Radio Losses for Concatenated Codes

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The advent of higher powered spacecraft amplifiers and better ground receivers capable of tracking spacecraft carrier signals with narrower loop bandwidths requires better understanding of the carrier tracking loss (radio loss) mechanism of the concatenated codes used for deep-space missions. In this article, we present results of simulations performed for a (7,1/2), Reed–Solomon (255,223), interleaver depth-5 concatenated code in order to shed some light on this issue. Through these simulations, we obtained the performance of this code over an additive white Gaussian noise (AWGN) channel (the baseline performance) in terms of both its frame-error rate (FER) and its bit-error rate at the output of the Reed–Solomon decoder (RS-BER). After obtaining these results, we curve fitted the baseline performance curves for FER and RS-BER and calculated the high-rate radio losses for this code for an FER of 10^{-4} *and its corresponding baseline RS-BER of* 2.1×10^{-6} *for a carrier loop signal-to-noise ratio (SNR) of 14.8 dB. This calculation revealed that even though over the AWGN channel the FER value and the RS-BER value correspond to each other (i.e., these values are obtained by the same bit SNR value), the RS-BER value has higher high-rate losses than does the FER value. Furthermore, this calculation contradicted the previous assumption that at high data rates concatenated codes have the same radio losses as their constituent convolutional codes. Our results showed much higher losses for the FER and the RS-BER (by as much as 2 dB) than for the corresponding baseline BER of the convolutional code. Further simulations were performed to investigate the effects of changes in the data rate on the code's radio losses. It was observed that as the data rate increased the radio losses for both the FER and the RS-BER approached their respective calculated high-rate values. Furthermore, these simulations showed that a simple two-parameter function could model the increase in the radio losses as the data rate increased for both the FER and the RS-BER. However, further simulations are required to obtain functions for the two parameters in terms of the loop SNR and the error rate for which the loss is calculated.*

I. Introduction

Concatenated codes were first used for deep-space missions on the Voyager I and II spacecraft [1]. Miller et al. [2] developed a methodology to evaluate the performance of these codes over an additive white Gaussian noise (AWGN) channel. This methodology modeled the output of the Viterbi decoder as

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a two-state Markov chain where the decoder is in either a waiting mode or a bursty mode. In the waiting mode, the decoder makes no errors. In the bursty mode, the decoder makes errors. These errors occur close enough to each other as to constitute a "burst" (for an exact definition of a burst see [2]). The duration of these bursts as well as the length of the waiting mode could be modeled as geometric random variables. Following this model, Miller et al. managed to produce a simulation approach that could quickly simulate the performance of concatenated codes without directly simulating the convolutional code. This approach was extremely attractive at the time because simulating convolutional codes is an extremely central processing unit (CPU)-intensive process.

While Miller's approach was ground breaking and gave a quick way of simulating the performance of concatenated codes over AWGN channels, it did not address the issue of radio losses that arise when there is imperfect carrier tracking by the ground receiver. However, at the time the study was published, knowing the radio losses for these codes was not as critical as it currently is. This was because the combination of the data rates (at most of the order of tens of kilobits per second) and the receiver carrier tracking-loop bandwidths (usually greater than 20 Hz) that were used at that time made the radio losses for the concatenated codes equivalent to the radio losses for the inner (convolutional) code of the concatenated code, which were well-known and rather easy to calculate [1]. As far as the outer (Reed–Solomon) code was concerned, the combination of low data rate and large carrier bandwidth made the channel operate in the low-rate regime. In this regime, the same bit-error rate (BER) at the output of the Viterbi decoder produced the same performance at the output of the outer code's decoder. (Briefly, in the low-rate regime, the coherence time of the carrier tracking loop is much smaller than the transmission time of a frame. In the high-rate regime, the tracking loop's coherence time is much larger than the transmission time of a frame. The medium-rate regime implies that the coherence time of the tracking loop is roughly the same as the transmission time of a frame. For convolutional codes, the descriptions of these regimes are identical except for the fact that the frame transmission time is defined as the time it takes to transmit five constraint lengths' worth of bits. More exact definitions of these three regimes are given in Section III).

However, with the advent of the Block V Receiver (BVR) and more powerful spacecraft amplifiers, the carrier tracking-loop bandwidths that are used can be quite narrow (about 3 Hz), and the data rates can be quite high ($\sim$1 Mb/s). Therefore, the low-rate model for the outer code no longer applies. Furthermore, the set of data rates, the loop bandwidths, and the interleaving depths that are used by most deep-space missions cause most spacecraft links to operate in the medium-rate regime, which does not have a universal theoretical model [1]. Therefore, it is necessary to simulate the concatenated codes in order to evaluate the radio losses for these higher data rates and narrower loop bandwidths.

In this article, we present the results of some simulations to evaluate the radio losses for a concatenated code with the NASA (7,1/2) convolutional inner code and the Reed–Solomon (255,223) outer code with an interleaving depth of 5. These simulations provide us with the baseline (AWGN) error-rate performance curves for this code in terms of both the bit-error rate at the output of the Reed–Solomon decoder (RS-BER) and the frame-error rate (FER). These baseline curves were in turn curve fitted, and the curve fits were used in the high-rate error-rate function [1] to analytically calculate the radio losses for a loop signal-to-noise ratio (SNR) of 14.8 dB for an FER of 10^{-4} and its corresponding baseline RS-BER of 2.1×10^{-6} (an FER value and an RS-BER value correspond to each other if they are obtained from the same bit SNR value over the AWGN channel). It was shown that, although in terms of the baseline performance the chosen values of the FER and the RS-BER correspond to each other, the high-rate radio loss for the RS-BER value was greater than the high-rate radio loss for the FER value. In addition, these values were compared to the high-rate radio loss for the corresponding baseline convolutional code BER value. This comparison indicated that the high-rate losses for the FER and the RS-BER were much greater (by as much as 2 dB for the RS-BER) than the high-rate losses for the convolutional code's BER. Further simulations were performed to evaluate the effects of changing the data rate on the medium-rate radio losses. For this purpose, the demodulation process was simulated for a carrier tracking loop SNR (LSNR) of 14.8 dB while the data rate was varied from 10 kb/s to 10 Mb/s. It was shown that as the data rate increases the radio losses for (7,1/2), RS-(255,223), interleaver depth-5 code approach the theoretical high-rate losses for both the FER and the RS-BER. Furthermore, this increase could be modeled rather well through the use of a simple two-parameter function. However, further simulations are necessary to formalize the relationship among the parameters of this function, the PLL loop SNR, and the error rate for which the radio loss is calculated.

This article is organized as follows: In Section II, a brief description of the simulation approach that was used is given. In Section III, the baseline (AWGN) performance of the (7,1/2), RS-(255,223), interleaver depth-5 code, the analytical methodology for evaluation of high-rate radio losses, and the interpolation function for evaluation of radio losses as a function of data rate are introduced. In Section IV, the results of the simulations for radio losses for different data rates are discussed. In Section V, conclusions are presented.

II. Description of the Simulations

The simulation setup that was used to obtain the radio losses is shown in Fig. 1. Random bits are encoded with the (7,1/2) convolutional code to generate the channel symbols. A simulated digital phase-locked loop (PLL) then is used to generate tracking phase errors. A predetermined number of channel symbols are then multiplied by the cosine of each generated phase error. After this a Gaussian random variable is added to each symbol. Finally, these symbols are fed to a Viterbi decoder and decoded. The decoded bits then are compared to the original bits, and the number and the positions of the bits in error are recorded. From the positions of bits in error, the positions of the bytes in error within Reed–Solomon frames are then calculated. Subsequently, based on the number of bytes in error per codeword, it is determined whether or not the frame is in error. Note that under real conditions the white Gaussian noise is added to the signal before the phase-locked loop. However, since the noise power in both the in-phase and the quadrature components of the signal is the same, any shift in the phase of the noise produces another white Gaussian noise process with the same power spectrum. Therefore, this simulation produces results equivalent to the real conditions at the input of the Viterbi decoder.

A Reed–Solomon frame consists of five (255,223) Reed–Solomon codewords. Each codeword is capable of correcting 16 bytes. If a codeword has more than 16 bytes in error, it is considered uncorrectable and in error. Furthermore, it is assumed that a frame is in error if *any* of its five codewords are in error. The Reed–Solomon bit-error rate (RS-BER) is calculated by assuming that if a frame is in error then the RS decoding process does not alter any of the bits received from the Viterbi decoder. Therefore, the bit-error

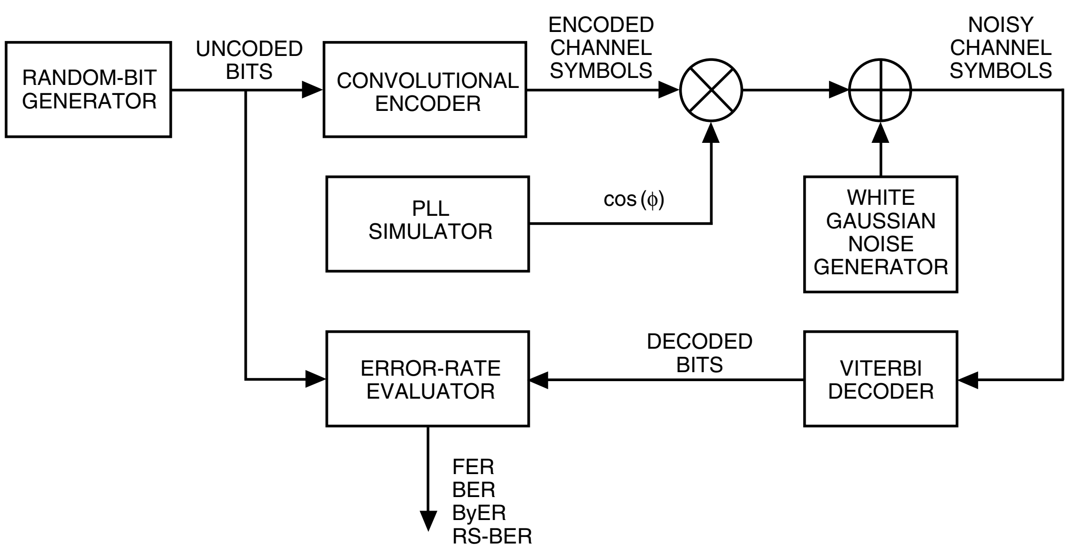


Fig. 1. The simulation setup.

rate at the output of the Reed–Solomon decoder is equal to the frame-error rate times the bit-error rate of the convolutional code over those frames that are in error. (Note that while the correctable codewords in the frame could correct their bits, in actual operations, if a frame is in error it is archived without any decoding.) From these assumptions, the simulation calculates the FER and the RS-BER. The input parameters to this simulation are the bit signal-to-noise ratio, E_b/N_0, the carrier-to-noise ratio, P_c/N_0, the PLL loop bandwidth, B_L, and the number of symbols per single update of the PLL. As the PLL is digital, the number of symbols that are affected by a single value of phase error depends on the data rate. Therefore, by changing the number of symbols per single PLL update, the data rate for the simulation is set.

The PLL that is simulated is a linear digital PLL similar to the one used in the DSN's Block V Receiver (BVR) [3]. The update rate for this PLL is set to 2000 Hz: exactly the same as the BVR. For the purpose of this article, P_c/N_0 was set to 24.8 dB-Hz and the loop bandwidth was kept at 10 Hz. The number of symbols per loop update was varied between 10 symbols per update and 10,000 symbols per update. This corresponds to varying the data rate from 10 kb/s to 10 Mb/s. In order to illustrate the radio loss characteristics of this code, we concentrate on the losses for a FER of 10^{-4} and its corresponding baseline RS-BER of 2.1×10^{-6}. The values for PLL settings were chosen because they produce a loop signal-to-noise ratio of 14.8 dB. This loop SNR value causes the radio losses for the inner (convolutional) code to be rather small (0.21 dB). Since on most current deep-space links a radio loss of 0.3 dB is deemed acceptable, these settings are likely for a link designed under the assumption that the radio losses for the concatenated code and its inner code are the same.

III. Baseline Performance and Analytical Calculations

Before we can evaluate radio losses for the (7,1/2), RS-(255,223), interleaver depth-5 concatenated code, we need to establish the performance of this code over the AWGN channel. This was done by setting the phase error at the output of the PLL to 0 (i.e., perfect tracking conditions). The results of this simulation are shown in Fig. 2. As we can see from this figure, the curve for the bit-error rate at the output of the Viterbi decoder (BER) and the curve for the byte-error rate at the output of the Viterbi decoder (ByER) have the same general profile except that the byte-error rate is roughly 2.5 times greater than the bit-error rate. Similarly, at higher bit SNR values, the curve for the bit-error rate at the output of the Reed–Solomon (RS) decoder (RS-BER) follows the curve for the frame-error rate at the output of that decoder (FER) except that the frame-error rate is roughly 45 times as great as the RS-BER. Note that, at low SNR values where the FER value is 1, the RS-BER and the BER curves are identical. This is due to the assumption that if a frame is in error the RS decoder does not alter any of the bits that it has received from the Viterbi decoder. Note also that, compared to the BER curves for the (7,1/2) code in [1] and [3], the BER curve in this article is shifted to the left by 0.3 dB. This is because the simulations here used real valued symbols instead of the three-bit quantized versions that normally are used in the DSN's Viterbi decoders.

These baseline error-rate curves can be curve fitted. For the frame-error-rate and the byte-error-rate curves, the curve-fit function takes the form

$$f(x) = \begin{cases} \exp(\alpha_0 - \alpha_1 x) & x > x_1 \\ \exp(\beta_0 + \beta_1 x + \beta_2 x^2) & x_1 > x > x_2 \\ 1 & \text{otherwise} \end{cases} \tag{1}$$

For the BER curve, the curve-fit function takes the form

$$f_{BER}(x) = \begin{cases} \exp(\alpha_0 - \alpha_1 x) & x > x_1 \\ \exp(\beta_0 + \beta_1 x + \beta_2 x^2) & x_1 > x > x_2 \\ 0.5 & \text{otherwise} \end{cases} \tag{2}$$

where x is the bit signal-to-noise ratio in unitless form (not in dB).

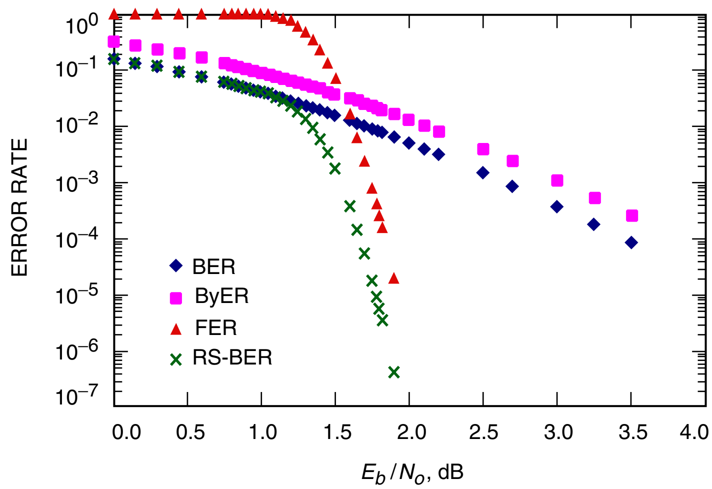


Fig. 2. Baseline FER, BER, ByER, and RS-BER versus E_b/N_o for (7,1/2), RS-(255,223), interleaver depth-5 concatenated code.

Due to our assumptions, at low SNR values the RS-BER curve is identical to the BER curve. Therefore, the curve fit that is used for the RS-BER is given by

$$f_{RSB}(x) = \begin{cases} \exp(\alpha_0 - \alpha_1 x) & x > x_1 \\ \exp(\beta_0 + \beta_1 x + \beta_2 x^2) & x_1 > x > x_2 \\ f_{BER}(x) & \text{otherwise} \end{cases} \tag{3}$$

The results of these curve fits are shown in Figs. 3(a) through 3(d) and Tables 1 through 3. As can be seen from Figs. 3(a) through 3(d), these curve fits are rather accurate for the error-rate regions of interest. Having these curve fits also allows us to rapidly calculate the high-rate losses for this code using the high-rate model equation for error-rate functions [1,3]:

$$f_{hr}(x) = \int_{-\pi}^{\pi} f_{AWGN}\left(\cos^2(\phi)x\right) p_\Phi(\phi) d\phi \tag{4}$$

where $p_\Phi(\phi)$ is the probability density function of the phase error at the output of the PLL. This is assumed to be a Tikhonov density function [1] given by

$$p_\Phi(\phi) = \frac{\exp(\rho\cos\phi)}{2\pi I_0(\rho)} \tag{5}$$

where $I_0(\rho)$ is the zeroth-order modified Bessel function and ρ is the loop signal-to-noise ratio of the PLL. When an ultrastable oscillator (USO) is used on the spacecraft for modulation of the telemetry, ρ is given by

$$\rho = \frac{P_c}{N_0 B_L} \tag{6}$$

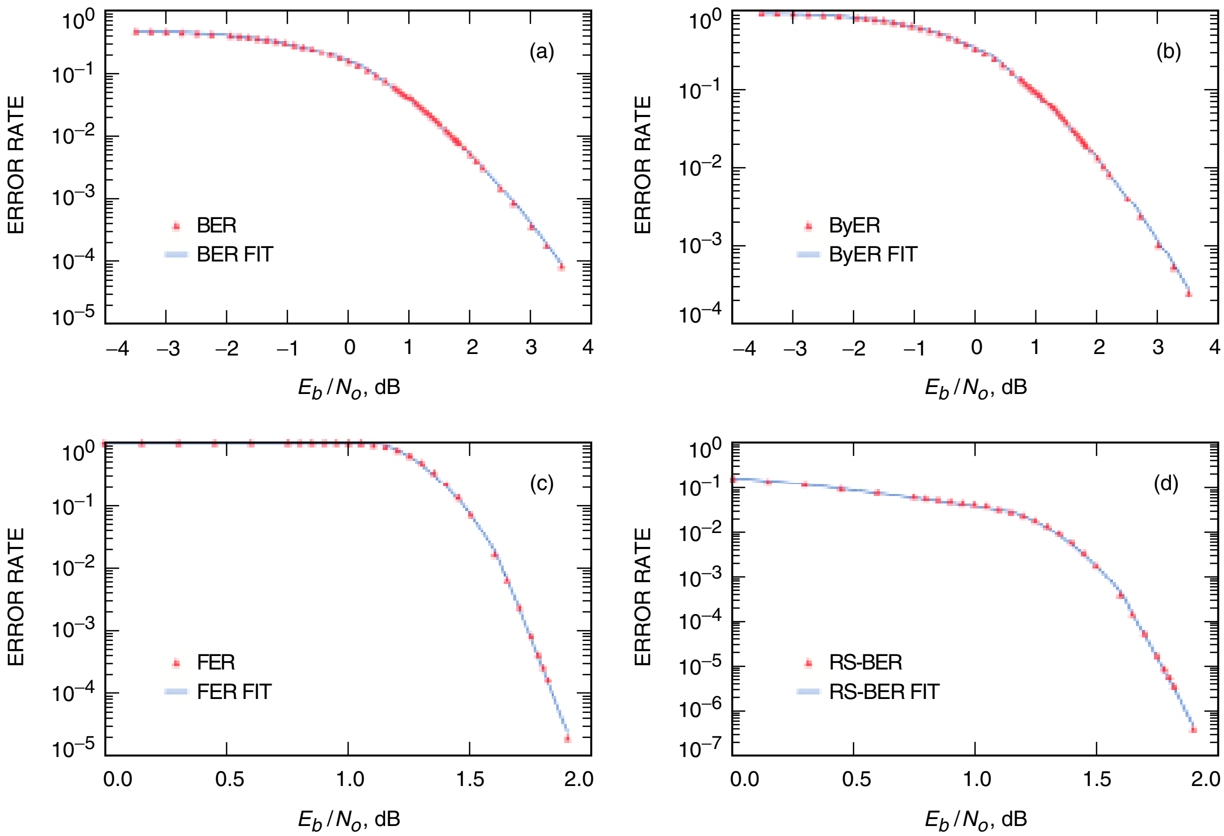


Fig. 3. Error rate and error rate curve fit versus E_b/N_o: (a) BER, (7,1/2) convolutional code, (b) ByER, (7,1/2) convolutional code, (c) FER, (7,1/2), RS-(255,223) interleaver depth-5 concatenated code, and (d) RS-BER, (7,1/2), RS-(255,223) interleaver depth-5 concatenated code.

where P_c/N_0 is the 1-Hz carrier power-to-noise ratio and B_L is the PLL's loop filter bandwidth. For non-USO types of spacecraft oscillators, the loop SNR depends on the phase spectrum of the oscillator and needs to be calculated on a case-by-case basis.

The high-rate error-rate equation is used to calculate the high-rate radio loss, L_{hr}, in the following manner: Let r_e be the desired error rate. Let x_{AWGN} be the value of E_b/N_0 for which $f_{AWGN}(x_{AWGN}) = r_e$, where $f_{AWGN}(x)$ is the error-rate function (either BER, ByER, RS-BER, or FER) over the AWGN channel. Now let x_{hr} be the value of E_b/N_0 such that $f_{hr}(x_{hr}) = r_e$, where $f_{hr}(x)$ is the high-rate error-rate function in Eq. (4). Then the high-rate loss is defined as

$$L_{hr} = \frac{x_{AWGN}}{x_{hr}} \tag{7}$$

Note that in unitless terms losses are always less than 1 and are, therefore, less than 0 dB. However, by convention, when referring to losses in terms of dB, the negative sign is dropped. In this article, this convention is followed.

It should be noted that, since the PLL has a filter, statistical dependence exists between consecutive phase estimates of the PLL. A PLL with a filter loop bandwidth of B_L has a coherence time of $T_L = 1/2B_L$. In other words, phase estimates within T_L seconds of each other are dependent. Otherwise they are independent.

Table 1. Curve-fit parameters for the log-linear portion of the curve fit for different error-rate curves for (7,1/2), RS-(255,223), interleave depth-5 concatenated code.

Function	α_0	α_1
BER	4.4649	6.161
ByER	4.9013	5.8389
RS-BER (low-error-rate portion)	88.0343	64.9705
FER	89.9583	66.2451

Table 2. Curve-fit parameters for the log-quadratic portion of the curve fit for different error-rate curves for (7,1/2), RS-(255,223), interleave depth-5 concatenated code.

Function	β_0	β_1	β_2
BER	-1.4347	3.077	-3.4661
ByER	-0.742	2.9382	-3.2697
RS-BER (low-error-rate portion)	-159.7969	254.4975	-103.2711
FER	-198.7073	314.5291	-124.4003

Table 3. Curve-fit boundary points for different error-rate curves for (7,1/2), RS-(255,223), interleave depth-5 concatenated code.

Function	x_1	x_2
BER	1.06097	0.44668
ByER	1.06736	0.44668
RS-BER (low-error-rate portion)	1.44459	1.30765
FER	1.44701	1.29304

The high-rate error-rate equation, Eq. (4), assumes that a single independent PLL estimate of the phase affects an entire frame. Since this is an asymptotic behavior, it implies that when the high-rate model applies, $T_L \gg T_F$, where T_F is the time it takes to transmit one frame. Conversely, the low-rate model assumes that many independent phase estimates affect a single frame. In other words, $T_L \ll T_F$. Therefore, the frame sees the average reduction in SNR due to phase errors. This leads to the low-rate model equation for error rates [1,3]:

$$f_{lr}(x) = f_{AWGN}\left(x \int_{-\pi}^{\pi} \cos^2(\phi) p_\Phi(\phi)\, d\phi\right) \tag{8}$$

and the low-rate model loss equation:

$$L_{lr} = \int_{-\pi}^{\pi} \cos^2(\phi) p_\Phi(\phi)\, d\phi \tag{9}$$

Note that Eq. (9) is independent of the type of code, the type of error (frame, bit, RS-bit, or byte), or the error rate. This simply indicates that the error-rate curve is shifted to the right by L_{lr}.

Typically, the radio losses are defined in terms of an interpolation between the high-rate radio loss, L_{hr}, and the low-rate radio loss, L_{lr}. The interpolation usually takes the form of [1,3]

$$L_{\text{actual}}(\text{dB}) = (1-a)L_{lr}(\text{dB}) + aL_{hr}(\text{dB}) \tag{10}$$

where a is dependent on the ratio of loop coherence time, T_L, to frame transmission time, T_F. Kinman [3] has proposed using the following equation for a:

$$a = \frac{1}{1 + c_1 \left(\dfrac{T_L}{T_F}\right)^{-c_2}} \tag{11}$$

where c_1 and c_2 are determined based on the code, the loop SNR, and the error rate of interest. This equation will be used to curve fit the results of our radio loss simulations.

In order to illustrate the effects of data-rate changes on radio loss, we will evaluate the radio loss for a frame-error rate of 10^{-4} and its corresponding baseline RS-BER of 2.1×10^{-6} for all the data rates under consideration. Over an AWGN channel, these error rates are achieved with an E_b/N_o of 1.837 dB. For the same value of E_b/N_o, the values of BER and ByER are given in column 2 of Table 4. Using Eq. (7) to evaluate the high-rate losses for these error measures with a 14.8-dB loop SNR, column 3 of Table 4 is obtained. As we can see from Table 4, the lower the error-rate values are over the AWGN channel, the higher are the high-rate losses for the error measure. The low-rate loss at a 14.8-dB loop SNR as obtained by Eq. (9) is 0.144 dB. These numbers are used to analytically validate the results of our simulations.

Table 4. Different error measures, their values for 1.837 dB E_b/N_0 over an AWGN channel, and their high-rate and low-rate losses for a loop SNR of 14.8 dB.

Error measure	Error-rate value	High-rate loss with 14.8-dB loop SNR, dB	Low-rate loss with 14.8-dB loop SNR, dB
BER	7.2×10^{-3}	0.213	0.144
ByER	1.8×10^{-2}	0.212	0.144
FER	1.0×10^{-4}	2.020	0.144
RS-BER	2.1×10^{-6}	2.376	0.144

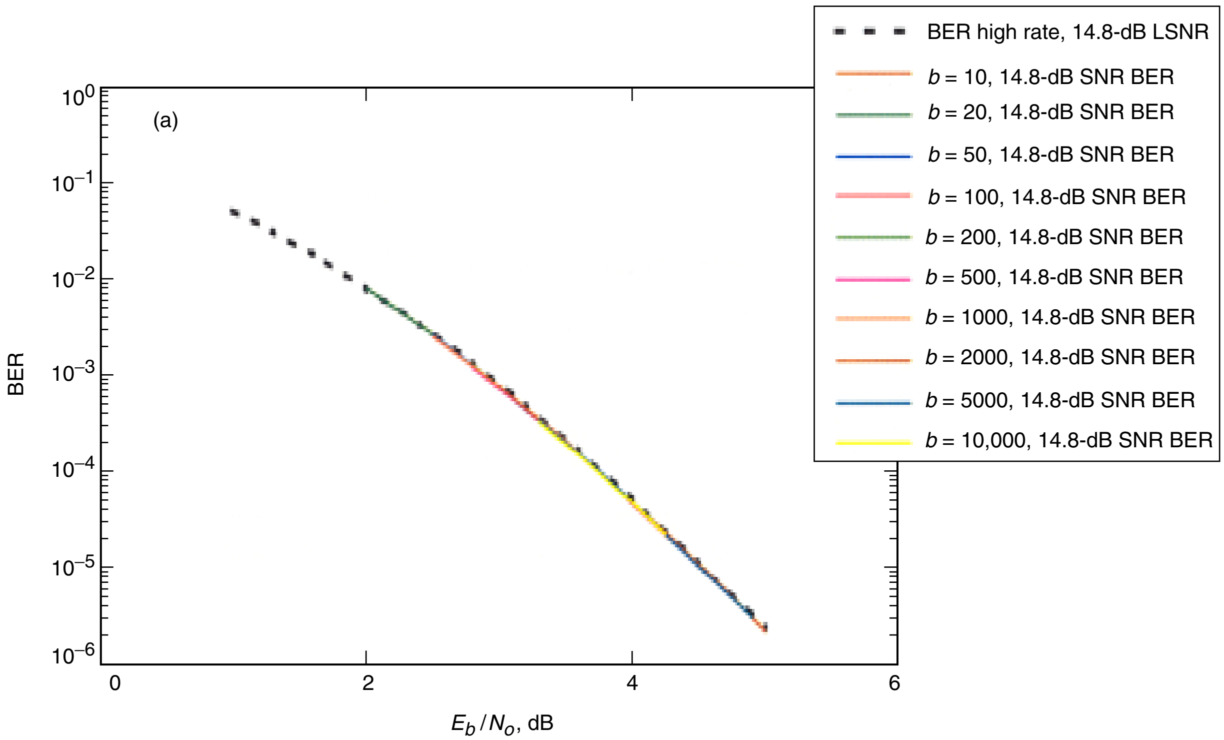
IV. Radio Loss Simulation Results for Different Data Rates

The results of these simulations are shown in Figs. 4(a) through 4(d). These figures depict the BER, the ByER, the FER, and the RS-BER curves, respectively, as the number of symbols per PLL phase update (and consequently the data rate) is increased. It should be noted that for this particular set of simulations the number of symbols per PLL phase update happens to correspond to the data rate in kb/s due to the code rate and the update rate of the PLL. These figures also include respective analytical high-rate model error-rate curves obtained by using Eq. (4). In addition, Figs. 4(c) and 4(d) include the low-rate error-rate curves obtained from Eq. (8). Figures 4(a) and 4(b) indicate that the BER and the ByER curves trace their respective high-rate error-rate curves rather closely. This indicates that, for all the data rates under consideration, the constituent (7,1/2) convolutional code operates in the high-rate regime. According to [1], for a loop bandwidth of 10 Hz, the high-rate regime applies for the (7,1/2) code at roughly 1 kb/s. This corresponds well with the results shown in Figs. 4(a) and 4(b).

However, the story is different for Figs. 4(c) and 4(d). These figures show that for both the FER and the RS-BER at lower data rates, the error-rate curves are far to the left of their respective analytical high-rate error-rate curves. However, as the number of symbols per phase update increases, the error-rate curves approach the analytical high-rate error-rate curves. When the number of symbols per phase update is greater than 500 (corresponding to a data rate of 500 kb/s), Figs. 4(c) and 4(d) indicate that, for a given error rate, the simulation results are within 0.2 dB of the theoretical high-rate curves. Note that some of the FER and the RS-BER curves for high values of the number of symbols per update are rather rough and irregular. This is because for these values several frames at a time are affected by a large error in the PLL estimate of the phase. This causes frame errors to occur in bursts. Therefore, even though a simulation result may include 100 frame errors, these errors could occur in only a few bursts (10 to 20). Therefore, no statistical smoothing occurs. This is especially true at low error rates where a burst of errors could significantly skew the error rate. To smooth these curves, more simulations are needed. However, due to the amount of time that it takes to perform these simulations, it was not deemed practical to do so.

In order to characterize the increase in the radio losses as the data rate increases, for each of the values of symbols per update, we evaluated the radio losses for an FER of 10^{-4} and its corresponding AWGN RS-BER of 2.1×10^{-6}. These results are plotted in Figs. 5(a) and 5(b). As these figures indicate, as the data rate increases so does the radio loss until the radio loss plateaus at a data rate of approximately 500 kb/s. This corresponds to a T_L/T_F of 2.45. This agrees well with our definition of the high-rate regime provided in the previous section. Note that the high-rate radio losses on these curves are within 0.2 dB of the analytical high-rate losses calculated in Section III.

Finally, we attempted to curve fit the changes in radio losses as the data rate increases as a function of the ratio of the loop coherence time, T_L, to the frame transmission time, T_F. The results of these curve fits also are shown in Figs. 5(a) and 5(b). The curve-fit parameters, c_1 and c_2, are shown in Table 5. As we can see from the figures, the curve fits are rather accurate. Therefore, the function in Eq. (11) is a rather good model for the interpolation factor. However, unless further simulations for different error rates and different loop SNR values are performed, we cannot define specific functions for c_1 and c_2 in terms of loop SNR and error rates nor can we reach any further conclusions regarding the dependence of radio losses on T_L/T_F value. Unfortunately, such simulations will take a long time to perform and may not be practical at this time.



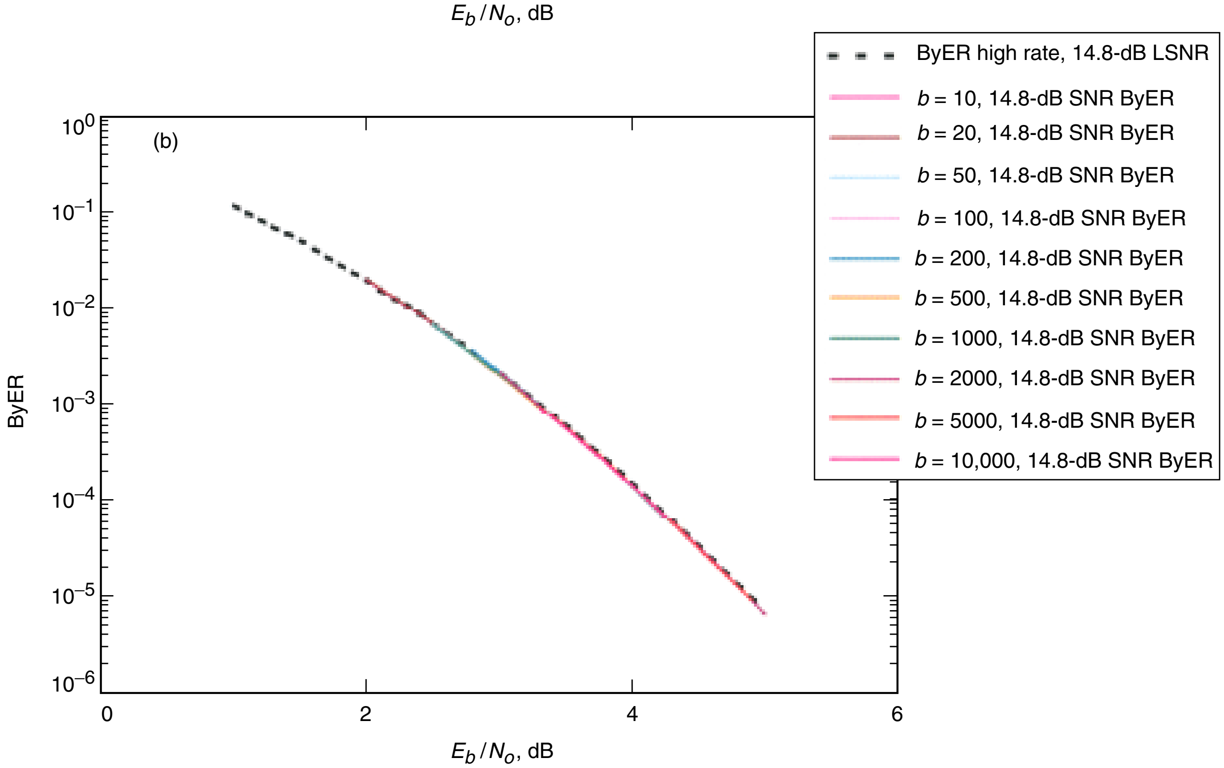
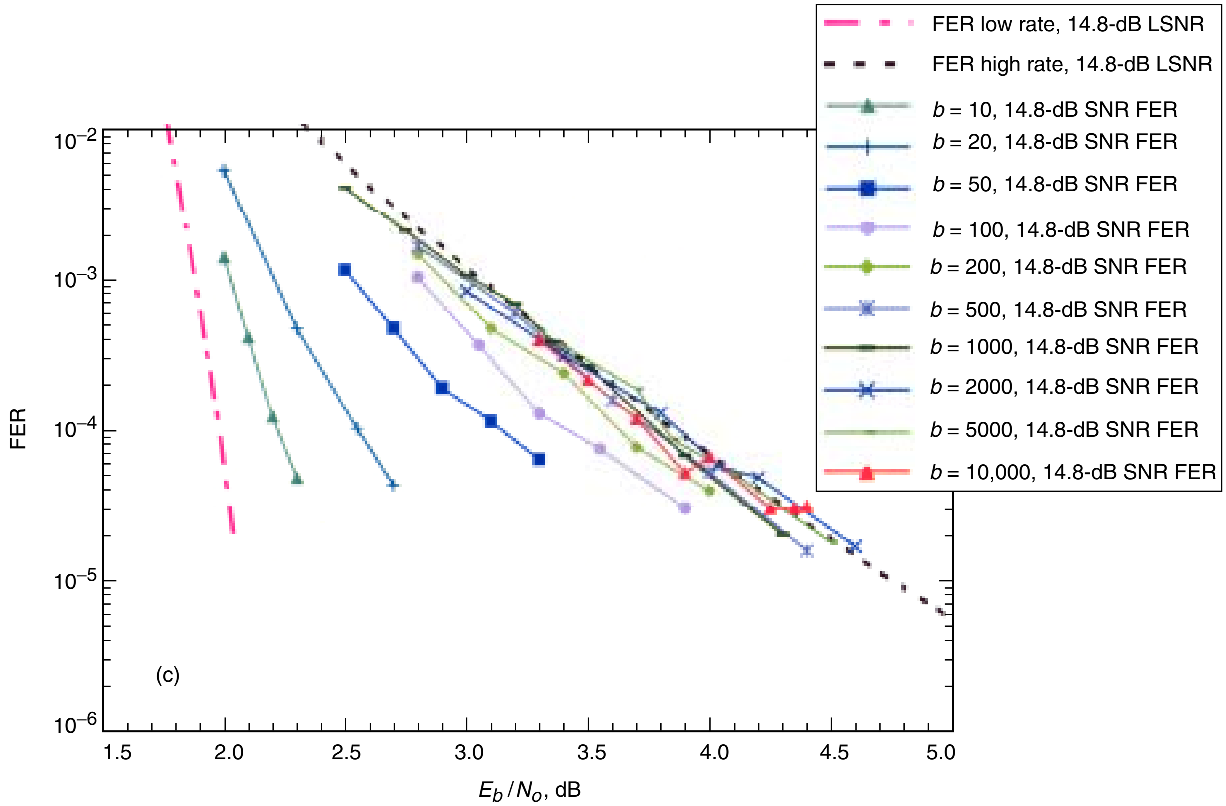
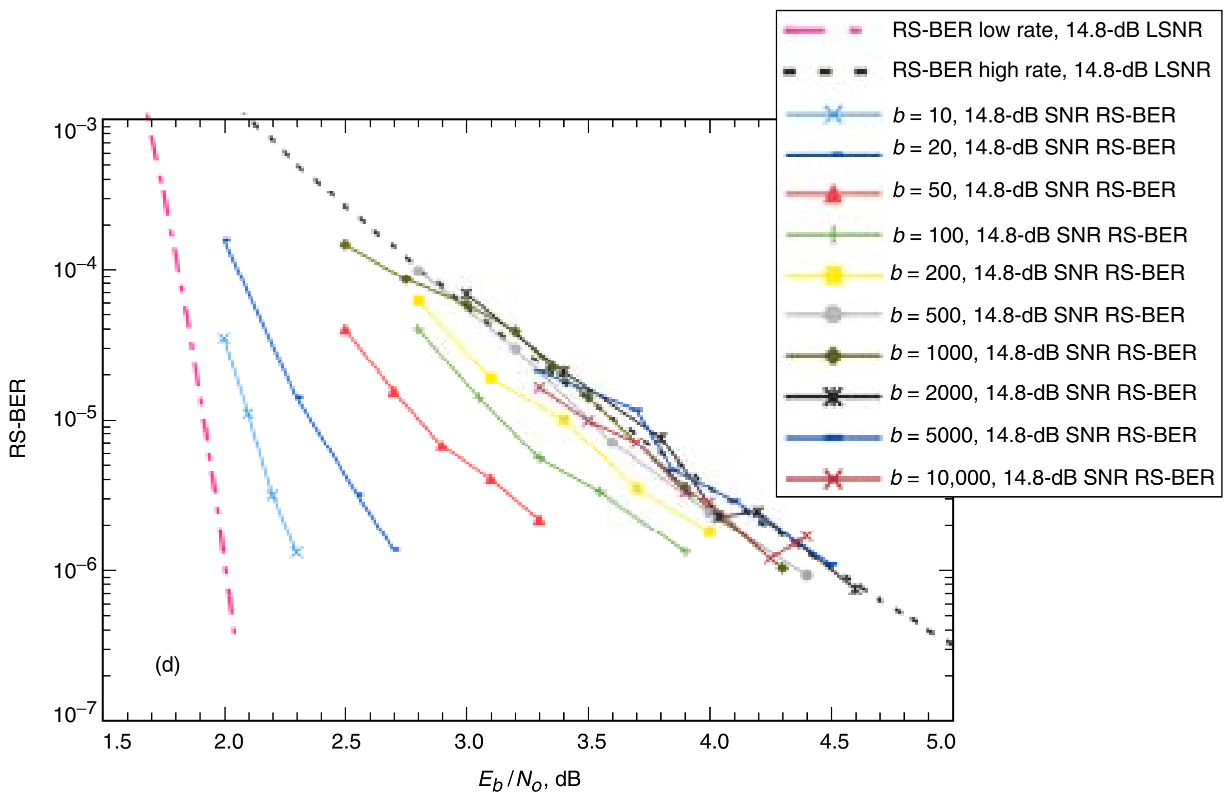


Fig. 4. Error rate versus E_b/N_o for different values of symbols per PLL phase update, 14.8-dB loop SNR, 10-Hz loop bandwidth: (a) BER, (7,1/2) convolutional code, (b) ByER, (7,1/2) convolutional code, (c) FER, (7,1/2), RS-(255,223) interleaver depth-5 concatenated code, and (d) RS-BER, (7,1/2), RS-(255,223) interleaver depth-5 concatenated code.



FER low rate, 14.8-dB LSNR
FER high rate, 14.8-dB LSNR
b = 10, 14.8-dB SNR FER
b = 20, 14.8-dB SNR FER
b = 50, 14.8-dB SNR FER
b = 100, 14.8-dB SNR FER
b = 200, 14.8-dB SNR FER
b = 500, 14.8-dB SNR FER
b = 1000, 14.8-dB SNR FER
b = 2000, 14.8-dB SNR FER
b = 5000, 14.8-dB SNR FER
b = 10,000, 14.8-dB SNR FER
FER
(c)
Eb / No, dB



RS-BER low rate, 14.8-dB LSNR
RS-BER high rate, 14.8-dB LSNR
b = 10, 14.8-dB SNR RS-BER
b = 20, 14.8-dB SNR RS-BER
b = 50, 14.8-dB SNR RS-BER
b = 100, 14.8-dB SNR RS-BER
b = 200, 14.8-dB SNR RS-BER
b = 500, 14.8-dB SNR RS-BER
b = 1000, 14.8-dB SNR RS-BER
b = 2000, 14.8-dB SNR RS-BER
b = 5000, 14.8-dB SNR RS-BER
b = 10,000, 14.8-dB SNR RS-BER
RS-BER
(d)
Eb / No, dB

Fig. 4. Cont'd.

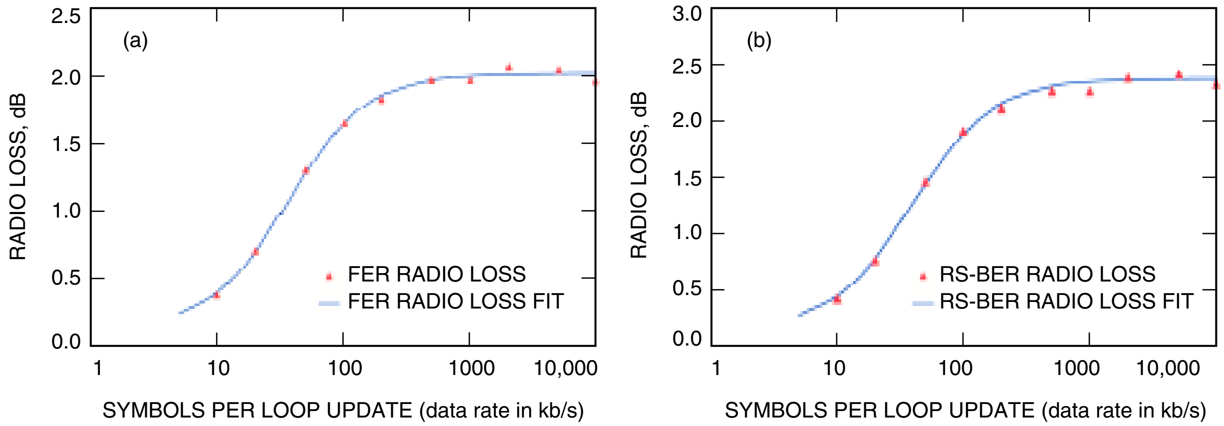


Fig. 5. Radio loss and radio loss curve fit versus symbols per PLL phase update (data rate in kb/s), 14.8-dB loop SNR, 10-Hz loop bandwidth, (7,1/2), RS-(255,223) interleaver depth-5 concatenated code: (a) FER = 1×10^{-4} and (b) RS-BER = 2.1×10^{-6}.

Table 5. Curve-fit parameters for the interpolation function for FER and RS-BER radio losses.

Error measure	c_1	c_2
FER	0.089	1.405
RS-BER	0.121	1.304

V. Conclusions

In this article, results of a series of simulations performed for evaluation of radio losses for the (7,1/2), RS-(255,223), interleaver depth-5 concatenated code have been presented. These results show that

(1) Accurate curve-fit functions could be obtained for both the Reed–Solomon bit-error rate (RS-BER) and the frame-error rate (FER) for this code. These curve fits could then be used to calculate high-rate radio losses rather accurately.

(2) At very high data rates, the assumption that concatenated codes have the same radio losses as their constituent convolutional codes is wrong. This assumption could lead to very large underestimation of the radio losses in the power budget for the link (by about 2 dB in the case of an RS-BER of 2.1×10^{-6} and a 14.8-dB loop SNR).

(3) A simple two-parameter function could be used to interpolate between the high-rate and low-rate radio losses for the medium-rate radio losses. Further simulations are required to find further relationships between the parameters of this function and the PLL loop SNR and the error-rate value for which the loss is calculated.

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