Three Models of Wind-Gust Disturbances for the Analysis of Antenna Pointing Accuracy

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This article develops and compares three wind models that are used to simulate antenna servo errors. The first model represents wind forces acting on the antenna dish, and it closely represents actual wind action. The second model represents wind gusts as torque disturbances applied to the antenna drives. The third model represents gusts acting at the antenna rate input. A wind filter introduced in this article makes the wind gusts of the third model equivalent to the gusts of the second model. The third model is of recent interest, since it is applicable to the currently developed models of the antenna closed-loop systems.

Simulink models of the 34-m and 70-m antennas' closed-loop systems were developed to simulate antenna servo errors in wind gusts. The article shows that for the 34-m and 70-m antennas all three models give similar results, and that the results are comparable to the field measurements of the servo errors taken in wind.

I. Introduction

There are two known ways to model wind gusts acting on an antenna structure. See [2,3,5] and Fig. 1(a) for the first one, where the wind-gust disturbances are modeled as force acting on the antenna dish. The white noise of unit standard deviation is filtered by the Davenport filter (on the spectrum of the Davenport wind gusts, see [6]) and appropriately scaled, with scale factor k_f. This model is used when an antenna finite-element model is available, and it includes forces at the dish panels. It is mainly used during the design stage. The second model is presented as a time-varying torque, $T_w(t)$, acting at the antenna drives [see Fig. 1(b)]. The wind torque adds up to the antenna drive torque, T_c, as illustrated in Fig. 1(b), producing the total torque, T, acting at the structure. The Davenport filter again is used to shape white noise into wind gusts, but a different scaling factor, k_t, is now used. This model is often used in antenna analyses, giving adequate approximation of servo errors. It is used, for example, when antenna structure and antenna drives are modeled as separate units. There are models, however, where antenna and drives are inseparable. Such is the case of the antenna rate loop model obtained from field tests and system identification, where one obtains an integrated model of the antenna structure and the attached drives. In this model, as shown in Fig. 1(c), the rate command is the input to the model, and the encoder reading is its output, while the drive torque is an internal variable. In this situation, the
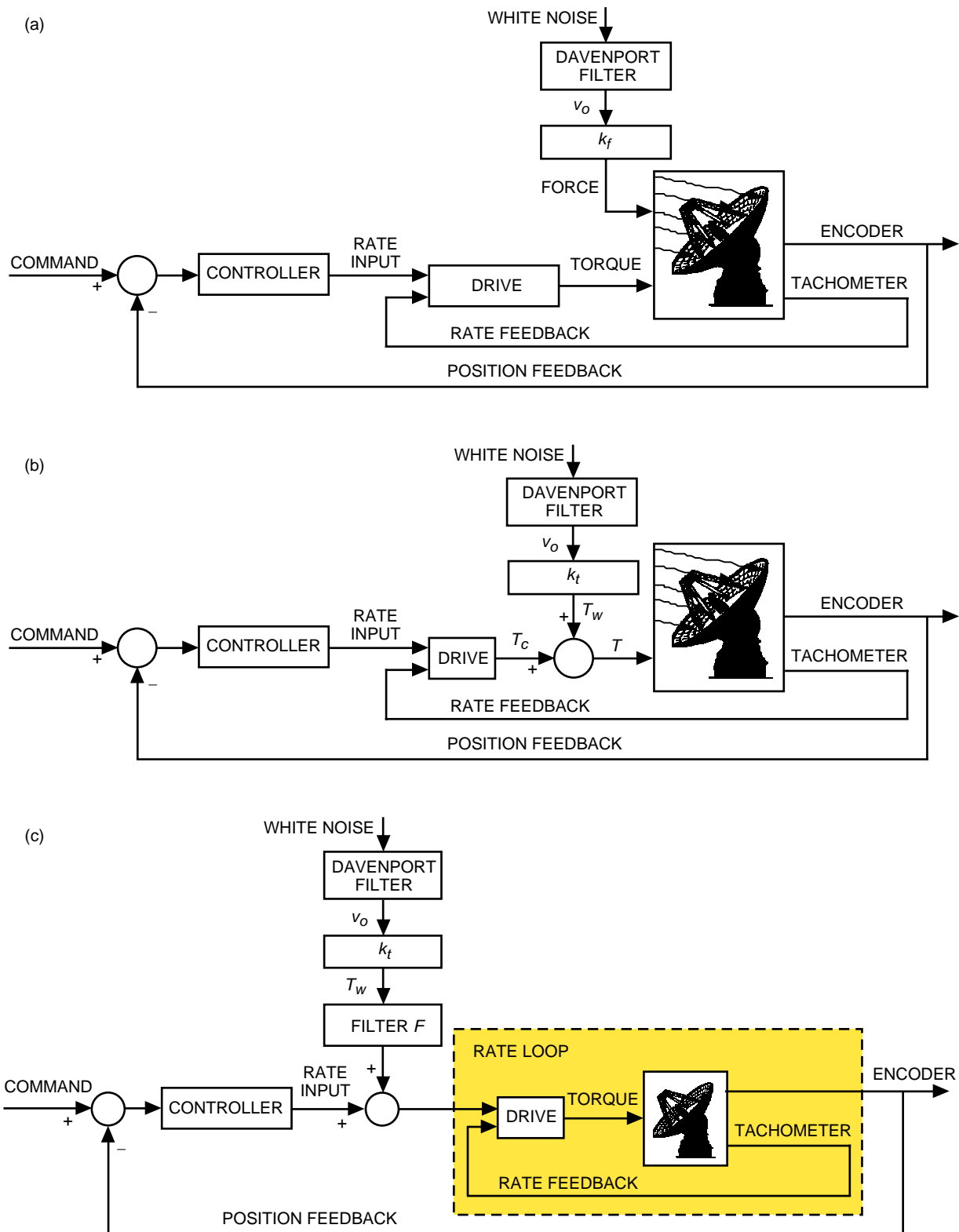
Fig. 1. Wind-gust models: (a) Model 1, wind forces acting on the dish, (b) Model 2, wind torques acting at the drive motors, and (c) Model 3, wind rate acting at the rate input.

only possible entry point for the wind-gust disturbances is the rate input. In this article we determine the model of wind disturbances equivalent to the previous model. The equivalent model is obtained by adding a filter with transfer function F [see Fig. 1(c)]. The wind torques are now filtered to obtain an equivalent rate input signal. The filter transfer function F is such that the action of white noise [see the model in Fig. 1(c)] causes the same servo error as the action of white noise in the model shown in Fig. 1(b).

This article presents the development of the Davenport filter; the determination of force, k_f, and torque, k_t, gains; and the determination of the filter transfer function, F, for 34-m and 70-m antennas. It also compares servo errors in wind gusts produced by the three wind models and measured at the stations.

II. The First Wind Model: Wind Forces Acting on the Dish

In this model, as shown in Fig. 1(a) and described in [2,3,5], the wind gust is represented as a uniformly distributed force acting on the antenna dish, either from its front, its back, or its side. The gust force is obtained from the wind-gust velocity of unit standard deviation, Δv_o; on the other hand, the velocity, Δv_o, is obtained from the Davenport spectrum. Consider first the determination of velocity Δv_o.

A. Obtaining Wind-Gust Velocity from the Davenport Spectrum

The total wind velocity, v, is a combination of a steady-state or mean velocity, v_m, and a turbulence (gust) velocity, Δv, i.e.,

$$v = v_m + \Delta v \tag{1}$$

The gust component is a random process with zero mean and a spectrum called the Davenport spectrum. The Davenport spectrum, $S_v(\omega)$, depends on average wind speed and terrain roughness and is given by

$$S_v(\omega) = 4800 v_m \kappa \frac{\beta \omega}{(1 + \beta^2 \omega^2)^{4/3}} \tag{2}$$

where $\beta = 600/\pi v_m$, and κ is the surface drag coefficient, obtained from the roughness of the terrain (see [5]):

$$\kappa = \frac{1}{\left(2.5 \ln \left(\frac{z}{z_o}\right)\right)^2} \tag{3}$$

In these equations, z is the distance from the ground to the antenna dish center ($z = 17$ m for a 34-m antenna), and z_o is the height of the terrain roughness ($z_o = 0.1$ to 0.3 m at Goldstone, California). Thus, $\kappa = 0.006$ to 0.010 for the 34-m antenna. For the 70-m antenna, $z = 35$ m and $z_o = 0.1$ to 0.3 m; thus, $\kappa = 0.005$ to 0.007.

The wind-gust velocity, Δv_o, of unit standard deviation is obtained by applying a white-noise input of unit standard deviation to a filter that approximates the Davenport spectrum. The unit standard deviation is obtained by appropriate scaling of the filter gain. This filter is called a Davenport filter. The filter transfer function is of fourth order and was obtained in [3] and [5] by adjusting the filter parameters such that the magnitude of the filter transfer function best fits the Davenport spectrum within the antenna bandwidth of [0.001, 20] Hz. The resulting filter transfer function is

$$H = \frac{3.9021s^3 + 230.1426s^2 - 686.3151s + 3.4197}{0.331s^4 + 38.2997s^3 + 224.7118s^2 + 22.7788s + 0.3538} \tag{4}$$

The corresponding digital filter for a sampling time of 0.02 s is as follows:

$$H_d = \frac{0.1584z^3 - 0.3765z^2 + 0.2716z - 0.0534}{z^4 - 2.9951z^3 + 3.0893z^2 - 1.1930z + 0.0988} \tag{5}$$

This filter is scaled such that, applying white noise of unit standard deviation, one obtains an output, Δv_o, of unit standard deviation as well.

The plots of the square root of the Davenport spectrum and of the magnitude of the filter transfer function are shown in Fig. 2, and a sample of the wind speed generated by the filter is shown in Fig. 3.
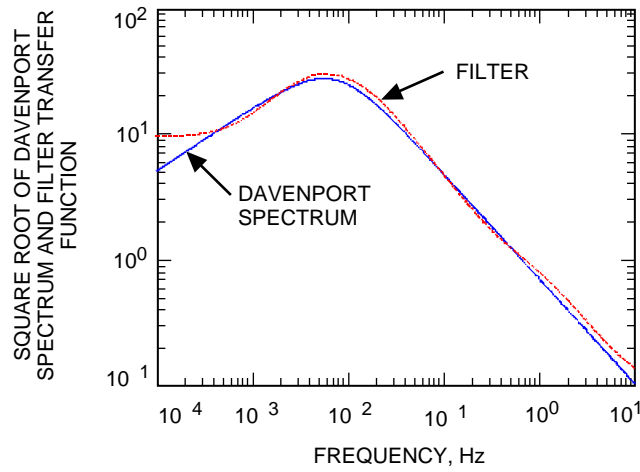

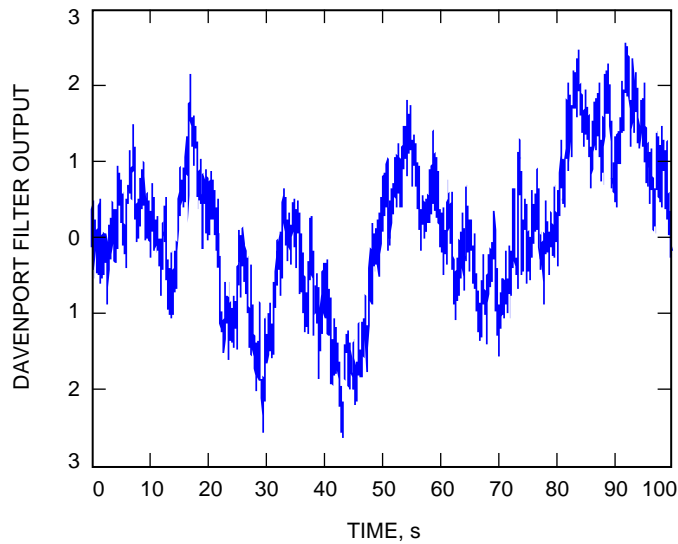**Fig. 2. The Davenport spectrum of the wind velocity and the magnitude of the filter transfer function.**

Fig. 3. Generic wind speed of unit standard deviation.

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B. Obtaining Wind Force from Wind Velocity

In this section, we derive the time history of the wind-gust force. In order to do this, consider first a steady wind for which the quadratic law relates its velocity, v_n, and force, F_n:

$$F_n = k_F v_n^2 \tag{6}$$

The constant k_F depends on the scaling of the structural modal model. In our case, modes were scaled such that a 161 km/h wind corresponds to a force of 4.4482 N. Thus, from the above equation, for this case we obtain

$$k_F = 0.0022 \text{ Ns}^2/\text{m}^2 \tag{7}$$

where speed is in m/s and force in N. Equation (6) represents a steady (or static) wind force.

The next step is to obtain time-varying forces generated by wind gusts using the velocity time history, Δv_o. Consider a long enough time interval (e.g., 120 s or more). The wind speed over this interval can be decomposed into its constant (or mean) component, v_m, and variable component, Δv, of zero mean value [see Eq. (1)]. We similarly decompose the corresponding wind forces:

$$F = F_m + F_w \tag{8}$$

Variable F_m represents the steady-state (static-force) component. The wind-gust force variations, F_w, are related to wind velocity variations, Δv, by expanding Eq. (6) in Taylor series. We obtain $F_w = (\partial F/\partial v)|_{v=v_m} \Delta v$, and, since $(\partial F/\partial v)|_{v=v_m} = 2v_m$, therefore

$$F_w = 2k_F v_m \Delta v \tag{9}$$

We scale the wind-gust Δv obtained in the previous section to obtain its unit standard deviation, i.e., we find the speed $\Delta v_o(t)$ such that

$$\Delta v_o(t) = \frac{\Delta v(t)}{\sigma_v} \tag{10}$$

where σ_v is the standard deviation of Δv. However, the standard deviation of the wind gust is proportional to the mean wind speed (see [4]):

$$\sigma_v = \alpha \, v_n \tag{11}$$

Combining Eqs. (10) and (11), one obtains

$$\Delta v = \alpha v_m \Delta v_o \tag{12}$$

In the above equations,

$$\alpha = \sqrt{6\kappa} \tag{13}$$

and κ is the surface drag coefficient [see Eq. (3)].

Introducing Eq. (12) to Eq. (9), one obtains the final relationship between wind-gust velocity and force:

$$F_w = k_f \Delta v_o \tag{14}$$

where

$$k_f = 2k_F \alpha v_n^2 \tag{15}$$

For the 34-m and 70-m antennas, $k_F = 0.00223 \text{ Ns}^2/\text{m}^2$, and $\alpha = 0.20$; thus,

$$k_f = 8.9200 \times 10^{-4} v_m^2 \tag{16}$$

for speed in m/s and force in N. For 8.94-m/s wind, the force gain is $k_f = 0.0713 \text{ Ns/m}$.

III. The Second Wind Model: Wind Torque Acting at the Drives

In this model, the wind torque disturbance is added to the drive torque. It is a time function, $T_w(t)$, determined from the velocity gusts, Δv_o. This model is shown in Fig. 1(b), where the white noise is applied to the Davenport filter. The filter output is the velocity gust Δv_o of unit standard deviation, which is consequently scaled to obtain wind torque that is added to the antenna drive torque. In this model, the Davenport filter is identical to the filter presented in Section II. The scaling factor, k_t, from the velocity to torque is obtained from the wind quadratic law for torques, T_n, and for steady wind speed, v_m:

$$T_n = k_T v_m^2 \tag{17}$$

The constant k_T in this equation is particular for an antenna, antenna elevation position, antenna site, terrain profile, and wind direction. It is determined as follows. First, the wind torque depends on wind pressure, and the pressure–torque relationship was determined experimentally in wind tunnels[2,3,4] and in field tests [4]:

$$T_n = c_t A D p_n \tag{18}$$

where D is the antenna dish diameter, m; A is the antenna dish frontal area, $A = \pi D^2/4 \text{ m}^2$; and c_t is a dimensionless torque coefficient. This coefficient depends on the wind direction and the antenna elevation position, and it varies from -0.05 to 0.25.

[2] R. B. Blaylock, "Aerodynamic Coefficients for a Model of a Paraboloidal-Reflector Directional Antenna," JPL Interoffice Memorandum (internal document), CP6, Jet Propulsion Laboratory, Pasadena, California, May 1, 1964.

[3] N. L. Fox, "Load Distribution on the Surface of Paraboloidal Reflector Antennas," JPL Interoffice Memorandum (internal document), CP4, Jet Propulsion Laboratory, Pasadena, California, July 1962.

[4] N. L. Fox and B. Layman, Jr., "Preliminary Report on Paraboloidal Reflector Antenna Wind Tunnel Tests," JPL Interoffice Memorandum (internal document), CP3, Jet Propulsion Laboratory, Pasadena, California, February 28, 1962.

Next note that, similarly to the torque, the dynamic pressure of wind, p, depends on wind speed[5] [4]:

$$p_n = \alpha_p v_m^2 \tag{19}$$

where α_p is the static air density and $\alpha_p = 0.6126$ Ns^2/m^4 (pressure is in N/m^2 and velocity is in m/s). Introducing Eq. (19) to Eq. (18), one obtains

$$T_n = c_t \alpha_p A D v_m^2 \tag{20}$$

Comparing Eqs. (20) and (17), we obtain the quadratic law coefficient k_T:

$$k_T = c_t \alpha_p A D = c_t \alpha_p \frac{\pi D^3}{4} \tag{21}$$

For a 34-m antenna, $D = 34$ m, and for $c_t = 0.25$ (front and side wind), one obtains $k_T = 4661$ Ns^2/m. The plot of the wind quadratic law, Eq. (17), for the above coefficient is shown in Fig. 4(a). For a 70-m antenna, $D = 70$ m, and for $c_t = 0.25$ (front and side wind), one obtains $k_T = 41,257$ Ns^2/m. The plot of the wind quadratic law, Eq. (17), for the 70-m antenna is shown in Fig. 4(b).

The torque equation, Eq. (17), is valid for steady wind only. However, the wind-gust torque can be derived from it using linear expansion. Indeed, it is determined from the velocity gusts, Δv_o, by linearizing Eq. (17), which gives

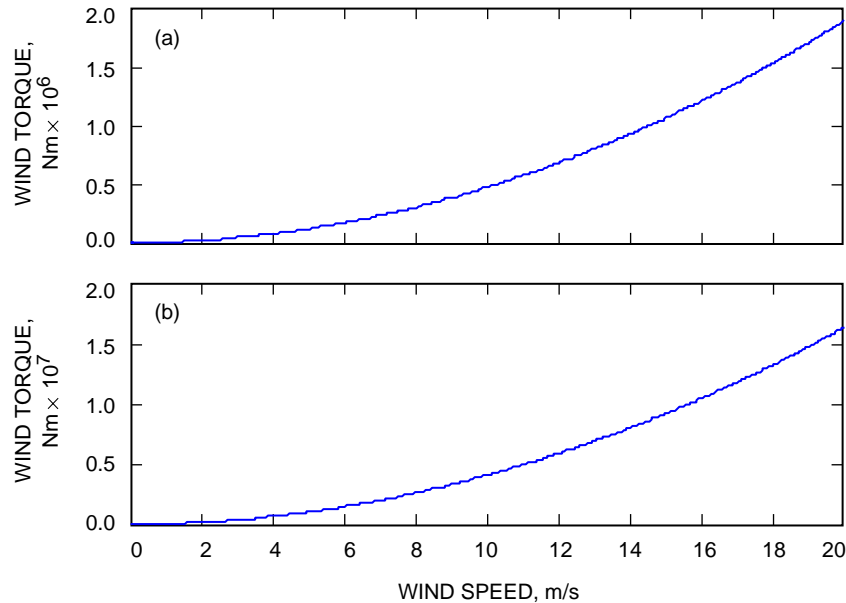$$\Delta T = 2 k_T v_m \Delta v \tag{22}$$

Fig. 4. The quadratic relationship between wind speed and antenna torques for (a) 34-m antennas and (b) 70-m antennas.

[5] Ibid.

The above represents the antenna axis torque. The wind torque at the pinion axis, T_w, is the axis torque, ΔT, divided by the axis-to-pinion ratio, N_p; thus, the pinion torque is also proportional to the velocity:

$$T_w = \frac{\Delta T}{N_p} = \frac{2k_T v_n}{N_p} \Delta v \tag{23}$$

In simulations, we will use a wind-speed model with unit standard deviation $\Delta v_o(t)$. It was previously derived that $\Delta v_o(t)$ is related to an arbitrarily scaled wind speed as in Eq. (12).

Introducing Eq. (12) to Eq. (23), one obtains

$$T_w = k_t \Delta v_o \tag{24}$$

where

$$k_t = \frac{2k_T \alpha}{N_p} v_m^2 \tag{25}$$

For the 34-m antenna, $k_T = 4661$ Ns^2/m, $\alpha = 0.20$, and $N_p = 42$; thus,

$$k_t = 44.4 v_m^2 \tag{26}$$

where speed is in m/s and torque in Nm. For the 70-m antenna, $k_T = 41,257$ Ns^2/m, $\alpha = 0.20$, and $N_p = 47$ in azimuth and $N_p = 56$ in elevation; thus, for the azimuth axis,

$$k_t = 351.1 v_m^2 \tag{27}$$

where speed is in m/s and torque in Nm; and, for the elevation axis,

$$k_t = 294.7 v_m^2 \tag{28}$$

where speed is in m/s and torque in Nm.

The algorithm to generate a wind-gust torque for Model 2 is given in the Appendix.

IV. The Third Wind Model: Wind Acting at the Rate Input

In this model, the wind gusts are applied at the rate input. It is shown in Fig. 1(c), where the white noise is applied to the Davenport filter; its output is the velocity gust Δv_o of unit standard deviation, appropriately scaled with gain k_t to obtain wind torque. Next, the torque is filtered with filter F, which produces a rate that is added to the rate input of the antenna. In this model, the Davenport filter and the scaling factor, k_t, are identical to the filter and factor presented in Section III. The task of this section is to find the filter transfer function F such that the encoder response to the wind disturbances is almost the same as the response for wind Model 2.

In order to have the encoder response the same as in wind Model 2, the transfer function of the filter should be the inverse of the drive transfer function. The drive transfer function, $F_d(s)$, from the rate

input to the torque output is shown in Fig. 5 (solid line). Within the antenna bandwidth it can be approximated with the integral, i.e., with $F_d(s) = k_d/s$, where k_d is the drive gain. The magnitude of the approximate transfer function is shown in Fig. 5 (dashed line). Thus, the filter transfer function, as an inverse of the approximate drive transfer function, is a derivative with the inverse gain

$$F(s) = \frac{s}{k_d} \tag{29}$$

The corresponding discrete-time filter of sampling time Δt is as follows:

$$F(z) = \frac{1}{k_d} \frac{z-1}{z} \frac{1}{\Delta t} \tag{30}$$

For the 34-m antenna, the drive gains are

$$k_d = \begin{cases} 7.11 \times 10^6 & \text{azimuth drive} \\ 7.58 \times 10^6 & \text{elevation drive} \end{cases} \quad \text{Nms/deg} \tag{31}$$

For the 70-m antenna, the drive gains are

$$k_d = \begin{cases} 1.07 \times 10^8 & \text{azimuth drive} \\ 1.02 \times 10^8 & \text{elevation drive} \end{cases} \quad \text{Nms/deg} \tag{32}$$

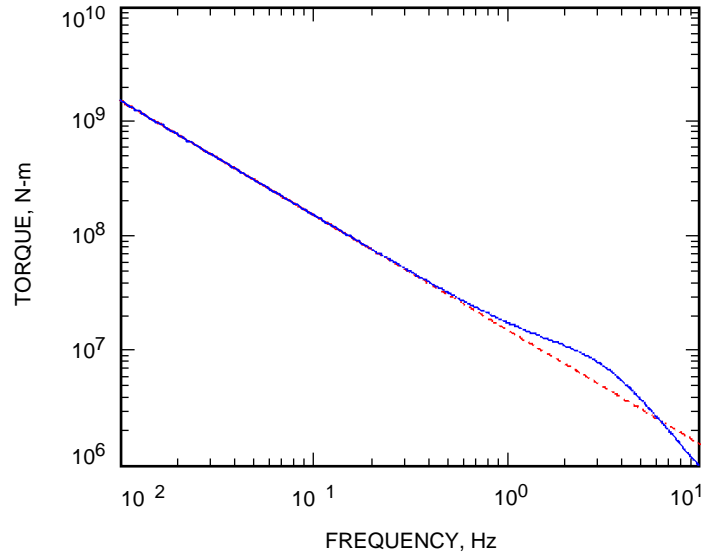The algorithm to generate the wind-gust rate for Model 3 is given in the Appendix.

Fig. 5. The transfer function of the elevation drive (solid line) and its approximate (dashed line).

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V. Equivalence of the Wind Torque and the Wind Rate Models

The nature of the first model—which consists of wind forces acting on the dish—is different from the second and third models, which consist of torque and rate signals acting on the antenna drives. Thus, the first and second model and the first and third model can be compared only in statistical terms, such as standard deviation of the resulting antenna servo error. However the second and third models are closely related and can be compared directly. Namely, we compare the antenna encoder output due to the wind gusts that are generated by the second model (wind torque) and by the third model (wind input rate). In order to do this, we apply the white noise as shown in Fig. 1(b), with the position loop open. The resulting elevation encoder reading is shown in Figs. 6(a) and 6(b). The plots show the encoder outputs for the two wind cases are very close, indicating close equivalence of the second and third models.

VI. Closed-Loop Pointing Accuracy with Wind Disturbances

We simulated wind gusts using all three models, determined the antenna servo error in azimuth and elevation for the front and side wind directions, and compared the obtained results with the field data. The Simulink model was developed for the 34-m antennas and allows for simulation of three wind models, shown in Figs. 7(a) and 7(b). The simulation results are given in Table 1, where they are compared with the measured servo errors from [1]. The servo errors were measured repeatedly, and the table shows the minimal and maximal values of the standard deviation of the errors. The table shows that the simulation results are within the range of measurements, except for the elevation error in the front wind of Model 1, which exceeds the measurements.

A similar model was developed for the 70-m antenna, and the simulation results are given in Table 2. The field data were collected by David Girdner in November 1999; however, no wind direction was recorded. The simulation results are close to the data, except for Model 1 in azimuth, for the reason explained in [1].
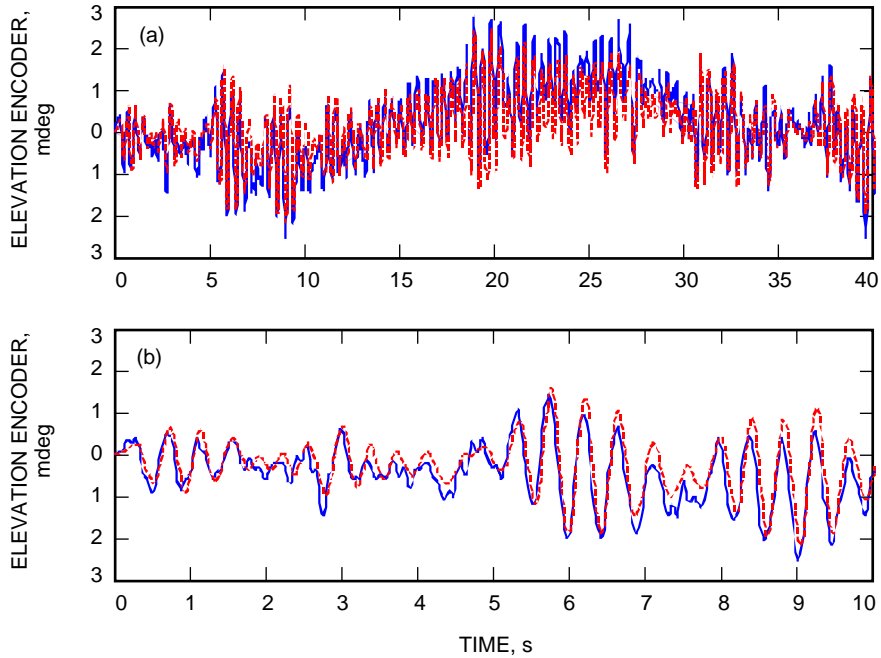

Fig. 6. Open-loop elevation encoder response to the wind: (a) 40-s response and (b) 10-s segment. Torque input is the solid line, and rate input is the dashed line.
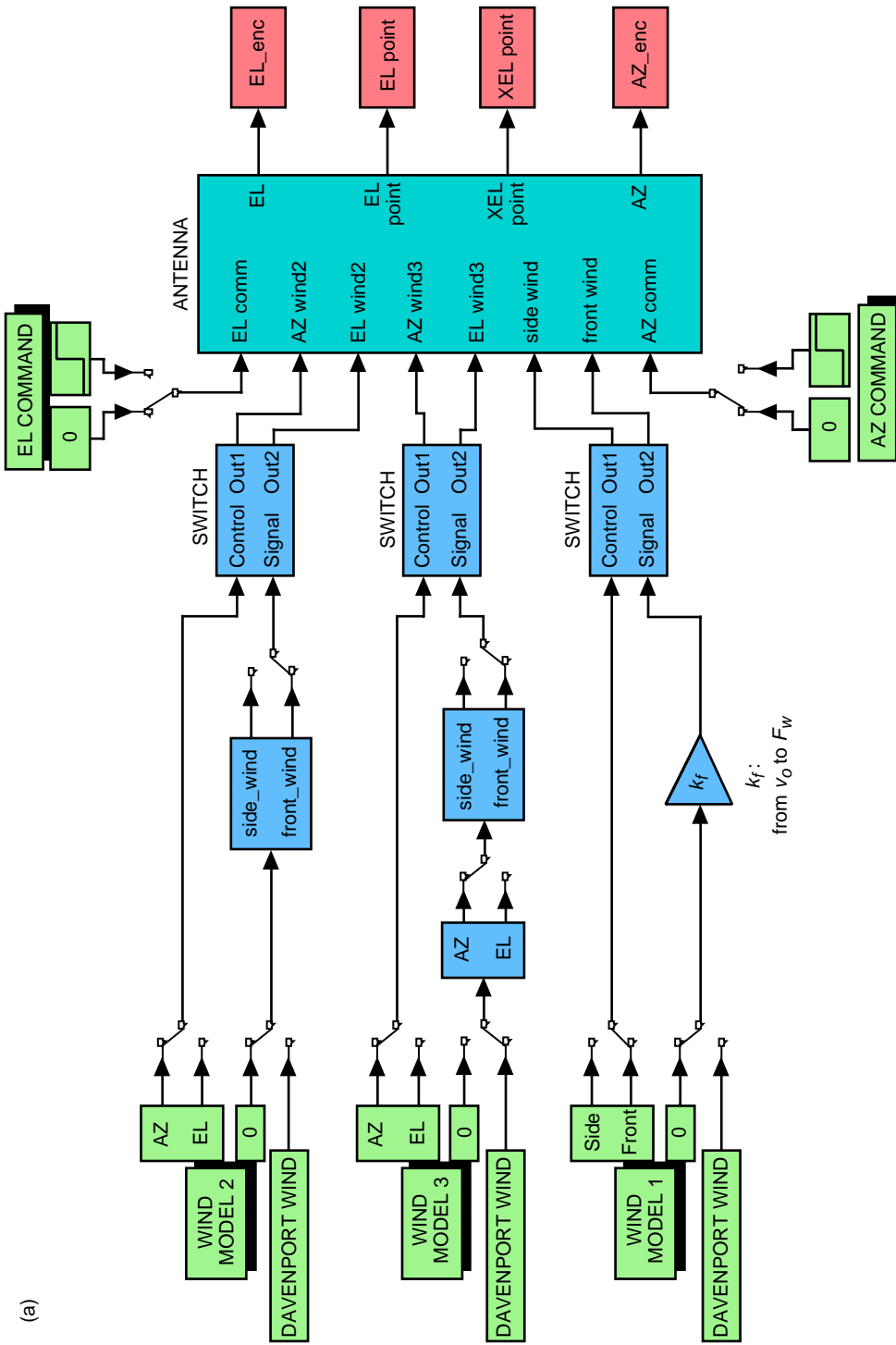
Fig. 7. Simulink models of the 34-m antenna: (a) with three wind models acting on the closed-loop antenna (denoted ANTENNA) and (b) the ANTENNA subsystem with three wind-model inputs—front wind1 and side wind1 for Model 1, AZ wind2 and EL wind2 for Model 2, and AZ wind3 and EL wind3 for Model 3. (AZ = azimuth, EL = elevation, and XEL = cross-elevation.)

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Fig. 7 (cont'd.).

(b)

EL comm — 1

AZ comm — 8

EL CONTROLLER (servo error, rate comm)

AZ CONTROLLER (servo error, rate comm)

EL wind3 — 5

AZ wind3 — 4

EL wind2 — 3

AZ wind2 — 2

EL DRIVE — 4

AZ DRIVE — 8

side wind — 6

front wind — 7

$$x(n+1) = Ax(n) + Bu(n)$$
$$y(n) = Cx(n) + Du(n)$$

STRUCTURE

EL ENCODER

EL PINION RATE

AZ PINION RATE

AZ ENCODER

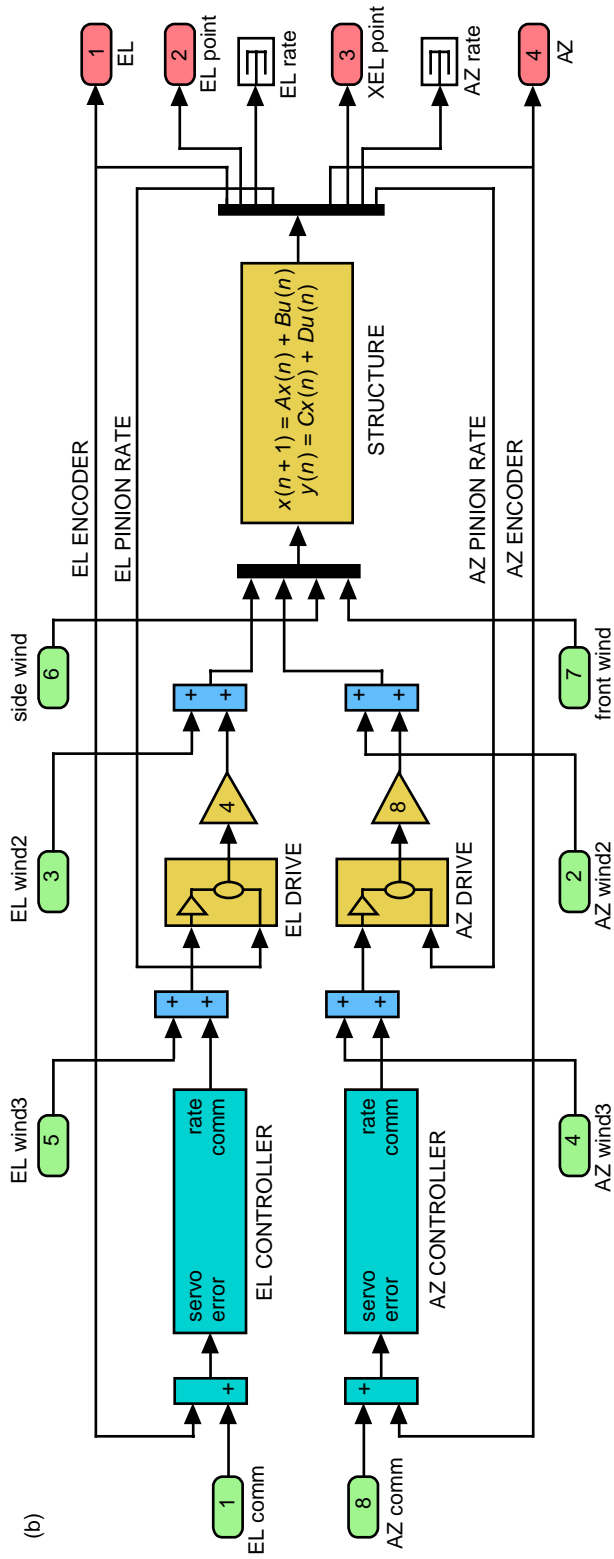1 — EL

2 — EL point

EL rate

3 — XEL point

AZ rate

4 — AZ

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Table 1. Standard deviations of 34-m antenna servo errors, mdeg, due to 12.5-km/h wind gusts.

Model/ field data	Front wind		Side wind	
	Azimuth, mdeg	Elevation, mdeg	Azimuth, mdeg	Elevation, mdeg
Field data[a]	0.23–0.48	0.6–1.2	0.23–0.48	0.9–1.5
Model 1	0.33	1.73	0.30	1.02
Model 2	0.32	1.20	0.32	1.20
Model 3	0.44	1.21	0.44	1.07

[a] From [1].

Table 2. Standard deviations of 70-m antenna servo errors, mdeg, due to 12.5-km/h wind gusts.

Model/ field data	Front/side wind	
	Azimuth, mdeg	Elevation, mdeg
Field data[a]	1.3	2.1
Model 1	0.02	2.0
Model 2	1.7	2.0
Model 3	1.6	1.9

[a] Measured and reported by David Girdner at the DSS-14 antenna in November 1999.

VII. Conclusions

In this article, wind-gust velocity of unit standard deviation and of the spectrum of the Davenport filter was derived. This velocity was used to create three models of the wind forces acting on the antenna dish and the torques acting on the antenna drives. The three wind-gust models were used in the analysis of the antenna closed-loop model and the standard deviation of the servo errors in azimuth and elevation, and for simulations of side and front winds, which were compared with previously collected wind data. The simulations show that Models 2 and 3 are equivalent and that all three models produce servo errors that match the field data.

References

[1] W. Gawronski, "Wind Gust Models Derived From Field Data," *The Telecommunications and Data Acquisition Progress Report 42-123, July–September 1995*, Jet Propulsion Laboratory, Pasadena, California, pp. 30–36, November 15, 1995. http://tmo.jpl.nasa.gov/tmo/progress_report/42-123/123G.pdf

[2] W. Gawronski, B. Bienkiewicz, and R. E. Hill, "Pointing-Error Simulations of the DSS-13 Antenna Due to Wind Disturbances," *The Telecommunications and Data Acquisition Progress Report 42-108, October–December 1991*, Jet Propulsion Laboratory, Pasadena, California, pp. 109–134, February 15, 1992.
http://tmo.jpl.nasa.gov/tmo/progress_report/42-108/108J.PDF

[3] W. Gawronski, B. Bienkiewicz, and R. E. Hill, "Wind-Induced Dynamics of a Deep Space Network Antenna," *Journal of Sound and Vibration*, vol. 178, no. 1, 1994.

[4] W. K. Gawronski and J. A. Mellstrom, "Field Verification of the Wind Tunnel Coefficients," *The Telecommunications and Data Acquisition Progress Report 42-119, July–September 1994*, Jet Propulsion Laboratory, Pasadena, California, pp. 210–220, November 15, 1994.
http://tmo.jpl.nasa.gov/tmo/progress_report/42-119/119G.pdf

[5] W. Gawronski and J. A. Mellstrom, "Control and Dynamics of the Deep Space Network Antennas," in *Control and Dynamics Systems*, C. T. Leondes, ed., vol. 63, San Diego, California: Academic Press, pp. 289–412, 1994.

[6] E. Simiu and R. H. Scanlan, *Wind Effects on Structures*, New York: Wiley Interscience, 1978.

Appendix

Algorithms to Generate Wind Gusts

I. Algorithm 1

To generate a time profile of wind-gust torque (for Model 2), do the following:

(1) Apply white noise to the filter, H, with the following transfer function:

$$H_d = \frac{0.1584z^3 - 0.3765z^2 + 0.2716z - 0.0534}{z^4 - 2.9951z^3 + 3.0893z^2 - 1.1930z + 0.0988}$$

and obtain time series of wind $\Delta v(t)$. The above filter represents the Davenport wind-gust model.

(2) Calculate σ_v, the standard deviation of $\Delta v(t)$.

(3) Divide $\Delta v(t)$ by σ_v, obtaining wind speed with unit standard deviation

$$\Delta v_o(t) = \frac{\Delta v(t)}{\sigma_v}$$

The standard deviation of $\Delta v_o(t)$ is 1.

(4) Scale $\Delta v_o(t)$ by k_t to obtain $T_w(t)$:

$$T_w(t) = k_t \Delta v_o(t)$$

where

$$k_t = \frac{2k\alpha}{N_p} v_m^2$$

and v_n is the mean wind speed, N_p is the gear ratio between the antenna main axis and the drive pinion, k is the "wind quadratic law," $\alpha = \sqrt{6\kappa}$, and $\kappa = \left(2.5\ln(z/z_o)\right)^{-2}$. In the latter equation, z is the distance from the ground to the antenna dish center and z_o is the height of the terrain roughness.

II. Algorithm 2

To generate a time profile of wind-gust rate (for Model 3), do the following:

(1) Follow the steps of Algorithm 1 to obtain $T_w(t)$.

(2) Filter $T_w(t)$ to obtain $u_w(t)$. Use filter F with the transfer function as follows:

$$F(z) = \frac{1}{k_d} \frac{z-1}{z\,\Delta t}$$

where k_d is the drive gain from input u to torque T and Δt is the sampling time.