Calculation of Antenna System Noise Temperatures at Different Ports—Revisited

T. Y. Otoshi[1]

With the advent of cryogenically cooled front-end assemblies, it has become the practice to define antenna system noise temperature at the horn aperture rather than at the low-noise amplifier (LNA) input port. It is not generally known that when the reference port is moved towards the horn aperture, the values of the operating system noise temperature, T_{op}, and effective receiver noise temperature, T_e, will always increase. It is also not generally known that, when the horn and waveguide are at cryogenic rather than ambient temperature, as will be shown in two examples in this article, the value of T_{ia} defined at the LNA reference port can be either larger or smaller than the value of T_{ia} defined at the horn aperture. This fact is not obvious from studying approximate formulas that neglect products of the loss factors of the components between the horn aperture and the LNA.

In this article, the exact expressions for T_{op} at different ports are derived. The exact and approximate formulas are compared, and calculations are made on a DSN 8.4-GHz (X-band) and DSN 32-GHz (Ka-band) feed system showing the magnitudes of errors that result when using the approximate formula. The magnitudes of errors were about 0.1 K and 1 K for the X- and Ka-band systems, respectively.

I. Introduction

With the advent of cryogenically cooled front-end assemblies, it has become the practice to define antenna system noise temperature at the horn aperture rather than at the low-noise amplifier (LNA) input port. A front-end assembly usually consists of only the horn, waveguide, and LNA. For convenience and conciseness, the symbol T_{op} instead of $T_{op,a}$ will be used to mean *antenna* system operating noise temperature.

It is not generally known that when the reference port is moved towards the horn aperture, the T_{op} value and effective receiver noise temperature, T_e, become larger and the antenna input termination noise temperature, T_{ia}, becomes smaller. This fact is not obvious from the use of approximate formulas that neglect loss factors of the associated system components. For most applications, the loss factors are sufficiently close to unity that it is permissible to use the approximate formulas. However, when deriving

[1] Communications Ground Systems Section.

The research described in this publication was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

exact formulas of T_{op} at different ports, it is necessary to retain all loss factors, L, between the source and the receiver input.

In 1968, in the Appendix of [1], co-author Stelzried derived the basic formula for transferring system noise temperature from the receiver port to the horn aperture port, where the two ports were separated by a matched lossy transmission line. In the same special issue on noise as [1], Otoshi derived a similar expression that applied to mismatched cascaded components [2]. In a follow-up internal memorandum,[2] the formulas for transfer (between ports) for all components of a receiving system were presented for the matched case. The formula from [1] appeared again in a tutorial reference document in 1982 [3]. Although these basic formulas were derived more than 30 years ago, their applications were not widely utilized until recently, when it was decided by developers of low-noise systems at JPL that T_{op} should be defined at the antenna aperture rather than at the LNA input port. One of the early applications of defining T_{op} at the feedhorn aperture was in 1989, when Gatti et al. measured the performance of the DSS-14 70-m antenna at 32 GHz [4].

The purpose of this article is to provide a more current reference on the derivation of exact formulas for calculating T_{op} at different reference ports and to show the magnitude of errors that result if the new formula is not used. In the following, it will first be shown how the exact T_{op} expressions are derived when the reference port is the LNA input port. Then the exact T_{op} expression will be derived when the reference port is moved to the horn aperture. The exact and approximate formulas will be compared, and sample calculations will be made on a DSN 8.4-GHz (X-band) feed system and a DSN 32-GHz (Ka-band) feed system, showing the magnitude of errors that result when using the approximate formula instead of the exact formula.

II. Derivations

A. Antenna Operating System Noise Temperature Defined at Port B

Figure 1(a) shows a block diagram of a simple antenna receiving system where the antenna is a horn connected to a waveguide section, which in turn is connected to a receiver consisting of an LNA and a follow-up receiver. For this configuration, the antenna input termination (per IEEE standard terminology [5]) at port B consists of the sky and the horn waveguide assembly. In a typical system, the horn assembly consists of a corrugated horn and additional waveguide components, such as a spacer, rotary joint, polarizer or orthomode, and finally the circular-to-rectangular waveguide transition, which connects to the LNA. For purposes of this article, the corrugated horn section will be called the "horn," and the additional waveguide components between the horn output and the LNA input will be called the "waveguide." For a more complete system that consists of a Cassegrain main reflector, subreflector, mirrors, and the beam-waveguide (BWG) antenna shroud, one can refer to a more general BWG antenna system operating noise temperature equation that was presented by Otoshi et al. [6]. For the tutorial purpose of this article, only noise temperature equations for the simple antenna system [shown in Fig. 1(a)] will be derived.

A subscript A will be used for defining noise temperature components belonging to the system where the reference port is the horn aperture, and a subscript B will be used to identify noise temperature components belonging to the system where the reference port is the LNA input port. When the T_{op} reference port is port B [shown in Fig. 1(a)], T_{op} is expressed as

$$(T_{op})_B = (T_{ia})_B + (T_e)_B \tag{1}$$

[2] C. Stelzried, T. Otoshi, and B. Seidel, "Noise Temperature Symbols and Definitions," JPL Interoffice Memorandum 3333-68-508 (internal document), Jet Propulsion Laboratory, Pasadena, California, November 11, 1968.

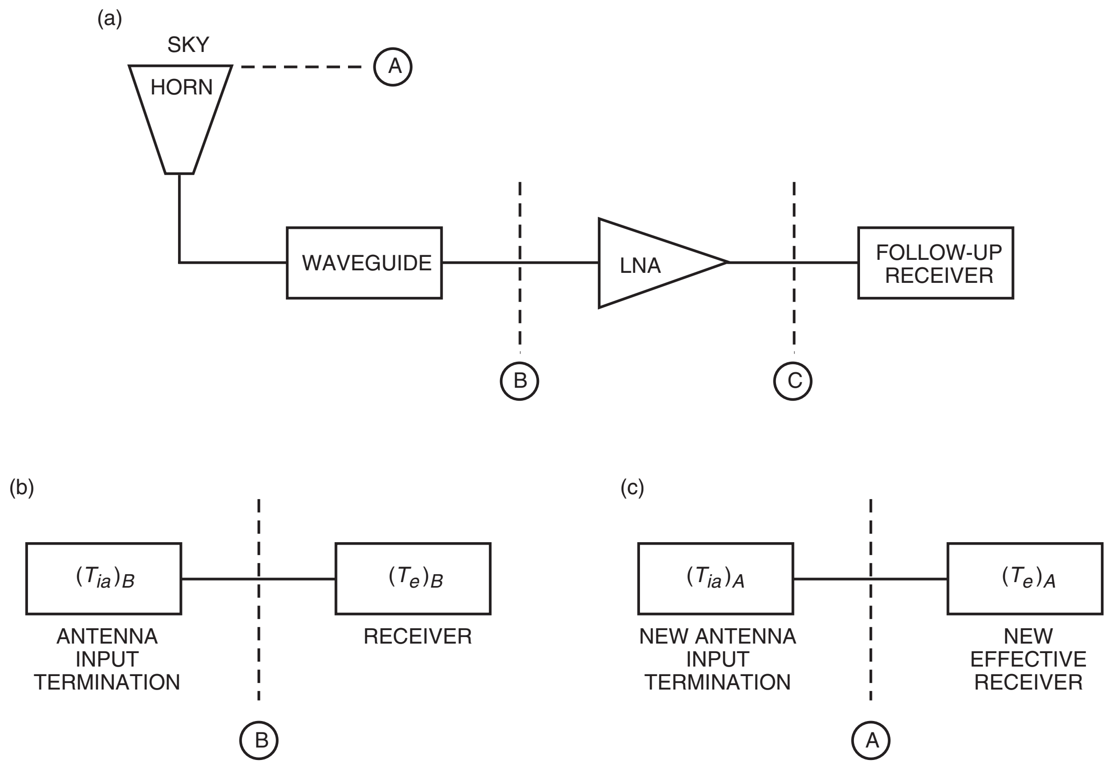

Fig. 1. Typical antenna receiving system: (a) all components of the system shown, (b) equivalent antenna input termination and receiver defined at LNA input port B, and (c) equivalent antenna input termination and receiver defined at the horn aperture.

where T_{ia} is the noise temperature of the antenna input termination and T_e is the effective input noise temperature of the receiver [5,7]. Since these two components of T_{op} at port B will be compared later to corresponding antenna and receiver components at port A, they will be discussed separately in the following two subsections.

1. Antenna Input Termination at Port B. In Fig. 1(b), the antenna input termination consists of the sky and the horn plus the waveguide. Its noise temperature can be expressed as

$$(T_{ia})_B = (T_{\rm sky})_B + (T_{h,wg})_B \tag{2}$$

where

$$(T_{\rm sky})_B = T_{cb}\left(L_{atm}L_{\rm horn}L_{wg}\right)^{-1} + T_{atm}\left(L_{\rm horn}L_{wg}\right)^{-1} \tag{3}$$

and

$$(T_{h,wg})_B = T_{\rm horn}L_{wg}^{-1} + T_{wg} \tag{4}$$

where

$$T_{cb} = \text{ noise temperature of the cosmic background radiation, K}$$

$$T_{atm} = \text{ noise temperature of the atmosphere defined at its output, K}$$

$$T_{\text{horn}} = \text{ noise temperature of the horn defined at its output port, K}$$

$$T_{wg} = \text{ noise temperature of the waveguide defined at its output port, K}$$

and L_{atm}, L_{horn}, and L_{wg} are the loss factors ($\geq$1) of the atmosphere, horn, and waveguide, respectively. For a general-case matched two-port network, the reciprocal of L is equal to a power ratio of $P_{\text{out}}/P_{\text{in}}$. Note in Eq. (3) that T_{sky} is just the sum of the cosmic background noise temperature, T_{cb}, and the atmosphere noise temperature, T_{atm}, attenuated by the losses of the particular components/media in front of them before reaching port B.

Substitutions of Eqs. (3) and (4) into Eq. (2) give the exact expression for the *antenna input termination at port B* of

$$(T_{ia})_B = T_{cb}\left(L_{atm}L_{\text{horn}}L_{wg}\right)^{-1} + T_{atm}\left(L_{\text{horn}}L_{wg}\right)^{-1} + T_{\text{horn}}L_{wg}^{-1} + T_{wg} \tag{5}$$

If one wishes to calculate noise temperature contributions of each component at its individual output port, the following formulas may be used:

$$T_{atm} = \left(1 - L_{atm}^{-1}\right) T_{p,atm} \tag{6}$$

$$T_{\text{horn}} = \left(1 - L_{\text{horn}}^{-1}\right) T_{p,\text{horn}} \tag{7}$$

$$T_{wg} = \left(1 - L_{wg}^{-1}\right) T_{p,wg} \tag{8}$$

where

$T_{p,atm}$ = effective or mean physical temperature of the atmosphere, K (a value sometimes used is 277.5 K for 90 percent cumulative distribution [8])

$T_{p,\text{horn}}$ = physical temperature of the horn, K

$T_{p,wg}$ = physical temperature of the waveguide components, K

Derivation of Eq. (5) is given in Appendix A for an equivalent configuration of a noise source followed by three lossy passive networks. Note that to calculate the individual contributions given in Eqs. (6) through (8) it is necessary to know only the individual loss factors and the physical temperatures of the components.

Special Case: If an aperture load at physical temperature T_p is placed over the horn, T_{cb} and T_{atm} are blocked out, and if $T_{p,\text{horn}} = T_{p,wg} = T_p$, then Eq. (5) becomes

$$(T_{ia})_B = T_p\left(L_{\text{horn}}L_{wg}\right)^{-1} + T_{\text{horn}}L_{wg}^{-1} + T_{wg}$$

and substitution of Eqs. (7) and (8) leads to

$$\begin{aligned}(T_{ia})_B &= T_p\left(L_{\text{horn}}L_{wg}\right)^{-1} + T_p\left(1 - L_{\text{horn}}^{-1}\right)L_{wg}^{-1} + \left(1 - L_{wg}^{-1}\right)T_p \\ &= T_p \end{aligned} \tag{5a}$$

which shows why an aperture load can still be used as the ambient load noise standard for calibrating antenna system T_{op} defined at Port B. For the calibration of antenna system T_{op} at port B using the aperture load, it is required that (1) T_e be defined at Port B and (2) the aperture load, horn, and waveguide be at the same physical temperature, T_p. If any of these components are not at the same physical temperature, an error will occur in calibrating antenna system noise temperatures when using the aperture load for the ambient load thermal noise standard method described in [6] and [9].

General Case: If the losses are known in dB, then loss factors are calculated from the basic formula

$$L = 10^{L_{\mathrm{dB}}/10} \tag{9}$$

where L_{dB} is the dissipative loss of the component in positive dB. It is assumed that the system is matched so that it can be assumed that the total loss and dissipative loss are the same. If the loss in dB is less than 0.1 dB, the following useful approximate formulas can be derived from a series expansion of $\log_{10} L$ in the exact expression $L_{\mathrm{dB}} = 10 \log_{10} L$. Dropping off higher-order terms assuming $L \approx 1$, then

$$L \approx 1 + 0.23 L_{\mathrm{dB}} \tag{10a}$$

and

$$L^{-1} \approx 1 - 0.23 L_{\mathrm{dB}} \tag{10b}$$

For example, if the waveguide component has a loss of 0.05 dB and a physical temperature of 300 K, then

$$L_{wg} \approx 1.0115$$

and from Eq. (8)

$$T_{wg} = \left(1 - L_{wg}^{-1}\right) T_p = 3.41 \text{ K} \approx 0.23 L_{\mathrm{dB}} T_p = 3.45 \text{ K}$$

In Eq. (6), L_{atm} and T_{atm} can be calculated from knowledge of frequency, measured ground-level barometric pressure, ground-level relative humidity, and the physical temperature of air at the ground level. Slobin has written a BASIC program named SDSATM4.BAS (later modified by Otoshi to become SDSATM7.BAS [10]) to enable these calculations to be done easily. Values of T_{atm} for different cumulative distributions for Goldstone, Canberra, and Madrid can be obtained for different frequency bands from Table 1 in [8].

2. Effective Input Noise Temperature of the Receiver at Port B. The effective input noise temperature of the receiver at port B is expressed as the sum of the contributions from the LNA and the follow-up receiver as follows:

$$(T_e)_B = T_{\mathrm{LNA}} + T_{fu} \tag{11}$$

where

T_{LNA} = effective input noise temperature of the LNA at port B, K

T_{fu} = effective input noise temperature of the follow-up receiver at port B, K

If one wishes to express T_{fu} in terms of the follow-up receiver noise temperature contribution at its own input port, the following expression is used:

$$T_{fu} = \frac{T_f}{G_{\text{LNA}}} \tag{12}$$

where

$T_f =$ effective input noise temperature of the follow-up receiver at its own input port, K (see port C in Fig. 1)

$G_{\text{LNA}} =$ gain of the LNA

Substitution of Eq. (12) into Eq. (11) gives an alternate expression for $(T_e)_B$ of

$$(T_e)_B = T_{\text{LNA}} + \frac{T_f}{G_{\text{LNA}}} \tag{13}$$

3. Antenna System Operating Noise Temperature at Port B. Substitution of the expression for $(T_{ia})_B$ given in Eq. (5) and $(T_e)_B$ given in Eq. (11) into Eq. (1) gives the exact equation for the antenna system operating noise temperature at Port B of

$$(T_{op})_B = T_{cb}\left(L_{atm}L_{\text{horn}}L_{wg}\right)^{-1} + T_{atm}\left(L_{\text{horn}}L_{wg}\right)^{-1} + T_{\text{horn}}L_{wg}^{-1} + T_{wg} + T_{\text{LNA}} + T_{fu} \tag{14}$$

The term “exact” means that in Eq. (14) the loss factor products have not been omitted. In practice, it is commonly assumed that L_{atm}, L_{horn}, and L_{wg} are each very close to unity so that the expression for $(T_{ia})_B$ shown in Eq. (5) becomes

$$\text{Approx}\,(T_{ia})_B = T_{cb} + T_{atm} + T_{\text{horn}} + T_{wg} \tag{15}$$

In Eq. (13), $(T_e)_B$ remains unchanged, so Eq. (14) becomes

$$\begin{aligned}\text{Approx}\,(T_{op})_B &= \text{Approx}\,(T_{ia})_B + \text{Approx}\,(T_e)_B \\ &= T_{cb} + T_{atm} + T_{\text{horn}} + T_{wg} + T_{\text{LNA}} + T_{fu}\end{aligned} \tag{16}$$

and the error resulting from use of this approximate formula for Port B is

$$E_B = \text{Approx}\,(T_{op})_B - (T_{op})_B \tag{17}$$

Even though the above approximate formula for $(T_{op})_B$ may be sufficiently accurate for application to some low-noise systems, it can be misleading to the lay person and can be mistaken for the exact equation. The approximate formula may not be accurate for some antenna systems where the horn and waveguide losses have significant values (>0.2 dB) or at Ka-band frequencies where L_{atm} is not close to unity. In the following subsection, the expression for T_{op} at port A will be derived, and then comparisons will be made of the components of $(T_{op})_A$ to corresponding components of $(T_{op})_B$.

B. Antenna System Operating System Noise Temperature Defined at the Horn Aperture

If one wishes to define the antenna system operating noise temperature at the antenna aperture and to use port A as the reference port rather than the LNA input (port B) as shown in Fig. 1, the following relationship applies:

$$(T_{op})_A = (T_{ia})_A + (T_e)_A \tag{18}$$

where, from inspection of Fig. 1(a), it can be seen that the new expression for *antenna input termination* is simply

$$(T_{ia})_A = T_{\text{sky}} = T_{cb} L_{atm}^{-1} + T_{atm} \tag{19}$$

As is pointed out in Appendix B, noise temperatures defined at a particular output port can be transferred to the antenna aperture by multiplying those noise temperatures by the loss factors of the components between the old and new reference ports. Then, application of the method given in Appendix B, and inspection of Fig. 1(c), leads to the following expression for *effective input receiver noise temperature at port A*:

$$(T_e)_A = \left[(T_{h,wg})_B + (T_e)_B \right] L_{\text{horn}} L_{wg} \tag{20}$$

Substitution of Eqs. (4) and (11) into Eq. (19) gives

$$(T_e)_A = \left(T_{\text{horn}} L_{wg}^{-1} + T_{wg} + T_{\text{LNA}} + T_{fu} \right) L_{\text{horn}} L_{wg} \tag{21}$$

Equation (21) can also be derived from a cascade formula often used by designers of LNAs. This alternate derivation is given in Appendix C.

Substitutions of Eqs. (19) and (21) into Eq. (18) result in

$$(T_{op})_A = T_{cb} L_{atm}^{-1} + T_{atm} + T_{\text{horn}} L_{\text{horn}} + (T_{wg} + L_{\text{LNA}} + T_{fu}) L_{\text{horn}} L_{wg} \tag{22}$$

Examination of Eqs. (14) and (22) reveals that

$$(T_{op})_A = (T_{op})_B L_{\text{horn}} L_{wg} \tag{23}$$

This equation is identical to the equation derived by Stelzried (in the Appendix of [1]) if the symbol L in [1] is used to represent the product $L_{\text{horn}} L_{wg}$.

It is now of interest to compare the individual terms of $(T_{op})_A$ given above in Eq. (22) to the individual terms of $(T_{op})_B$ shown previously in Eq. (14). They are noticeably different. For convenience and future reference, definitions of all symbols used thus far are tabulated in Table 1, and the formulas for computing an individual component's contribution of $(T_{op})_B$ and $(T_{op})_A$ are given in Table 2. Formulas for the components of T_{ia} and T_e at each of the two ports are also shown in Table 2.

Table 1. Definitions of symbols.[a]

Symbol	Definition
$(T_{op})_A, (T_{op})_B$	Antenna system operating noise temperatures at ports A and B, respectively, K.
$(T_{ia})_A, (T_{ia})_B$	Antenna input termination noise temperatures at ports A and B, respectively, K.
$(T_e)_A, (T_e)_B$	Effective input noise temperatures of the receiver at ports A and B, respectively, K.
T_{cb}	Cosmic background noise temperature, K. See [8] for values at S-, X-, and Ka-bands.
T_{atm}	Noise temperature of the atmosphere defined at the horn aperture port, K.
T_{sky}	Noise temperature looking at the sky from the horn aperture port. It is equal to the sum of T_{atm} and T_{cb} attenuated by the atmosphere, K.
T_{horn}	Noise temperature of the horn only at its output port, K.
T_{wg}	Noise temperature of the waveguide (components) at its output port, K.
$T_{p,atm}$	Effective or mean physical temperature of the atmosphere [8], K.
$T_{p,\text{horn}}$	Physical temperature of the horn, K.
$T_{p,wg}$	Physical temperature of the waveguide components between the horn output and LNA input, K.
T_{fu}	Effective input noise temperature of the follow-up receiver noise defined at the LNA input port, K.
T_f	Effective input noise temperature of the follow-up receiver at its own input port, K.
G_{LNA}	Gain of the LNA.
$L_{atm}, L_{\text{horn}}, L_{wg}$	Loss factors of the atmosphere, horn, and waveguide, respectively. The values of these loss factors are each greater than or equal to one.

[a] Underlined words are the IEEE definitions given in [5]. Normally, the symbol $T_{op,a}$ would be used to indicate the antenna system operating noise temperature. For conciseness and convenience in this article, the a will be omitted from $T_{op,a}$, which will be shown simply as T_{op}. Subscripts A and B outside the parentheses identify the port at which the noise temperature is defined.

In practice, it is often assumed that L_{atm}, L_{horn}, and L_{wg} are each very close to unity so that

$$\text{Approx}\,(T_{ia})_A = T_{cb} + T_{atm} \tag{24}$$

and

$$\text{Approx}\,(T_e)_A = T_{\text{horn}} + T_{wg} + T_{\text{LNA}} + T_{fu} \tag{25}$$

so that

$$\begin{aligned}\text{Approx}\,(T_{op})_A &= \text{Approx}\,(T_{ia})_A + \text{Approx}\,(T_e)_A \\ &= T_{cb} + T_{atm} + T_{\text{horn}} + T_{wg} + T_{\text{LNA}} + T_{fu} \end{aligned} \tag{26}$$

and the error resulting from use of this approximate formula for port A is

$$E_A = \text{Approx}\,(T_{op})_A - (T_{op})_A \tag{27}$$

Table 2. Comparisons of individual contributions to T_{op} when T_{op} is defined at the horn aperture versus at the LNA input.[a]

Contributor	Contributions to T_{op} when defined at LNA input port B	Contributions to T_{op} when defined at horn aperture port A
Cosmic background	$T_{cb}\left(L_{atm}L_{\mathrm{horn}}L_{wg}\right)^{-1}$	$T_{cb}L_{atm}^{-1}$
Atmosphere	$T_{atm}\left(L_{\mathrm{horn}}L_{wg}\right)^{-1}$	T_{atm}
Horn	$T_{\mathrm{horn}}L_{wg}^{-1}$	$T_{\mathrm{horn}}L_{\mathrm{horn}}$
Waveguide	T_{wg}	$T_{wg}L_{wg}L_{\mathrm{horn}}$
LNA	T_{LNA}	$T_{\mathrm{LNA}}L_{wg}L_{\mathrm{horn}}$
Follow-up	$T_{fu} = T_f/G_{\mathrm{LNA}}$	$T_{fu}L_{wg}L_{\mathrm{horn}}$
Antenna input termination	$\left(T_{ia}\right)_B$ = sum of contributions from cosmic background + atmosphere + horn + waveguide	$\left(T_{ia}\right)_A$ = sum of contributions from cosmic background + atmosphere
Effective receiver	$\left(T_e\right)_B$ = sum of contributions of the LNA + follow-up	$\left(T_e\right)_A$ = sum of contributions of the horn + waveguide + LNA + follow-up
Antenna operating system	$\left(T_{op}\right)_B = \left(T_{ia}\right)_B + \left(T_e\right)_B$	$\left(T_{op}\right)_A = \left(T_{ia}\right)_A + \left(T_e\right)_A$

[a] Note that $\left(T_{op}\right)_A = \left(T_{op}\right)_B \times L_{\mathrm{horn}}L_{wg}$.

Comparison of Eqs. (25) and (16) shows that

$$\mathrm{Approx}\left(T_{op}\right)_A = \mathrm{Approx}\left(T_{op}\right)_B \tag{28}$$

It will be shown by the following examples that the approximate formulas might be sufficiently accurate for the case of a cryogenically cooled horn and an ultra-low-loss system, but not accurate for some lossier systems. If it is important to calculate $\left(T_{op}\right)_A$ values to within a 0.2-K accuracy, then the exact formula of Eq. (22) should be used.

III. Sample Cases

In the following, calculations will be made for two examples, showing how much T_{ia}, T_e, and T_{op} differ at the LNA input port versus at the horn aperture. These calculations will be done for a typical DSN cryogenically cooled horn system developed for (1) 8.4 GHz (X-band) and (2) 32 GHz (Ka-band). It will be shown that, for typical DSN low-noise systems, the error in using approximate formulas instead of the exact expressions is small (about 0.1 K) at X-band but can be significant (as large as 1 K) at Ka-band.

Example 1. DSN Cryogenically Cooled Horn System at 8.4 GHz. The input data are as follows:

$T_{cb} =$ 2.5 K adjusted for frequency

$T_{atm} =$ 2.29 K for average clear weather at Goldstone and cumulative distribution (CD) = 0.25 [8], zenith

$\left(L_{atm}\right)_{\mathrm{dB}} =$ 0.038 dB (for CD = 0.25 [8]), zenith

$\left(L_{\mathrm{horn}}\right)_{\mathrm{dB}} =$ 0.035 dB

$$\left(L_{wg}\right)_{\mathrm{dB}} = 0.057 \text{ dB}$$

$$T_{p,\mathrm{horn}} = 6 \text{ K (note that the horn is at cryogenic temperature)}$$

$$T_{p,wg} = 6 \text{ K (note that the waveguide is at cryogenic temperature)}$$

The horn and waveguide losses are not actual values for a current DSN X-band system, but are estimates for example purposes only. For the receiver,

$$T_{\mathrm{LNA}} = 4.9 \text{ K}$$

$$\text{LNA gain} = 25 \text{ dB or } G_{\mathrm{LNA}} = 10^{2.5} = 316.2$$

$$T_f = 31.62 \text{ K}$$

From use of Eq. (12),

$$T_{fu} = \frac{31.62}{316.2} = 0.1 \text{ K}$$

The output data are as follows.[3] Use of the formula for converting loss factors in dB to power ratios [see Eq. (9)] gives

$$L_{atm} = 1.0088$$

$$L_{\mathrm{horn}} = 1.0081$$

$$L_{wg} = 1.0132$$

From use of Eqs. (7) and (8),

$$T_{\mathrm{horn}} = 0.048 \text{ K (note that the horn is cooled to a 6-K cryogenic physical temperature)}$$

$$T_{wg} = 0.078 \text{ K (note that the waveguide is cooled to a 6-K cryogenic physical temperature)}$$

Now all of the values are known for calculation of T_{op} at ports A and B. Substitution of these values into Eqs. (5), (11), and (14) for port B (also see Table 2) gives

$$\left(T_{ia}\right)_B = 4.79 \text{ K}$$

$$\left(T_e\right)_B = 5 \text{ K}$$

$$\left(T_{op}\right)_B = 9.79 \text{ K}$$

Substitution of values into Eqs. (19), (21), and (22) for port A respectively gives

$$\left(T_{ia}\right)_A = T_{\mathrm{sky}} = T_{cb} L_{atm}^{-1} + T_{atm} = 2.5\,(1.0088)^{-1} + 2.29 = 4.768 \text{ K}$$

$$\left(T_e\right)_A = 5.236 \text{ K}$$

$$\left(T_{op}\right)_A = 10.003 \text{ K (from use of Eq. (22) instead of adding the two above results)}$$

Values are shown to three decimal places because rounding off to two decimal places will make it appear that the stated $(T_{op})_A$ value is incorrect. Note that $\left(T_{op}\right)_A - \left(T_{op}\right)_B = 0.21$ K and that $\left(T_{op}\right)_A = L_{\mathrm{horn}} L_{wg} \times \left(T_{op}\right)_B$. If the approximate formulas of Eqs. (16) and (26) are used, then

[3] A copy of an Excel program for performing calculations of $(T_{ia})_B$, $(T_e)_B$, and $(T_{op})_B$, and $(T_{ia})_A$, $(T_e)_A$, and $(T_{op})_A$ can be obtained from the author upon request.

$$\text{Approx}\,(T_{op})_A = \text{Approx}\,(T_{op})_B = T_{cb} + T_{atm} + T_{\text{horn}} + T_{wg} + L_{\text{LNA}} + T_{fu} = 9.92 \text{ K}$$

The errors resulting from use of this approximate formula as calculated from Eqs. (17) and (27) are $E_B = 0.12$ K and $E_A = -0.09$ K. A plus/minus error sign means the approximate value is higher/lower than the exact value. The approximate formula is useful for obtaining quick estimates of T_{op} since only arithmetic addition is required. If T_{op} values are to be documented to two decimal places in a formal report, one should use the correct formulas. One way to check the correctness of the T_{op} value is to use the rule that there should always be a difference in the values of T_{op} if they are defined at different port locations. In addition, the T_{op} value at the horn aperture must always be higher than the T_{op} value at the LNA input port. In this X-band example, $(T_{op})_A$ was higher than $(T_{op})_B$ by 0.21 K. *It is important to point out that the differences between T_{op} values at different ports are not errors. Only the difference between the approximate value and the exact value is defined as an error.*

If $T_{p,\text{horn}}$ and $T_{p,wg}$ were at ambient 295 K instead of 6 K, and assuming loss factors remain the same, new calculations will show that $(T_{op})_A$ would be higher than $(T_{op})_B$ by 0.34 K and the errors due to using the approximate formula would be $E_B = 0.15$ K and $E_A = -0.19$ K. If the horn and waveguide are made from high-conductivity copper, their loss factors typically decrease at cryogenic temperatures. For simplicity, their loss factors are assumed to remain constant between 295 K and 6 K.

Even though the errors in neglecting loss factors are small (about 0.1 K) for X-band, the following example for Ka-band will show that, due to higher atmospheric losses and waveguide losses, the error resulting from use of the approximate formula can be as large as 1 K.

Example 2. DSN Cryogenically Cooled Horn System at 32 GHz and at Goldstone, California. The input data are as follows:

$$\begin{aligned}
T_{cb} &= 2.0 \text{ K adjusted for frequency [8]} \\
T_{atm} &= 9.12 \text{ K for average clear weather, CD} = 0.25 \text{ [8], zenith} \\
(L_{atm})_{\text{dB}} &= 0.154 \text{ dB for average clear weather, CD} = 0.25 \text{ [8], zenith} \\
(L_{\text{horn}})_{\text{dB}} &= 0.15 \text{ dB} \\
(L_{wg})_{\text{dB}} &= 0.2 \text{ dB (purposely made high for this example)} \\
T_{p,\text{horn}} &= 6 \text{ K (note that the horn is at a cryogenic temperature)} \\
T_{p,wg} &= 6\ K \text{ (note that the waveguide is at a cryogenic temperature)}
\end{aligned}$$

The horn and waveguide losses are not actual losses for a typical Ka-band system, but are estimates for example purposes only.

For the receiver, the example values are as follows:

$$\begin{aligned}
T_{\text{LNA}} &= 15 \text{ K} \\
\text{LNA gain} &= 25 \text{ dB} \\
T_f &= 63.25 \text{ K} \\
T_{fu} &= 0.2 \text{ K}
\end{aligned}$$

The output data are as follows. Use of the formula for converting loss factors in dB to power ratios gives

$$
\begin{aligned}
L_{atm} &= 1.0361 \\
L_{\text{horn}} &= 1.0351 \\
L_{wg} &= 1.0471
\end{aligned}
$$

and substitutions into Eqs. (7) and (8) give

$$
\begin{aligned}
T_{\text{horn}} &= 0.204 \text{ K (note that the horn is cooled to a 6-K cryogenic physical temperature)} \\
T_{wg} &= 0.27 \text{ K (note that the waveguide is cooled to a 6-K cryogenic physical temperature)}
\end{aligned}
$$

Now all of the values are known for calculation of T_{op} at ports A and B. Ignoring small differences that could occur due to round-off, substitution of these values into Eqs. (5), (11), and (14) for port B gives

$$
\begin{aligned}
(T_{ia})_B &= 10.66 \text{ K} \\
(T_e)_B &= 15.2 \text{ K} \\
(T_{op})_B &= 25.86 \text{ K}
\end{aligned}
$$

and substitutions into Eqs. (19), (21), and (22) for port A give

$$
\begin{aligned}
(T_{ia})_A &= T_{\text{sky}} = 11.05 \text{ K} \\
(T_e)_A &= 16.98 \text{ K} \\
(T_{op})_A &= 28.03 \text{ K}
\end{aligned}
$$

Note that $(T_{op})_A - (T_{op})_B = 2.17$ K and, as stated by Eq. (22), $(T_{op})_A = L_{\text{horn}} L_{wg} \times (T_{op})_B$. If the approximate formula is used ,

$$
\text{Approx}\,(T_{op})_A = \text{Approx}\,(T_{op})_B = T_{cb} + T_{atm} + T_{\text{horn}} + T_{wg} + L_{\text{LNA}} + T_{fu} = 26.79 \text{ K}
$$

and the errors resulting from use of this approximate formula, as calculated from Eqs. (17) and (27), are $E_B = 0.93$ K and $E_A = -1.24$ K.

If $T_{p,\text{horn}}$ and $T_{p,wg}$ were 295 K instead of 6 K, and assuming loss factors remain constant with physical temperature, new calculations would show that $(T_{op})_A$ would be higher than $(T_{op})_B$ by 4.05 K, and the errors due to use of the approximate formula would be $E_B = 1.38$ K and $E_A = -2.67$ K. These errors are unacceptably large.

IV. Summary and Concluding Remarks

It is hoped that this article will serve as a useful tutorial guide for DSN microwave engineers being introduced to the subject of system and component noise temperature definitions for the first time. Equations have been presented for calculating the exact T_{op} when defined at the input to the LNA and also at the horn aperture.[4] Examples showed that, when the T_{op} reference port was moved from the LNA input port to the horn aperture, the T_{op} value at the horn aperture was higher than the T_{op} value at the LNA input port by as much as 0.21 K for a typical DSN X-band and 2.17 K for a Ka-band horn–receiver system. A verification rule to use is that T_{op} at the horn aperture will always be larger than T_{op} calculated

[4] This methodology is general and can be used to derive T_{op} equations at any reference port further up the system or below the horn aperture.

at the LNA input. The difference between T_{op} values at different ports is not an error. An error results only if one uses the approximate formula to calculate T_{op} at different ports.

The magnitudes of errors resulting from use of the approximate formula were about 0.1 K and 1 K, respectively, for the examples given at X-band and Ka-band. It appears that for quick calculation purposes and to obtain a rough value, the approximate formula can be useful. However, for reporting purposes, and especially for Ka-band systems, the correct equations need to be used to calculate T_{op}. In addition, when reporting T_{op} values, it is important to name the reference port for which the T_{op} values were calculated or measured. If this is not done, T_{op} values measured or reported for the horn aperture reference port will become mixed with T_{op} values measured or reported for the LNA input reference port, and lead to an erroneous conclusion that something in the system has changed.

It was brought to this author's attention that the current method being used by some at JPL for calibrating $(T_e)_A$ (of cryogenically cooled horn front-end receiving systems) is to measure $(T_{op})_A$ with the front-end system on the ground and with the horn pointed at zenith. Then, from measured ground-level barometric pressure, relative humidity, and air temperature, the zenith T_{atm} is calculated through the use of computer programs similar to SDSATM7.BAS [10]. Then, $(T_e)_A$ is calculated from

$$(T_e)_A = (T_{op})_A - (T_{ia})_A = (T_{op})_A - T_{\text{sky}} \tag{29}$$

where from Eq. (19)

$$T_{\text{sky}} = T_{cb} L_{atm}^{-1} + T_{atm}$$

and from Eq. (6)

$$(L_{atm})^{-1} = 1 - \frac{T_{atm}}{T_{p,atm}}$$

and values for T_{cb} and $T_{p,atm}$ can be obtained from [8]. An improvement of this method would be to simultaneously measure T_{atm} with a water vapor radiometer similar to that described by Keihm [11]. Another method would be to tip the horn to a 30-deg elevation angle and measure T_{op}. The difference of T_{op} measured at 30 deg and T_{op} at zenith would result in a measurement of T_{atm}.

The analysis presented in this article assumes a matched system where reflection coefficients of the antenna input termination and receiver are zero. If these reflection coefficients are non-zero and are different at ports A and B, mismatch error analyses similar to the one discussed in [12] may have to be done.

Acknowledgments

The author would like to thank C. T. Stelzried of JPL for critiquing this article and W. Veruttipong for encouragement and support for writing this article.

References

[1] B. Seidel and C. T. Stelzried, "A Radiometric Method for Measuring the Insertion Loss of Radome Materials," *IEEE Trans. on Microwave Theory and Techniques*, Special Issue on Noise, vol. MTT-16, no. 9, pp. 625–628, September 1968. (Equations in the Appendix, p. 628, were derived by C. T. Stelzried.)

[2] T. Y. Otoshi, "The Effect of Mismatched Components on Microwave Noise Temperature Calibrations," *IEEE Transactions on Microwave Theory and Techniques*, Special Issue on Noise, vo1. MTT-16, no. 9, pp. 675–686, September 1968.

[3] C. T. Stelzried, *The Deep Space Network—Noise Temperature Concepts, Measurements, and Performance*, JPL Publication 82-33, Jet Propulsion Laboratory, Pasadena, California, p. 8-2, September 15, 1982. (Note that in this reference the second term on the right-hand side of Eq. (8-5) should be preceded by a minus sign rather than a plus sign, and the second term on the right-hand side of Eq. (8-6) should be preceded by a plus sign rather than a minus sign. This correction will be included in any future distribution of this publication.)

[4] M. S. Gatti, M. J. Klein, and T. B. H. Kuiper, "32-GHz Performance of the DSS-14 70-Meter Antenna: 1989 Configuration," *The Telecommunications and Data Acquisition Progress Report 42-99, July–September 1989*, Jet Propulsion Laboratory, Pasadena, California, pp. 206–219, November 15, 1989.
http://tmo.jpl.nasa.gov/tmo/progress_report/42-99/99R.PDF

[5] "IRE Standards on Electron Tubes: Definition of Terms 1962 (62 IRE 7.S2)," *Proc. IEEE*, vol. 51, pp. 434-435, March 1963.

[6] T. Y. Otoshi, S. R. Stewart, and M. M. Franco, "A Portable Ka-Band Front-End Test Package for Beam-Waveguide Antenna Performance Evaluation—Part II: Tests on the Antenna," *The Telecommunications and Data Acquisition Progress Report 42-106, April–June 1991*, Jet Propulsion Laboratory, Pasadena, California, pp. 266–282, August 15, 1991.
http://tmo.jpl.nasa.gov/tmo/progress_report/42-106/106T.PDF

[7] W. W. Mumford and E. H. Sheibe, *Noise Performance Factors in Communication Systems*, Dedham, Massachusetts: Horizon House-Microwave Inc., 1968.

[8] S. D. Slobin, "Atmosphere and Environmental Effects," *DSMS Telecommunications Link Design Handbook*, 810-005, Rev. E, Module 105, Jet Propulsion Laboratory, Pasadena, California, pp. 1–30, November 30, 2000.

[9] C. T. Stelzried, "Operating System Noise-Temperature Calibrations of Low-Noise Receiving Systems," *Microwave Journal*, vol. 14, no. 6, pp. 41–47, June 1971.

[10] T. Y. Otoshi, S. R. Stewart, and M. M. Franco, "A Portable X-Band Front-End Test Package for Beam-Waveguide Antenna Performance Evaluation, Part II: Tests on the Antenna," *The Telecommunications and Data Acquisition Progress Report 42-105, January–March 1991*, Jet Propulsion Laboratory, Pasadena, California, pp. 54–68, May 15, 1991. (See p. 56 for a description of the SDSATM7.BAS program.)
http://tmo.jpl.nasa.gov/tmo/progress_report/42-105/105G.PDF

[11] S. J. Keihm, "Water Vapor Radiometer Measurements of the Tropospheric Delay Fluctuations at Goldstone Over a Full Year," *The Telecommunications and Data Acquisition Progress Report 42-122, April–June 1995*, Jet Propulsion Laboratory, Pasadena, California, pp. 1–11, August 15, 1995.
http://tmo.jpl.nasa.gov/tmo/progress_report/42-122/122J.pdf

[12] T. Y. Otoshi, "Antenna System Noise-Temperature Calibration Mismatch Errors Revisited," *The Interplanetary Network Progress Report 42-148, October–December 2001*, Jet Propulsion Laboratory, Pasadena, California, pp. 1–31, February 15, 2002.
http://ipnpr.jpl.nasa.gov/tmo/progress_report/42-148/148B.pdf

Appendix A

Equivalent Source Noise Temperature at the Output of Several Cascaded Two-Port Networks

The methodology for deriving the basic equation of the antenna input noise temperature at the output of several cascaded networks may be unfamiliar or sometimes forgotten. Therefore, the step-by-step derivation is documented here for reference purposes.

Figure A-1(a) shows a noise source followed by three matched passive two-port networks. Beginning with the first network, the output noise temperature at port 2 is the well-known formula

$$T_s' = T_s L_1^{-1} + \left(1 - L_1^{-1}\right) T_{p1} \tag{A-1}$$

where T_s is the source noise temperature, T_{p1} is the physical temperature of network 1, L_1 is the loss factor (dissipative attenuation) of network 1, and T_s' is the equivalent source temperature at the output port of network 1. The term "equivalent source temperature" as used here means that the source and first network can be combined and represented by a single new equivalent source temperature, as shown in Fig. A-1(b). Then, using this same methodology, the equivalent source temperature at port 3 shown in Fig. A-1(c) is

$$T_s'' = T_s' L_2^{-1} + \left(1 - L_2^{-1}\right) T_{p2} \tag{A-2}$$

and the new equivalent source temperature at port 4 shown in Fig. A-1(d) is

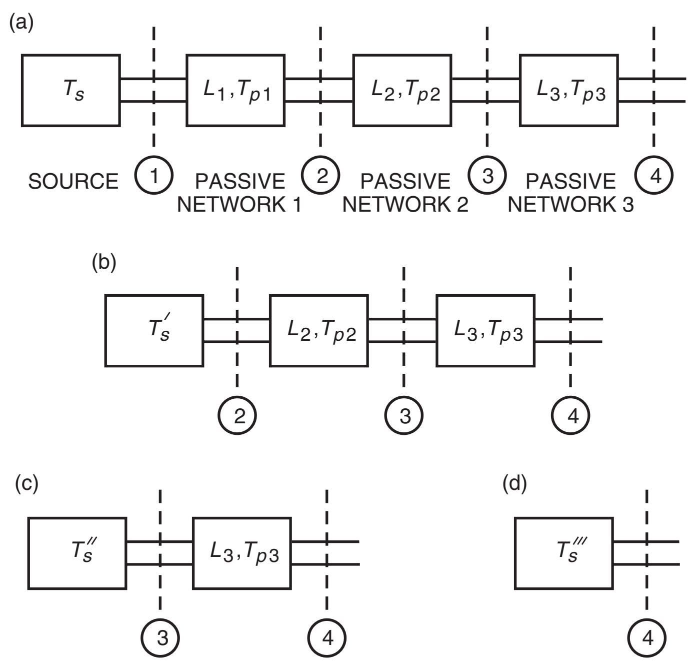

Fig. A-1. Equivalent source temperatures: (a) actual source followed by three passive cascaded two-port networks, (b) equivalent source temperature at port 2, (c) equivalent source temperature at port 3, and (d) equivalent source temperature at port 4.

$$T_s''' = T_s'' L_3^{-1} + \left(1 - L_3^{-1}\right) T_{p3} \tag{A-3}$$

Substitution of Eqs. (A-1) and (A-2) into Eq. (A-3) gives the expression for the equivalent source temperature at port 4:

$$T_s''' = T_s \left(L_1 L_2 L_3\right)^{-1} + \left(1 - L_1^{-1}\right) T_{p1} \left(L_2 L_3\right)^{-1} + \left(1 - L_2^{-1}\right) T_{p2} L_3^{-1} + \left(1 - L_3^{-1}\right) T_{p3} \tag{A-4}$$

Notice that, in Eq. (A-4), if we let $T_s = T_{cb}$, $T_{p1} = T_{p,atm}$, $T_{p2} = T_{p,\mathrm{horn}}$, and $T_{p3} = T_{p,wg}$, and $L_1 = L_{atm}$, $L_2 = L_{\mathrm{horn}}$, and $L_3 = L_{wg}$, then T_s''' is the same as $\left(T_{ia}\right)_B$ given in Eq. (5) of the main article.

Appendix B

General Equation for Transfer of Noise Temperatures from One Reference Port to Another

This appendix is an extension by Otoshi of the proof already given by Stelzried in the Appendix of [1]. For the simple case, it can be stated that the method for transferring the noise temperature at the output port of a lossy network to the input port of the same network is to use the general relationship

$$T_{\text{in}} = \frac{T_{\text{out}}}{G} \tag{B-1}$$

where G for a lossy network is $1/L$, where L is the loss factor of the network. For example, for the case of a single lossy network,

$$T_{\text{out}} = \left(1 - L^{-1}\right) T_p \tag{B-2}$$

Then, at the input,

$$T_{\text{in}} = \frac{T_{\text{out}}}{G} = \frac{T_{\text{out}}}{(1/L)} = LT_{\text{out}} = (L-1)T_p \tag{B-3}$$

which is an expression that can be found in Mumford [7]. A similar expression was derived by Otoshi [2, Eq. 35] for the mismatched case, where G is replaced by τ, the available transmission factor.

The general transfer equation for n cascaded lossy networks is

$$T_{\text{in}} = (L_1 L_2 \cdots L_n)\, T_{\text{out}} \tag{B-4}$$

where $(L_1 L_2 \cdots L_n)$ is the product of loss factors of n networks. For the example in the main text [see Fig. 1(a)], the two networks between ports B and A are the waveguide transition and horn. Hence,

$$(T_{\text{in}})_A = L_1 L_2 \,(T_{\text{out}})_B \tag{B-5}$$

where $(T_{\text{in}})_A$ corresponds to $(T_{op})_A$, $L_1 = L_{\text{horn}}$, $L_2 = L_{wg}$, and $(T_{\text{out}})_B$ corresponds to $(T_{op})_B$.

One way to become convinced that Eq. (B-3) is correct is to multiply it by L^{-1}. The operation gives

$$T_{\text{out}} = \left(1 - L^{-1}\right) T_p \tag{B-6}$$

which is the familiar equation for a single lossy network.

Appendix C

Alternate Method for Deriving the Equation of the Effective Input Noise Temperature of a Receiver Preceded by Two Lossy Networks

The overall effective input noise temperatures of two networks in cascade with a receiver as shown in Fig. C-1 can be derived from the following general formula (Mumford [7], p. 22):

$$T_{e1,2,3} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} \tag{C-1}$$

where

$$\begin{aligned}
T_{e1}, T_{e2} &= \text{effective input noise temperature, respectively, of networks 1 and 2} \\
T_{e3} &= \text{effective input noise temperature of the receiver} \\
G_1, G_2 &= \text{gain of networks 1 and 2, respectively}
\end{aligned}$$

From comparison of networks in Fig. C-1 with those in Fig. 1(a), we can let networks 1 and 2 be the lossy horn and lossy waveguide, respectively, so that

$$\begin{aligned}
T_{e1,2,3} &= (T_e)_A \\
T_{e1} &= T_{\text{horn}} L_{\text{horn}} \\
T_{e2} &= T_{wg} L_{wg} \\
T_{e3} &= T_{\text{LNA}} + T_{fu} \\
G_1 &= L_{\text{horn}}^{-1} \\
G_2 &= L_{wg}^{-1}
\end{aligned}$$

Substitutions into Eq. (C-1) give

$$\begin{aligned}
(T_e)_A &= T_{\text{horn}} L_{\text{horn}} + T_{wg} L_{wg} L_{\text{horn}} + (T_{\text{LNA}} + T_{fu}) L_{\text{horn}} L_{wg} \\
&= \left(T_{\text{horn}} L_{wg}^{-1} + T_{wg} + T_{\text{LNA}} + T_{fu} \right) L_{\text{horn}} L_{wg}
\end{aligned} \tag{C-2}$$

which is the same as Eq. (21).

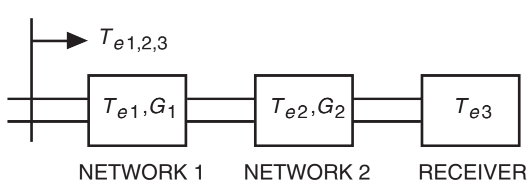

Fig. C-1. Two networks in cascade in front of a receiver.