the assumed matched-case antenna T_{op} value of 13.7 K. These curves show how mismatch errors affect the delivered T_{op} values as the ambient-load return loss moves away from its nominal value of -35 dB. Figure 5(b) shows delta $(T_{op,a})_d$ for the $CC = \pm 1$ cases. This plot

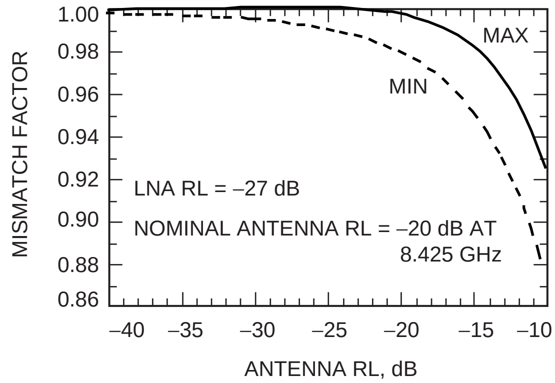

Fig. 4. Mismatch factor M_{ae} at the antenna–LNA interface as a function of antenna RL.

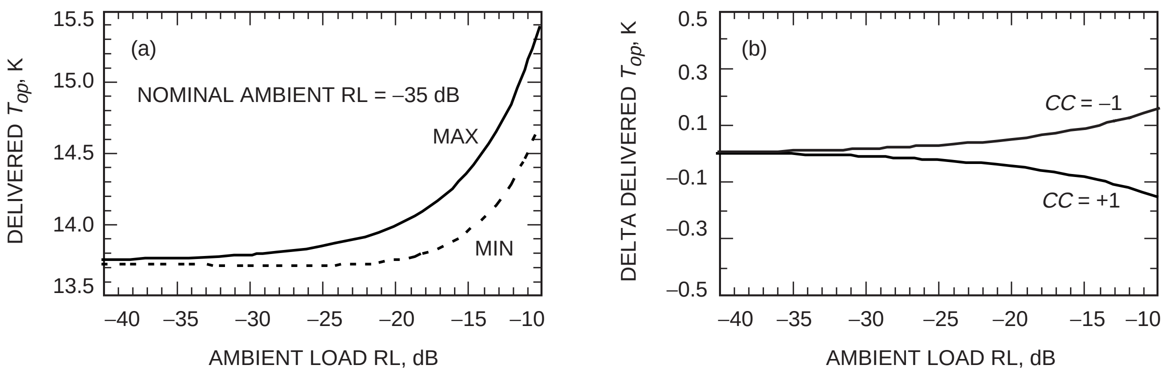

Fig. 5. *Delivered* antenna T_{op} versus *ambient- load* RL: (a) maximum and minimum delivered values when *CC* = 0 and all other nominal parameters are kept constant and (b) differences of T_{op} due to *CC* = ±1 instead of 0. Add the delta T_{op} values of the *CC* = –1 curve to the MAX curve of Fig. 5(a), and the delta T_{op} values of the *CC* = +1 curve to the MIN curve of Fig. 5(a).

shows the effects of correlation coefficient γ_p only. To get the maximum $(T_{op,a})_d$ for the $CC = -1$ case, add the positive deltas for the $CC = -1$ case to the maximum delivered antenna T_{op} curve shown in Fig. 5(a). To get the minimum delivered T_{op} curve for the $CC = +1$ case, add the negative deltas of the $CC = +1$ case to the minimum delivered antenna T_{op} curve shown in Fig. 5(a). The reader is reminded that, for the DSN X-band feed system, the correlation coefficients are zero, and the $CC = +1$ and $CC = -1$ contributions are really zero but are shown only for informational purposes to see what the effects would be if the correlation coefficients were really ± 1.

Figures 6(a) and 6(b) show plots similar to those in Fig. 5(a) and 5(b) except that the x-axis is LNA return loss rather than ambient-load return loss. The nominal LNA return loss of the LNA is -27 dB. The ambient-load return loss value is held constant at its nominal value of -35 dB, and only the LNA return loss values are changed. As the LNA return loss goes from -10 dB towards -40 dB, the delivered antenna operating-system noise temperature approaches the assumed matched-case T_{op} value of 13.7 K.

Figures 7(a) and 7(b) show how much larger the spread is between maximum and minimum values when both ambient-load and LNA return losses are allowed to vary simultaneously. The x-axes of Figs. 6(a) and 6(b) are labeled return loss of ambient load or LNA. This means that the ambient load and LNA each has the same return loss value simultaneously. As in the previous plots, as both return losses are varied from -10 dB to -40 dB and as they approach -40 dB, the delivered antenna T_{op} approaches

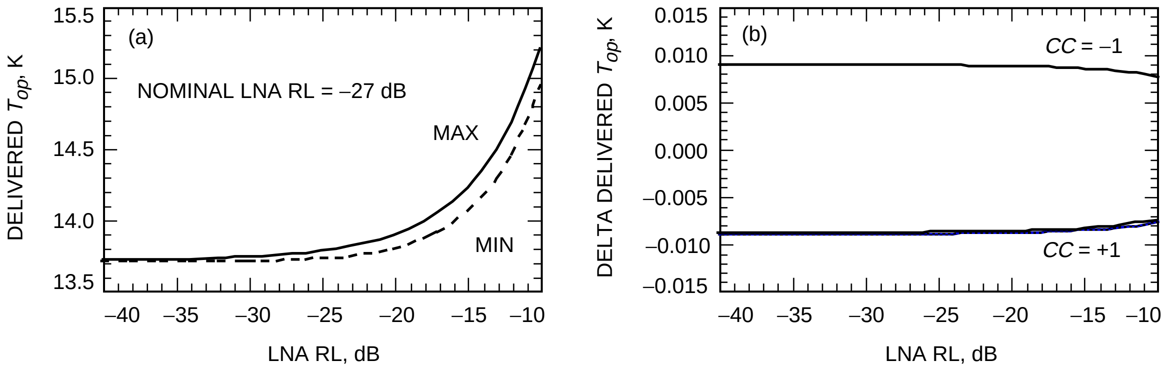

Fig. 6. ***Delivered*** **antenna** T_{op} **versus** ***LNA*** **RL: (a) maximum and minimum delivered values when** CC **= 0 and all other nominal parameters are kept constant and (b) changes of delivered antenna** T_{op} **when** CC **= ±1 instead of 0. Add the delta** T_{op} **values of the** CC **= –1 curve to the MAX curve of Fig. 6(a), and the delta** T_{op} **values of the** CC **= +1 curve to the MIN curve of Fig. 6(a).**

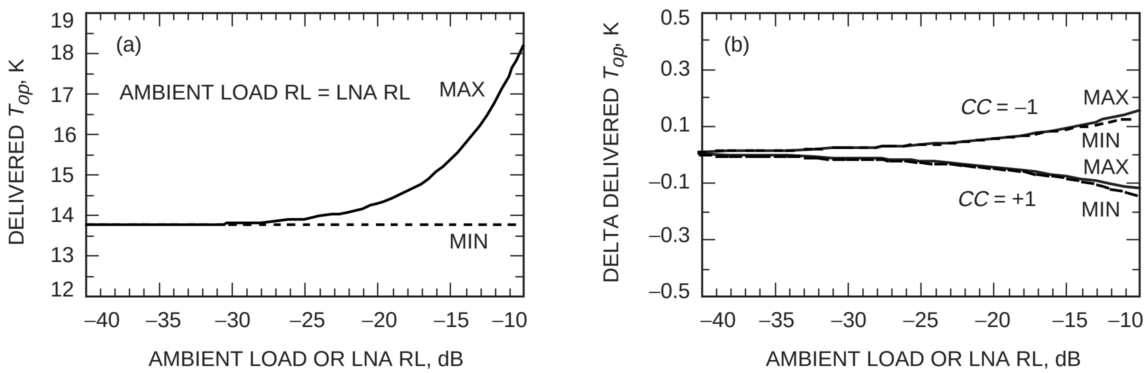

Fig. 7. ***Delivered*** **antenna** T_{op} **versus RL of both the** ***ambient load*** **and the** ***LNA*** **assuming both RLs have the same values and change at the same time: (a) maximum and minimum delivered values when** CC **= 0 and all other nominal parameters are kept constant and (b) changes of** T_{op} **due to** CC **= ±1 instead of 0. Add the delta** T_{op} **MAX values of the** CC **= –1 curve to the MAX curve of Fig. 7(a), and the delta** T_{op} **MIN values of the** CC **= +1 curve to the MIN curve of Fig. 7(a).**

the assumed matched-case T_{op} value of 13.7 K. Note that the limits are much larger than those of either Fig. 5(a) or Fig. 6(a). Also note that, for these curves in Fig. 7(b), the individual curves for the $CC = -1$ and $CC = +1$ cases have their own maximum and minimum values.

2. Available Antenna System Noise-Temperature Plots. Figure 8(a) plots are equivalent to those of Fig. 5(a) except that the y-axis is *available* rather than delivered antenna system noise temperatures when $CC = 0$. The nominal values of a -27-dB return loss of the LNA and a -20-dB return loss of the antenna are held constant as the return loss of the ambient load is varied from -10 dB to -40 db. For this system, the nominal value of the ambient load is -35 dB. Note that, as the ambient load goes towards -40 dB, there is still a spread of about 0.3 K at the ambient-load return loss of -40 dB. Comparison to Fig. 5 curves shows that the spread between maximum and minimum *available* system noise temperatures is significantly larger that that for *delivered* system noise temperatures and swings in a downward instead of upward direction as ambient return loss gets smaller towards -10 dB. These differences in characteristics are attributed to the fact that the values of $(T_{op,a})_a$ are affected by the additional mismatch factor M_{ae} while $(T_{op,a})_d$ is not explicitly a function of this mismatch factor. Figure 8(b) shows the differences from the $CC = 0$ case when the CC values are $+1$ and -1.

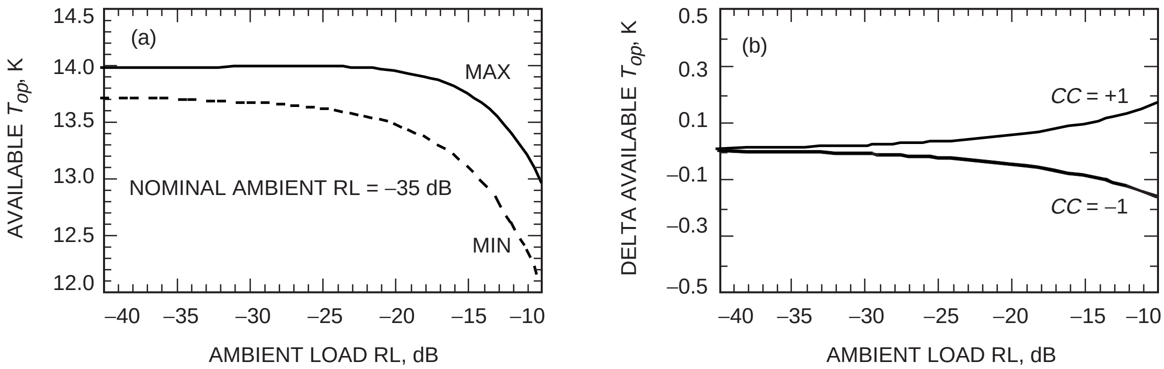

Fig. 8. ***Available* antenna T_{op} as a function of ambient-load RL: (a) maximum and minimum available values when CC = 0 and all other nominal parameters are kept constant and (b) differences when CC = ±1 instead of 0. Add the delta T_{op} values of the CC = +1 curve to the MAX curve of Fig. 8(a), and the delta T_{op} values of the CC = –1 curve to the MIN curve of Fig. 8(a).**

Figure 9(a) is a plot of *available* antenna T_{op} as a function of LNA return loss for the $CC = 0$ case. The nominal ambient-load and antenna return losses of -35 dB and -20 dB, respectively, are held constant as the LNA return loss is varied from -10 dB to -40 dB. As the return loss of the LNA approaches -40 dB, the available antenna Top approaches the apparent match case value of 13.7 K. Figure 9(b) shows the difference resulting from subtracting the available antenna T_{op} for the $CC = 0$ case from those for the $CC = \pm 1$ cases.

Figure 10(a) gives a plot of *available* antenna T_{op} as a function of antenna return loss. The ambient-load and LNA return losses of -35 dB and -27 dB, respectively, are held constant as the antenna return loss is varied from -10 dB to -40 dB. As the return loss of the antenna approaches -40 dB, the available antenna T_{op} approaches the apparent match case value of 13.7 K. Figure 10(b) shows the difference resulting from subtracting the available antenna T_{op} for the $CC = 0$ case from those for the $CC = \pm 1$ cases. The plots show that even though the antenna return loss is tuned to about -40 dB, some residual uncertainties remain. This residual amount is due to the errors introduced from the residual mismatch factor between the ambient load and the LNA.

Figure 11(a) shows the limits of *available* antenna T_{op} for the $CC = 0$ case when the antenna and LNA each has the same return loss at the same time and their return loss values are allowed to vary from -10 to -40 dB. Figure 11(b) shows the differences of available T_{op} due to subtracting T_{op} for the $CC = 0$ case in Fig. 11(a) from T_{op} of the $CC = \pm 1$ cases.

It is noted that the uncertainties of available antenna T_{op} are larger than those for delivered antenna T_{op} for the same return losses in the above-mentioned cases. The cause of the larger uncertainties is the additional involvement of the max and min of the M_{ae} mismatch factor for available antenna T_{op} but not for delivered antenna T_{op} determinations. Correlation effects are small for delivered antenna T_{op} cases but can become significant for the available antenna T_{op} cases. As the return losses become -40 dB or less, both the available and delivered antenna T_{op} values approach the assumed matched-case value of 13.7 K.

C. Sample-Case Antenna Efficiency Errors

It is possible to plot the curves for antenna efficiency errors, but additional plots would make this article unnecessarily long. One can subtract the example value of 13.7 K for $T_{op,a'}$ from all the curves and multiply the scales by the ratio of $T_{s'}/T_{op,a'}$ for errors on measured and available radio source noise temperatures, and by the ratio $\eta'/T_{op,a'}$ for antenna efficiencies. However, these procedures would lead to resolutions of extracted data that would be very poor.

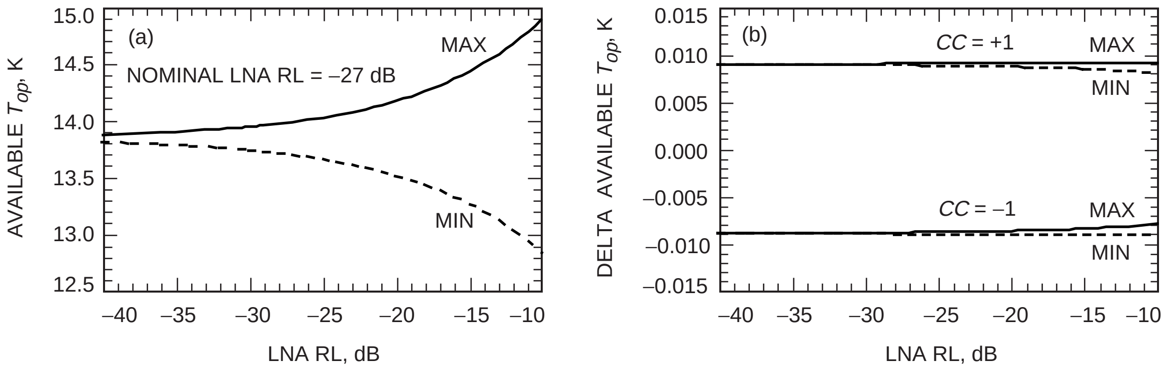

Fig. 9. ***Available*** **antenna T_{op} as a function of LNA RL: (a) maximum and minimum available values when CC = 0 and all other nominal parameters are kept constant and (b) differences when $CC = \pm 1$ instead of 0. Add the delta T_{op} values of the CC = +1 curve to the MAX curve of Fig. 9(a), and the delta T_{op} values of the CC = –1 curve to the MIN curve of Fig. 9(a).**

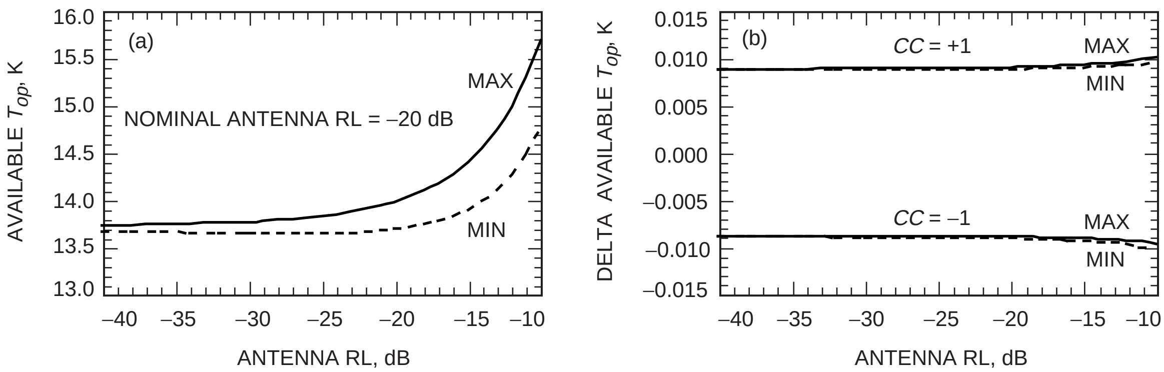

Fig. 10. ***Available*** **antenna T_{op} as a function of antenna RL: (a) maximum and minimum available values when CC = 0 and all other nominal parameters are kept constant and (b) differences when $CC = \pm 1$ instead of 0. Add the delta T_{op} values of the CC = +1 curve to the MAX curve of Fig. 10(a), and the delta T_{op} values of the CC = –1 curve to the MIN curve of Fig. 10(a).**

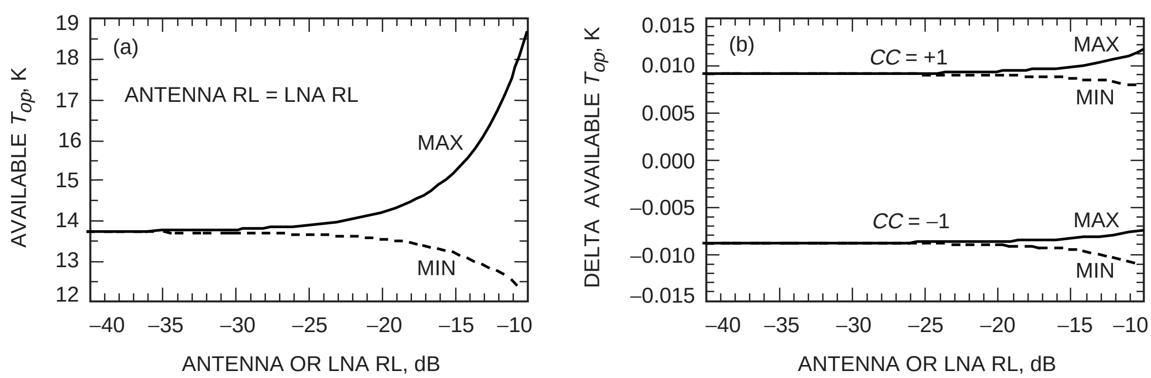

Fig. 11. ***Available*** **antenna T_{op} as a function of both antenna and LNA RL assuming both RLs have the same value and change at the same time: (a) maximum and minimum available values when CC = 0 and all other nominal parameters are kept constant and (b) differences when $CC = \pm 1$ instead of 0. Add the delta T_{op} values of the CC = +1 curve to the MAX curve of Fig. 11(a), and the delta T_{op} values of the CC = –1 curve to the MIN curve of Fig. 11(a).**

To extract more precise values of worst-case errors of $(T_{op,a})_d$ and $(T_{op,a})_a$, the errors $(\in_{mm})_d$ and $(\in_{mm})_a$ are tabulated in Tables 4 and 5, respectively, for the described receiving system. Only the values for a correlation coefficient equal to 0 are tabulated because they are representative of the actual DSN receiver. One need only multiply values in Tables 4 and 5 by the ratio $T_{s'}/T_{op,a'}$ to obtain worst-case mismatch errors on measured and available radio source noise temperatures, respectively. To obtain worst-case errors for measured and available antenna efficiencies, simply multiply the values in Tables 4 and 5 by the ratio $\eta'/T_{op,a'}$.

Table 4. Maximum and minimum delivered $T_{op,a}$ mismatch (MM) errors for the CC = 0 case.[a,b,c] Delivered $T_{op,a}$ *MM* error = $(\in_{mm})_d = T_{op,a'} - (T_{op,a})_d$.

Designated parameter RL, dB	RL of ambient load		RL of LNA		RL of ambient load and LNA	
	Max MM error, K	Min MM error, K	Max MM error, K	Min MM error, K	Max MM error, K	Min MM error, K
−40	−0.017	−0.041	−0.001	−0.010	0.000	−0.005
−38	−0.014	−0.045	0.000	−0.012	0.000	−0.009
−36	−0.012	−0.050	0.000	−0.015	0.000	−0.014
−35	−0.010	−0.053	0.000	−0.017	0.000	−0.017
−34	−0.009	−0.057	0.000	−0.019	0.000	−0.021
−32	−0.005	−0.066	−0.001	−0.025	0.000	−0.034
−30	−0.002	−0.078	−0.003	−0.033	0.000	−0.054
−29	−0.001	−0.086	−0.004	−0.038	0.000	−0.068
−28	0.000	−0.096	−0.007	−0.045	0.000	−0.085
−27	0.000	−0.107	−0.010	−0.053	0.000	−0.107
−26	0.000	−0.120	−0.015	−0.062	0.000	−0.135
−25	−0.001	−0.136	−0.021	−0.074	0.000	−0.169
−24	−0.004	−0.155	−0.029	−0.089	0.000	−0.213
−23	−0.008	−0.178	−0.039	−0.106	0.000	−0.267
−22	−0.015	−0.205	−0.053	−0.128	0.000	−0.336
−21	−0.025	−0.238	−0.071	−0.155	0.000	−0.421
−20	−0.039	−0.277	−0.094	−0.188	0.000	−0.528
−19	−0.059	−0.325	−0.123	−0.230	0.001	−0.661
−18	−0.086	−0.384	−0.162	−0.280	0.001	−0.827
−17	−0.122	−0.455	−0.211	−0.344	0.001	−1.033
−16	−0.171	−0.542	−0.274	−0.422	0.001	−1.288
−15	−0.234	−0.648	−0.354	−0.519	0.001	−1.601
−14	−0.318	−0.779	−0.457	−0.641	0.002	−1.984
−13	−0.428	−0.939	−0.588	−0.792	0.002	−2.449
−12	−0.571	−1.137	−0.755	−0.980	0.003	−3.009
−11	−0.755	−1.380	−0.967	−1.215	0.004	−3.676
−10	−0.995	−1.680	−1.236	−1.509	0.005	−4.458

[a] Only the RL of the designated parameter is varied, while all other nominal values are held constant.

[b] Values enclosed in lines are results for the nominal value case.

[c] Note: *Errors are defined as the negative of corrections.*

Table 5. Maximum and minimum available $T_{op,a}$ mismatch (MM) errors for the $CC = 0$ case.[a,b,c]
Available $T_{op,a}$ MM error = $(\in_{mm})_a = T_{op,a'} - (T_{op,a})_a$.

Designated parameter RL, dB	RL of ambient load		RL of LNA		RL of antenna		RL of antenna and LNA	
	Max MM error, K	Min MM error, K	Max MM error, K	Min MM error, K	Max MM error, K	Min MM error, K	Max MM error, K	Min MM error, K
−40	−0.002	−0.273	−0.102	−0.167	0.036	−0.031	0.010	−0.005
−38	0.002	−0.276	−0.093	−0.175	0.039	−0.035	0.012	−0.008
−36	0.007	−0.278	−0.083	−0.186	0.041	−0.040	0.015	−0.014
−35	0.011	−0.280	−0.076	−0.192	0.043	−0.043	0.017	−0.017
−34	0.014	−0.281	−0.069	−0.199	0.044	−0.047	0.019	−0.022
−32	0.023	−0.285	−0.053	−0.216	0.048	−0.057	0.025	−0.034
−30	0.036	−0.288	−0.032	−0.237	0.050	−0.070	0.033	−0.052
−29	0.044	−0.289	−0.019	−0.250	0.052	−0.078	0.038	−0.065
−28	0.054	−0.290	−0.005	−0.264	0.053	−0.088	0.045	−0.080
−27	0.065	−0.290	0.011	−0.280	0.053	−0.100	0.053	−0.100
−26	0.078	−0.290	0.028	−0.298	0.052	−0.113	0.062	−0.124
−25	0.094	−0.289	0.048	−0.318	0.051	−0.130	0.074	−0.154
−24	0.113	−0.286	0.070	−0.340	0.048	−0.150	0.089	−0.191
−23	0.136	−0.282	0.095	−0.366	0.043	−0.174	0.106	−0.238
−22	0.163	−0.275	0.122	−0.394	0.036	−0.202	0.128	−0.296
−21	0.196	−0.265	0.154	−0.426	0.026	−0.237	0.155	−0.370
−20	0.236	−0.250	0.188	−0.462	0.011	−0.280	0.188	−0.462
−19	0.284	−0.230	0.227	−0.502	−0.010	−0.332	0.230	−0.578
−18	0.342	−0.202	0.271	−0.547	−0.039	−0.395	0.280	−0.724
−17	0.414	−0.165	0.320	−0.598	−0.077	−0.474	0.344	−0.910
−16	0.501	−0.116	0.375	−0.656	−0.129	−0.571	0.422	−1.146
−15	0.608	−0.051	0.436	−0.720	−0.198	−0.691	0.519	−1.446
−14	0.739	0.035	0.505	−0.793	−0.289	−0.842	0.641	−1.830
−13	0.900	0.147	0.581	−0.874	−0.409	−1.031	0.792	−2.325
−12	1.098	0.292	0.667	−0.966	−0.570	−1.271	0.980	−2.967
−11	1.342	0.481	0.762	−1.070	−0.783	−1.578	1.215	−3.807
−10	1.643	0.725	0.869	−1.187	−1.068	−1.975	1.509	−4.919

[a] Only the RL of the designated parameter is varied, while all other nominal values are held constant.

[b] Values enclosed in lines are results for the nominal value case.

[c] Note: *Errors are defined as the negative of corrections.*

For close to a real case example, let $T_{s'} = 5$ K and $T_{op,a'} = 20$ K at an elevation angle of 35 deg. Then the mismatch error for the measured radio source temperature is $(5/20) \times (\in_{mm})_d$, where $(\in_{mm})_d$ is given in Table 4 for the return loss of interest. Note that if $T_{s'} = 50$ K and $T_{op,a'}$ is 20 K, the error on measurement of T_s is 10 times larger than that calculated for the $T'_s = 5$ K example.

A second practical example would be to let the assumed matched-case value of antenna efficiency be $\eta' = 0.50$ and $T_{op,a'} = 20$ K. The mismatch error on *delivered* antenna efficiency can be obtained by multiplying the ratio $(0.5/20) \times (\in_{mm})_d$, where $(\in_{mm})_d$ is given in Table 4 for the return loss of interest.

Similarly, the error on *available* antenna efficiency can by obtained by multiplying this (0.5/20) ratio by the $(\in_{mm})_a$ value in Table 5 for the applicable return loss of interest.

VI. Concluding Remarks

Mismatch error equations for the worst-case delivered and available antenna system noise temperatures were derived and presented in Tables 2 and 3 for easy identification of the individual noise-source contributions.

If M_{pe} is equal to unity, then the values of $(T_{op,a})_d$ and η_d as measured by the ambient-load method will be the true delivered values regardless of what antenna reflection coefficient value exists for the system. If $M_{pe} \neq 1$, mismatch errors (caused by non-zero reflection coefficients of the ambient load and receiver) will cause the slope of the T_{op} calibration curve to be erroneous and, hence, lead to erroneous values of $(T_{op})_d$ and η_d. The ambient-load calibration curve is independent of the value of the antenna reflection coefficient. Knowledge of the existing antenna reflection coefficient value is not required unless one wishes to improve antenna efficiency or radio source noise temperature towards its maximum attainable value. Also, if the measured performance does not agree with predictions, one should tune the antenna reflection coefficient to a smaller value.

It was shown that the equation for *available* antenna T_{op} is explicitly a function of antenna reflection coefficient. In general, it can be stated that, if the antenna is tuned for better VSWR, a better antenna efficiency will result. This was shown in Fig. 4, where lowering the antenna return loss caused the mismatch factor to go higher towards unity with smaller uncertainties, and, hence, to have the effect of improving the antenna efficiency. The term "lowering return loss" means more negative dB value.

In studying the applications of the mismatch equations for a typical DSN feed system, two conclusions can be made: (1) lowering the return losses of both the ambient load and LNA will result in better calibration of the *delivered* antenna T_{op} and (2) additional tuning of the antenna towards a lower return loss will result in raising the antenna efficiency. When antenna, ambient-load, and LNA-receiver reflection coefficients are all equal to zero, the mismatch errors go to zero and the measured T_{op} value is exactly equal to the "assumed matched-case" T_{op} value.

The mismatch error analyses for measurements of radio noise source temperatures and antenna efficiencies presented in this article have not previously been presented in the published literature. Some of the practical results are surprisingly larger than previously assumed. It would be profitable to go through the examples given in this article and apply actual values for other DSN feed systems and to report the mismatch errors as part of the overall errors.

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Appendix

Return Loss Versus VSWR and Reflection Coefficient

As was stated in the main body of the article, plots were made as functions of return losses because return loss is the data type often measured as a function of frequency when using a network analyzer. When reflection coefficient magnitudes become increasingly small, from 0.1 toward 0, or when VSWRs have values between 1.2 and 1.0, in order to use them as the x-axis variable on a plot, it is better to display reflection coefficient or VSWR converted to return loss in dB. For example, suppose that the ambient-load return loss was −20 dB before tuning and −30 dB after tuning. These values correspond to voltage reflection coefficients of 0.1 before tuning and 0.0316 after tuning. Suppose that the ambient load is further tuned to −42 dB instead of −30 dB. The voltage reflection coefficient magnitude corresponding to −42 dB is 0.0079. It can be seen that four decimal places are required to show this new voltage reflection coefficient value on a plot. As reflection coefficients are tuned to even smaller values, it is difficult to show these improvements on a plot. However, a return loss plot clearly shows small value improvements. For example, the VSWRs corresponding to return losses of −20 dB, −30 dB, and −42 dB are, respectively, 1.222, 1.065, and 1.016.

Many microwave engineers (including this author) are not as familiar with return loss in dB as they are with voltage reflection coefficient and VSWR magnitudes. Therefore, for convenience and reference purposes, plots of the return loss versus magnitude of voltage reflection coefficient and magnitude of VSWR are given in Figs. A-1 and A-2, respectively.

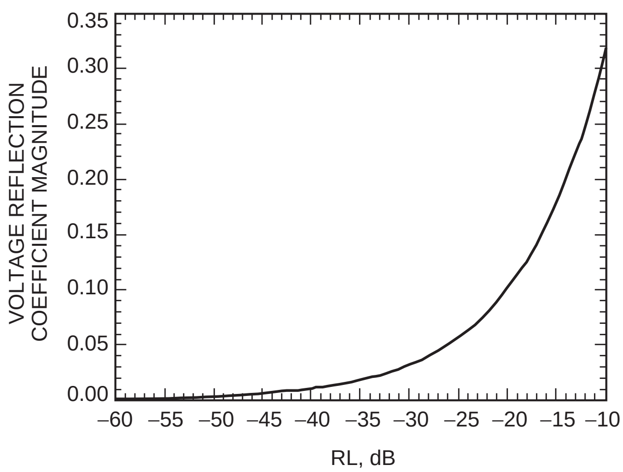

Fig. A-1. Relationship of RL to voltage reflection coefficient magnitude.

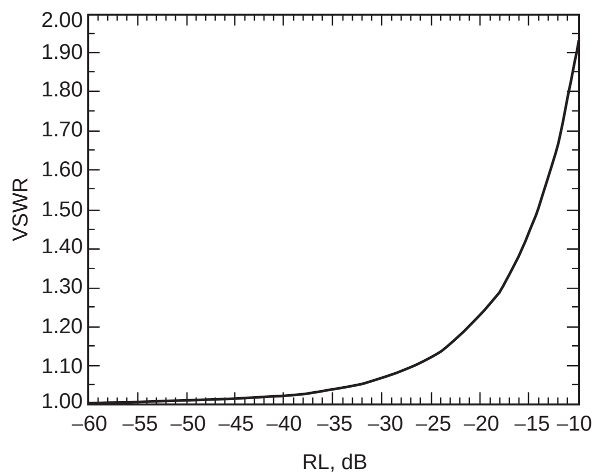

Fig. A-2. Relationship of RL to VSWR.