Noise Temperature due to Reflector Surface Resistivity

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This article presents the results of a study that shows that approximate formulas that have been used in the past for computing the noise temperatures of reflectors (due to surface resistivity losses) are more accurate than previously believed. Instead of being accurate only for incidence angles up to 40 deg, the recent study shows that at 8.45 GHz for a flat aluminum reflector the formulas are accurate to 0.0003 K and to 0.5 K for incidence angles up to 89.2 deg for perpendicular and parallel polarizations. Derivations of the exact and approximate noise temperature formulas also are given.

I. Introduction

In previous years, approximate formulas have been used for estimating the noise temperatures of mirrors caused by resistivity losses of the mirror surfaces as a function of incidence angle, polarization, frequency, and electrical conductivity of the metallic surface. The formulas were especially useful because their simplicity allowed calculations to be made through the use of a hand calculator rather than an extensive Fortran program. The approximate formulas were originally derived by Otoshi but not officially published because the actual accuracies of the formulas were not known but were thought to be accurate for incidence angles only up to about 40 deg.

The approximate formula was later used in an article by Veruttipong [1] for beam-waveguide mirror noise-temperature calculations where the incidence angles could be as high as 60 deg. Since the formulas will continue to be used for a variety of antenna applications, it was decided that the derivations be documented and the accuracies of the formulas be studied in depth. It is the purpose of this article to present the results of this in-depth study.

II. Approximate Formulas

The approximate formula for calculating noise temperature as a function of incidence angle, polarization, frequency, and electrical conductivity of the mirror surface is given in the following. Even though the derivations of the formulas are straightforward, the associated equations showing the accuracies are involved and, hence, the original derivations along with an error analysis are given in the Appendix rather than in the main text.

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For the case of a linearly polarized wave with the E-field polarized perpendicular to the incidence plane, the approximate noise temperatures due to resistivity losses of the mirror surface can be calculated from

$$(T'_n)_\perp = \left(\frac{4R_s}{\eta_o}\cos\theta_i\right) T_p \tag{1}$$

For a linearly polarized wave with the E-field polarized parallel to the incidence plane,

$$(T'_n)_{//} = \left(\frac{4R_s}{\eta_o \cos\theta_i}\right) T_p \tag{2}$$

and for a circularly polarized wave,

$$(T'_n)_{cp} = \frac{1}{2}\left[(T'_n)_\perp + (T'_n)_{//}\right] \tag{3}$$

where primes denote approximate formulas in contrast to exact formulas and θ_i, R_s, η_o, and T_p are, respectively, the angle of incidence, surface resistivity in ohms per square, the characteristic impedance of free space, and the physical temperature of the mirror surface in kelvins. The surface resistivity is calculated from

$$R_s = 0.02\pi\sqrt{\frac{F_{\mathrm{GHz}}}{10\sigma_n}} \tag{4}$$

where F_{GHz} is the frequency in gigahertz and σ_n is the normalized electrical conductivity of the metal calculated by dividing the actual electrical conductivity by 10^7. For example, for 6061-T6 aluminum, the actual electrical conductivity is 2.3×10^7 mhos/m, so that σ_n is equal to 2.3 and is assumed to be constant with frequency over the microwave frequency region of 1.0 GHz through 40 GHz.

A similar formula appeared in an early article by Otoshi [2] but applied to a perforated plate with holes. By taking the limiting case where the hole diameter is made to go to zero, the perforated plate becomes a solid plate and the resulting noise temperature equations become as shown by Eqs. (1) and (2). Equation (3) shows that the noise temperature for circular polarization is just the average of those of parallel and perpendicular polarizations. The theoretical proof that the average could be taken for circular polarization was presented in an article by Otoshi and Yeh [3].

Although Eq. (3) has been used for numerous noise temperature calculations for DSN reflectors in past years, the accuracy of the formula was not studied in depth. It was always thought that, because of the $1/\cos\theta_i$ factor in one of the terms of Eq. (3), the formula would be increasingly inaccurate for higher incidence angles. A study of the sum of $0.5 \times (\cos\theta_i + 1/\cos\theta_i)$ showed that it remained within 3.5 percent of unity up to $\theta = 40$ deg. This was the only basis for believing that Eq. (3) should not be used above a 40-deg incidence angle. An actual study of the errors had not been made until now.

III. Exact Formula

The exact values can be obtained from equations of reflection and transmission coefficients for parallel and perpendicular polarizations for a wave incident upon a dielectric sheet [4]. The parameters, which must be input to the exact equations, are frequency, incidence angle, the complex relative dielectric constant of the dielectric, and sheet thickness. For a solid metallic reflector, the real and imaginary parts of the complex dielectric constants are

$$\left.\begin{aligned} \epsilon' &= 1.0 \\ \epsilon'' &= \frac{18\sigma}{F_{\mathrm{GHz}}} \end{aligned}\right\} \tag{5}$$

where σ = actual electrical conductivity of the metal or $10^7 \times \sigma_n$ as defined previously in Eq. (4). For example, for aluminum at 8.45 GHz,

$$\sigma = 2.3 \times 10^7 \text{ mhos/m}$$

so from Eq. (5)

$$\epsilon'' = \frac{18 \times (2.3 \times 10^7)}{8.45} = 4.9 \times 10^7$$

Then let the thickness of the metallic surface be sufficiently large (at least 10 skin depths) so that the transmission coefficient is zero. For example, for 6061-T6 aluminum at 8.42 GHz, with $\sigma_n = 2.3$, 10 skin depths would correspond to 0.0114 mm (0.00045 in.) [5]. Then the dissipative power loss ratio (DPR) for a single mirror is simply [6]

$$DPR = 1 - |\Gamma|^2 \tag{6}$$

where Γ is the input voltage reflection coefficient and the vertical bars denote magnitude. Noise temperature then is calculated from

$$T_n = DPR \times T_p \tag{7}$$

where T_p was previously defined. The exact calculations become too cumbersome to perform with a hand calculator because of the complex parameters and, therefore, should be done through the use of a Fortran program such as the one called SLAB.FOR written for this purpose. It was discovered later that nearly identical exact values could be obtained with simpler formulas derived in the Appendix for metal sheets.

IV. Applications

A. Sample Case 1

For this analysis, let $T_p = 290$ K, $\eta_o = 120\pi$, and $F_{\mathrm{GHz}} = 8.45$, with the metal reflector made from 6061-T6 aluminum, which has a normalized electrical conductivity of 2.3 at 8.45 GHz based on actual measurements [5]. Approximate noise temperatures as calculated from Eqs. (1) through (3) are easy to calculate based on these input parameters. Although noise temperatures can be plotted, it is thought that displaying the errors associated with the formulas would be more informative.

The conventional practice is to define error as

$$\text{Error} = (\text{Approximate Value}) - (\text{Exact Value}) \tag{8}$$

Note that the negative of error is the correction term that can be added to the approximate value to get the exact value. However, staying with the conventional definition, errors, rather than corrections, will be plotted in this article.

Figure 1 shows that, for perpendicular polarization, the approximate formula is valid up to θ_i = 89.9 deg and that the errors fall between the bounds of ±0.0003 K and seem to be due mainly to numerical computation errors.

Figure 2 shows that, for parallel polarization, the error becomes greater than 0.1 K at $\theta_i = 89.1$ deg, and for circular polarization, the error becomes 0.1 K at $\theta_i = 89.4$ deg. In Fig. 2, values below $\theta = 85$ deg were purposely not plotted because the errors were less than 0.003 K and could not be seen on the linear scale of Fig. 2.

B. Sample Case 2

To see how accurate the approximate formulas are for the same aluminum sheet at a higher frequency, such as 32 GHz, a second case was studied. It is generally true that while surface resistivity, R_s, is a function of frequency [see Eq. (4)], the value of electrical conductivity of metals generally stays constant with frequency. The surface roughness, however, causes the effective resistivity to be higher than predicted when taking the square root of the ratios of frequencies and, hence, in practice makes the effective electrical conductivity seem lower than the predicted value at higher frequencies. However, for this study, it will be assumed that the normalized electrical conductivity of aluminum remained the same at 32 GHz as it was for 8.45 GHz. Therefore, for this second case, let $T_p = 290$ K, $\eta_o = 120\pi$, $F_{\text{GHz}} = 32$, and $\sigma_n = 2.3$ for 6061-T6 aluminum.

Figure 3 shows that at 32 GHz for perpendicular polarization the error is less than ±0.0003 K up to $\theta_i = 89.5$ deg. Figure 4 shows that at 32 GHz for parallel polarization the error becomes 0.1 K at $\theta_i = 88.2$ deg, and for circular polarization the error is 0.1 K at $\theta_i = 88.8$ deg. In Fig. 4, values below $\theta_i = 85$ deg were purposely not plotted because the errors were less than 0.01 K for $\theta_i < 85$ deg and could not be seen on the linear scale of Fig. 4.

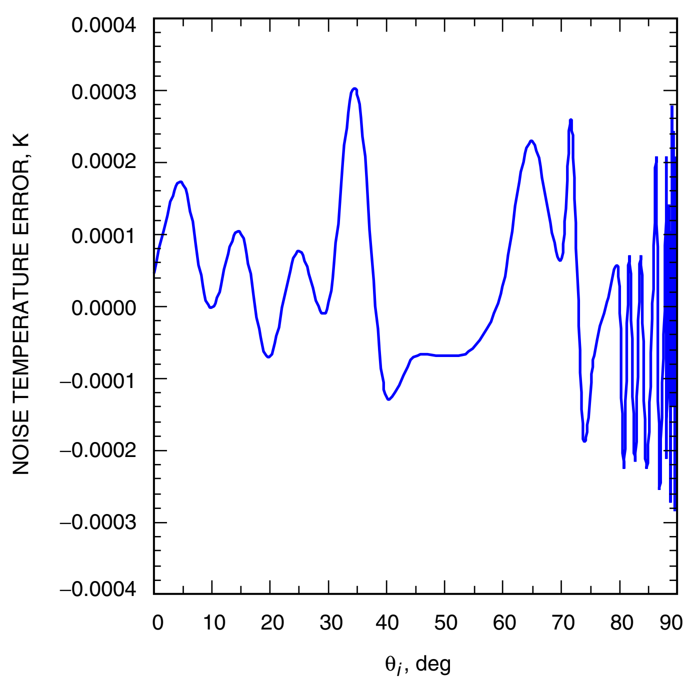

Fig. 1. Approximate formula error plot for an aluminum sheet at 8.45 GHz, perpendicular polarization.

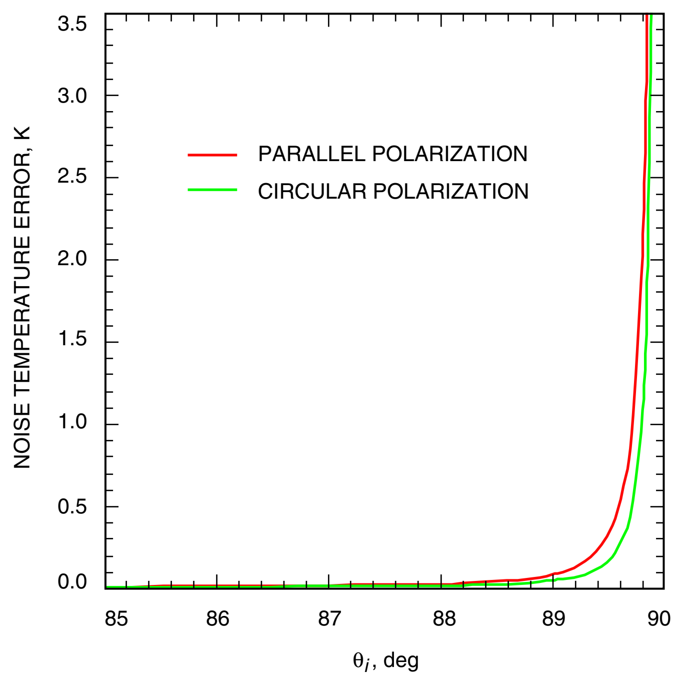

Fig. 2. Approximate formula error plots for an aluminum sheet at 8.45 GHz, parallel and circular polarizations.

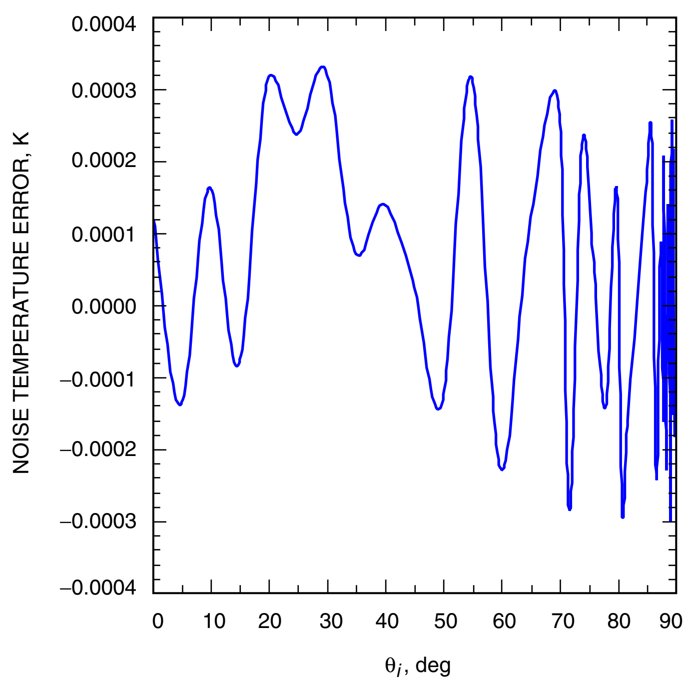

Fig. 3. Approximate formula error plot for an aluminum sheet at 32 GHz, perpendicular polarization.

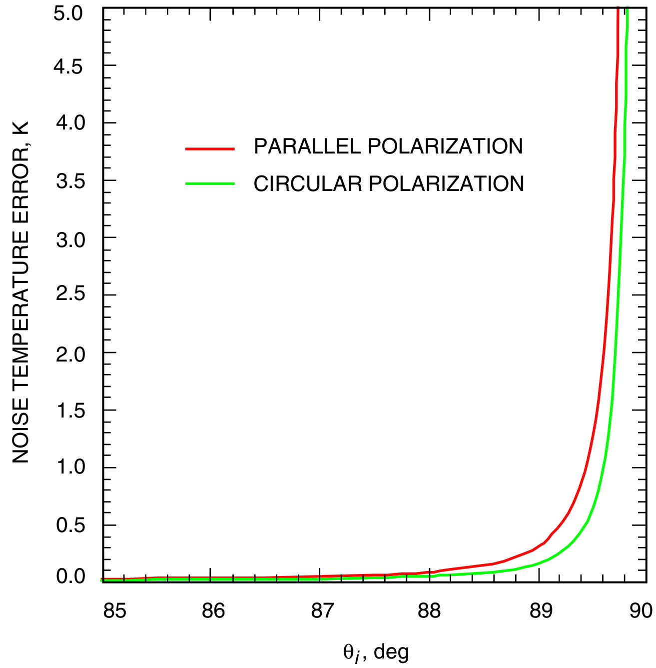

Fig. 4. Approximate formula error plots for an aluminum sheet at 32 GHz, parallel and circular polarizations.

Although not shown plotted, the errors are higher if the electrical conductivity of the metal is lower. For example, a case where $\sigma_n = 1.0$ rather than 2.3 for the metal showed that the error for parallel polarization is 0.2 K at 89.1 deg at 8.45 GHz as compared to an error of 0.1 K when $\sigma_n = 2.3$.

These results show that the errors as functions of incidence angles go up when frequency is higher and also go up if conductivity is lower. It is desirable to be able to predict the accuracies of the approximate formulas given by Eqs. (1) through (3) for any given set of parameters, such as incidence angle, frequency, and electrical conductivity of the metal, without having to resort to use of a Fortran program that generally is not available to workers in the field. It also would be desirable to be able to calculate these errors easily and accurately through the use of simple formulas and a hand calculator. Such simple formulas for this purpose have been derived and are given in the Appendix.

V. Conclusions

Approximate formulas have been presented and shown to be very accurate for incidence angles as high as 89.2 deg even at frequencies as high as 32 GHz. The maximum value at which the formulas can be used, therefore, is much higher than the 40-deg upper limit previously assumed. Most of the noise temperatures and associated errors can be calculated through the use of a hand calculator. A Fortran program will need to be used only if it is desired that accurate noise temperatures be calculated for incidence angles above the 89.5-deg region (or incidence angles close to grazing angles).

References

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Appendix A

Derivation of the Approximate Formulas for Thick Metallic Solid Sheets

I. General Case

For a lossy dielectric sheet, the air-to-dielectric interface input reflection coefficients for perpendicular and parallel polarizations, respectively, are given in [4] as

$$\Gamma_{\perp} = \frac{\cos\theta_i - \sqrt{\in_c' - \sin^2\theta_i}}{\cos\theta_i + \sqrt{\in_c' - \sin^2\theta_i}} \tag{A-1}$$

$$\Gamma_{//} = \frac{\sqrt{\in_c' - \sin^2\theta_i} - \in_c' \cos\theta_i}{\sqrt{\in_c' - \sin^2\theta_i} + \in_c' \cos\theta_i} \tag{A-2}$$

where

$\theta_i =$ incidence angle

$\in_c' =$ complex relative dielectric constant

If the dielectric sheet is metallic and at least 10 skin depths thick, the incident wave is reflected or attenuated completely inside the lossy metallic sheet before exiting the output interface. Then $\Gamma_{\perp}$ and $\Gamma_{//}$ become the input reflection coefficients for the entire sheet. If the metallic sheet has resistive losses, the magnitudes of $\Gamma_{\perp}$ and $\Gamma_{//}$ will be less than unity.

Let the complex relative dielectric constant be expressed as [7]

$$\in_c' = \in' - j\in'' = \in' - j\frac{\sigma}{\omega \in_o} \tag{A-3}$$

where σ is the electrical conductivity of the lossy media in mhos/m, ω is the radian frequency, and $\in_o$ is the free-space dielectric constant.

Furthermore, the expression for surface resistivity of a non-ferrous metal is given as

$$R_s = \sqrt{\frac{\omega\mu_o}{2\sigma}} \tag{A-4}$$

and the expression for free-space characteristic impedance is

$$\eta_o = \sqrt{\frac{\mu_o}{\in_0}} \tag{A-5}$$

So, for non-ferrous metals, manipulations of Eqs. (A-4) and (A-5) and substitutions into Eq. (A-3) give

$$\in'' = \frac{\sigma}{\omega \in_o} = \frac{1}{2}\left(\frac{\eta_o}{R_s}\right)^2 \tag{A-6}$$

Substitution of Eq. (A-6) into Eq. (A-3) and assuming $\in' = 1$ for metal [8],

$$\in'_c = 1 - j \in'' \tag{A-7}$$

Then

$$\in'_c - \sin^2\theta_i = \cos^2\theta_i - j \in'' \tag{A-8}$$

Note that thus far no assumptions or approximations have been made and Eqs. (A-1) through (A-8) are exact for non-ferrous metals.

Since for metals $\in'' >> 1$, the following approximations can be made in Eqs. (A-7) and (A-8):

$$\sqrt{\in'_c} \approx \sqrt{-j \in''} \tag{A-9}$$

and

$$\sqrt{\in'_c - \sin^2\theta_i} \approx \sqrt{\in'_c} \tag{A-10}$$

These will be the only two assumptions made in the derivations of the approximate formulas of the noise temperature of thick metals. Substitution of Eq. (A-6) into Eq. (A-9) gives

$$\sqrt{\in'_c} = \sqrt{-j}\frac{1}{\sqrt{2}}\left(\frac{\eta_o}{R_s}\right) = \frac{\eta_o}{\sqrt{2}R_s}\left[e^{-j(\pi/2)}\right]^{1/2}$$

$$= \frac{\eta_o}{2R_s}(1-j) \tag{A-11}$$

and

$$\frac{1}{\sqrt{\in'_c}} = \sqrt{2}\frac{R_s}{\eta_o}e^{j(\pi/4)}$$

$$= \frac{R_s}{\eta_o}(1+j) \tag{A-12}$$

II. Perpendicular Polarization Formula

It was shown in Eq. (A-10) that for metal sheets

$$\sqrt{\in'_c - \sin^2\theta_i} \approx \sqrt{\in'_c}$$

so Eq. (A-1) may be written as

$$\Gamma_\perp = \frac{\dfrac{1}{\sqrt{\in_c'}}\cos\theta_i - 1}{\dfrac{1}{\sqrt{\in_c'}}\cos\theta_i + 1} \qquad \text{(A-13)}$$

Substitution of Eq. (A-12) gives

$$\Gamma_\perp = \frac{\left(\dfrac{R_s}{\eta_o}\cos\theta_i - 1\right) + j\dfrac{R_s}{\eta_o}\cos\theta_i}{\left(\dfrac{R_s}{\eta_o}\cos\theta_i + 1\right) + j\dfrac{R_s}{\eta_o}\cos\theta_i} \qquad \text{(A-14)}$$

For simplification of notation, let

$$u = \frac{R_s}{\eta_o}\cos\theta_i \qquad \text{(A-15)}$$

Then

$$|\Gamma_\perp|^2 = \frac{1 - 2u + 2u^2}{1 + 2u + 2u^2}$$

$$= 1 - \frac{4u}{1 + 2u(1+u)} \qquad \text{(A-16)}$$

and

$$1 - |\Gamma_\perp|^2 = \frac{4u}{1 + 2u(1+u)} \qquad \text{(A-17)}$$

and the corresponding noise temperature from Eq. (7) of the main text is

$$(T_n)_\perp = \left(1 - |\Gamma_\perp|^2\right) T_p$$

$$= \frac{4uT_p}{1 + 2u(1+u)} \qquad \text{(A-18)}$$

where u was given in Eq. (A-15).

Note that the only approximations made to derive Eq. (A-18) were in obtaining Eqs. (A-9) and (A-10), where it was required that

$$\in'' >> 1$$

Substitution of Eq. (A-6) gives the equivalent requirement of

$$\left(\frac{\eta_o}{R_s}\right)^2 >> 2$$

which would hold true for all values of θ_i for most metals. Therefore, Eq. (A-18) may be considered to be the exact expression, and there is no need to run a separate Fortran computer program to compute the exact $(T_n)_\perp$. Note that all computations in Eq. (A-18) can be done on a hand calculator. The expression for R_s given in Eq. (A-4) instead can be calculated more easily from use of Eq. (4) in the main text, and 377 ohms may be used for η_o.

Since $u << 1$, Eq. (18) can be simplified to

$$(T_n')_\perp \approx 4uT_p = \left(\frac{4R_s}{\eta_o}\cos\theta_i\right) T_p \tag{A-19}$$

which is the same as Eq. (1) in the main text.

For perpendicular polarization, the noise temperature decreases as θ_i increases. If one wishes to know how accurate the approximate formula is as a function of R_s, frequency, and incidence angle, one can define the exact error of the $T'_{n\perp}$ formula to be

$$\begin{aligned}(\mathcal{E}_\perp)_{\text{exact}} &= T'_{n\perp} - T_{n\perp} \\ &= T'_{n\perp}\left[1 - \frac{1}{1+2u(1+u)}\right]\end{aligned} \tag{A-20}$$

which leads to an approximate error formula of

$$(\mathcal{E}_\perp)_{\text{approx}} = 2uT'_{n\perp} \tag{A-21}$$

which should be increasingly accurate as θ_i approaches 90 deg.

III. Parallel Polarization

Under the same assumption used for perpendicular polarization, let

$$\sqrt{\in_c' - \sin^2\theta_i} \approx \sqrt{\in_c'}$$

so that Eq. (A-2) becomes

$$\Gamma_{//} \approx \frac{\sqrt{\in_c'} - \in_c' \cos\theta_i}{\sqrt{\in_c'} + \in_c' \cos\theta_i}$$

$$= \frac{\dfrac{1}{\sqrt{\in_c'}\cos\theta_i} - 1}{\dfrac{1}{\sqrt{\in_c'}\cos\theta_i} + 1} \tag{A-22}$$

From Eq. (A-12), it was shown that

$$\frac{1}{\sqrt{\in_c'}} = \frac{R_s}{\eta_o}(1 + j) \tag{A-23}$$

Substitution into Eq. (A-22) and letting

$$v = \frac{R_s}{\eta_o}\frac{1}{\cos\theta_i} \tag{A-24}$$

leads to

$$\Gamma_{//} = \frac{v(1 + j) - 1}{v(1 + j) + 1} \tag{A-25}$$

so that

$$\left|\Gamma_{//}\right|^2 = 1 - \frac{4v}{1 + 2v(1 + v)} \tag{A-26}$$

and

$$(T_n)_{//} = \left(1 - \left|\Gamma_{//}\right|^2\right) T_p$$

$$= \frac{4v}{1 + 2v(1 + v)} T_p \tag{A-27}$$

Once again, the reader is reminded that the only assumption used in deriving $(T_n)_{//}$ above was that

$$\left(\frac{\eta_o}{R_s}\right)^2 >> 2$$

so that Eq. (A-27) may be considered to be sufficiently close to being exact for all values of θ_i except near the grazing angle where $\theta_i = 90$ deg.

Assuming that $2v << 1$ in the denominator of Eq. (A-27), the approximate formula is derived from Eq. (A-27) to be

$$(T_n')_{//} = 4vT_p = \left(\frac{4R_s}{\eta_o \cos\theta_i}\right) T_p \tag{A-28}$$

which is the same as Eq. (2) in the main text.

The exact error on the approximate formula given in Eq. (A-28) is

$$\begin{aligned} \left(\mathcal{E}_{//}\right)_{\text{exact}} &= (T_n')_{//} - (T_n)_{//} \\ &= (T_n')_{//} \left[1 - \frac{1}{1 + 2v(1+v)}\right] \end{aligned} \tag{A-29}$$

which leads to an approximate error formula of

$$\left(\mathcal{E}_{//}\right)_{\text{approx}} = 2v\,(T_n')_{//} \tag{A-30}$$

where v was given in Eq. (A-24) and for convenience is shown again here to be

$$v = \frac{R_s}{\eta_o}\frac{1}{\cos\theta_i}$$

For example, for aluminum, $\sigma_n = 2.3$ at 8.45 GHz. Then for $\theta_i = 89.6$ deg,

$$\frac{R_s}{\eta_o} = 1.01 \times 10^{-4}$$

$$v = 0.01447$$

$$(T_n')_{//} = 16.786 \text{ K}$$

so the approximate error on the $(T_n')_{//}$ value for this example is $2v\,(T_n')_{//} = 0.49$ K, which agrees with the results of Fig. 2. The reason the error on $(T_n')_{//}$ increases with θ_i is because $\cos\theta_i$ appears in the denominator. As θ_i goes close to 90 deg, the error becomes very large. If one is interested in calculating $(T_n)_{//}$ exactly in the region $87 \leq \theta_i \leq 89.9$ deg, the exact expression given by Eq. (A-27) should be used. What is so surprising is that the approximate formula for $(T_n')_{//}$, as given by Eq. (A-28), is so accurate for $\theta_i > 85$ deg even with $\cos\theta_i$ appearing in the denominator. The explanation is that even if $1/\cos\theta_i$ is large, v is still small for most highly conductive metals, such as aluminum used for reflector surfaces.