A Cross-Correlated Trellis-Coded Quadrature Modulation Representation of MIL-STD Shaped Offset Quadrature Phase-Shift Keying

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We show that MIL-STD shaped offset quadrature phase-shift keying (SOQPSK), a highly bandwidth-efficient constant-envelope modulation, can be represented in the form of a cross-correlated trellis-coded quadrature modulation, a generic structure containing both memory and cross-correlation between the in-phase and quadrature-phase channels. Such a representation allows identification of the optimum form of receiver for MIL-STD SOQPSK and at the same time, through modification of the equivalent I and Q encoders to recursive types, allows for it to be embedded as the inner code of a serial or parallel (turbo-like) concatenated coding structure together with iterative decoding.

I. Introduction

Shaped offset quadrature phase-shift keying (SOQPSK) is a constant-envelope modulation scheme that was developed as a variant of shaped binary phase-shift keying (SBPSK), introduced by Dapper and Hill [1] in the early 1980s as a means of bandlimiting a BPSK signal while, at the same time, keeping its envelope constant. The initial version of SOQPSK, which became adopted as part of a military standard, was referred to as MIL-STD SOQPSK and assumed a rectangular frequency-shaping pulse of duration equal to a bit time interval in its continuous phase modulation (CPM) representation. Other more spectrally efficient versions of SOQPSK were introduced later on by Hill [2] with frequency-shaping pulses that extend over several bit intervals. These variants offer spectral containment and power efficiency comparable to or better than Feher-patented quadrature phase-shift keying (FQPSK) [3], depending on the specifics of the comparison.

Cross-correlated trellis-coded quadrature modulation (XTCQM) was introduced by Simon and Yan [4] as a generic modulation scheme, containing both memory and cross-correlation between the in-phase (I) and quadrature-phase (Q) channels, that expands on the notion of combined bandwidth/power efficiency indigenous to trellis-coded modulation (TCM), but with particular emphasis on the spectral occupancy of the signal, while at the same time paying careful attention to the desirability of small envelope fluctuation.

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The research described in this publication was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

One specific embodiment of XTCQM is FQPSK, which, when viewed in this representation, makes the need for a memory type of receiver to achieve the best performance obvious. In this article, we show that MIL-STD SOQPSK also can be represented in the form of an XTCQM which then allows identification of the optimum form of receiver for this modulation scheme. Furthermore, analogous to the use of FQPSK (with encoders in its XTCQM representation modified to be of the recursive type) to form a serial or parallel (turbo-like) concatenated coding structure and its associated iterative decoding [5], it now becomes apparent that one can readily augment MIL-STD SOQPSK with the same form of concatenated coding/decoding. We will report the performance of coded offset quadrature phase-shift keying (OQPSK) and SOQPSK with iterative decoding in another article.

II. A Brief Review of the CPM Representation of MIL-STD SOQPSK

A conventional binary single-mode (one modulation index for all transmission intervals) CPM signal has the generic form (see the implementation in Fig. 1)

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left(2\pi f_c t + \phi(t, \boldsymbol{\alpha}) + \phi_0\right), \quad nT_b \leq t \leq (n+1)T_b \tag{1}$$

where E_b and T_b respectively denote the energy and duration of a bit ($P = E_b/T_b$ is the signal power), and f_c is the carrier frequency. In addition, $\phi(t, \boldsymbol{\alpha})$ is the phase modulation process that is expressible in the form

$$\phi(t, \boldsymbol{\alpha}) = 2\pi \sum_{i \leq n} \alpha_i h q\left(t - iT_b\right) \tag{2}$$

where $\boldsymbol{\alpha} = (\cdots, \alpha_{-2}, \alpha_{-1}, \alpha_0, \alpha_1, \alpha_2, \cdots)$ is an independent, identically distributed (i.i.d.) binary data sequence, with each element taking on equiprobable values ± 1, $h = 2\Delta f T_b$ is the modulation index (Δf is the peak frequency deviation of the carrier), and $q(t)$ is the normalized phase-smoothing response that defines how the underlying phase, $2\pi\alpha_i h$, evolves with time during the associated bit interval. Without loss of generality, the arbitrary phase constant, ϕ_0, can be set to zero.

It is convenient for our purpose here to identify the derivative of $q(t)$, namely,

$$g(t) = \frac{dq(t)}{dt} \tag{3}$$

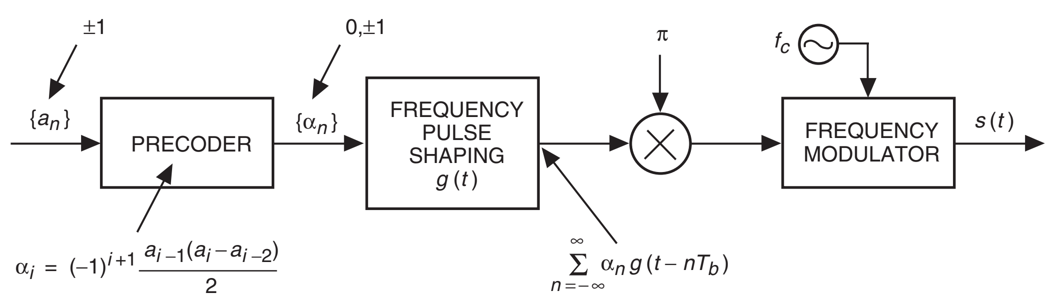


Fig. 1. Precoded CPM transmitter equivalent to OQPSK.

which represents the instantaneous frequency pulse (relative to the nominal carrier frequency, f_c) in the zeroth signaling interval. In view of Eq. (3), the phase-smoothing response is given by

$$q(t) = \int_{-\infty}^{t} g(\tau) d\tau \tag{4}$$

which, in general, extends over infinite time. For full-response CPM schemes, as will be the case of interest here, $q(t)$ satisfies the following:

$$q(t) = \begin{cases} 0, & t \leq 0 \\ \dfrac{1}{2}, & t \geq T_b \end{cases} \tag{5}$$

and, thus, the frequency pulse, $g(t)$, is nonzero only over the bit interval, $0 \leq t \leq T_b$. In view of Eq. (5), we see that the ith data symbol, α_i, contributes a phase change of $\pi \alpha_i h$ rad to the total phase for all time after T_b seconds of its introduction, and, therefore, this fixed phase contribution extends over all future symbol intervals. Because of this overlap of the phase-smoothing responses, the total phase in any signaling interval is a function of the present data symbol as well as all of the past symbols, and accounts for the memory associated with this form of modulation. Consequently, in general, optimum detection of CPM schemes must be performed by a maximum-likelihood sequence estimator (MLSE) form of receiver as opposed to bit-by-bit detection, which is optimum for memoryless modulations such as conventional binary frequency-shift keying (FSK) with discontinuous phase.

To best understand the concept that motivated MIL-STD SOQPSK, one must first recognize that conventional OQPSK can be represented as a CPM in the form of Eq. (1) together with Eq. (2), where $h = 1/2$, the frequency pulse, $g(t)$, of Eq. (3) is a delta function, i.e., $g(t) = (1/2)\delta(t)$ [equivalently, the phase pulse, $q(t)$, is a step function, and the ith element of the effective data sequence, α_i, is related to the true input binary data bit sequence $\mathbf{a} = (\cdots, a_{-2}, a_{-1}, a_0, a_1, a_2, \cdots)$ by[2] [6]

$$\alpha_i = (-1)^{i+1} \frac{a_{i-1}\left(a_i - a_{i-2}\right)}{2} \tag{6}$$

In the following paragraphs, we will explain how Eq. (6) can be obtained through an eight-state trellis representation of OQPSK. In Eq. (6), the a_i's take on ± 1 values, then the α_i's come from a ternary $(-1, 0, +1)$ alphabet. Since $h = 1/2$ together with the factor of $1/2$ in $g(t)$ corresponds to a phase change of $\pi/2$ rad over a bit interval, then a value of $\alpha_i = 0$ suggests no change in carrier phase (no transition occurs in the I (or Q) data symbol sequence at the midsymbol time instant of the Q (or I) data symbol), whereas a value of $\alpha_i = \pm 1$ suggests a carrier phase change of $\pm\pi/2$ (a transition occurs in the I (or Q) data symbol sequence at the midsymbol time instant of the Q (or I) data symbol) over this same interval. Finally, note that since the duration of the frequency pulse does not exceed the baud (bit) interval, then, in accordance with the above discussion, the CPM representation is full response and can be implemented with the cascade of a *precoder* satisfying Eq. (6) and a conventional CPM modulator (see Fig. 1).

We now explain how Eq. (6) can be obtained through an eight-state (three-bit-state) trellis diagram of OQPSK as well as provide a means for demodulation of OQPSK using a Viterbi algorithm. The three-bit

[2] Note that the I and Q data symbols a_{In}, a_{Qn} of conventional offset QPSK are respectively obtained as the even and odd bits of the sequence $\mathbf{a}$, and because of the offset between them the phase is allowed to change every bit time. Also note that, whereas the I-Q representation of OQPSK contains I and Q data sequences at the symbol rate $1/T_s$, the effective data sequence for the CPM representation occurs at the half-symbol (bit) rate, $1/\left(T_s/2\right) = 1/T_b$.

trellis state variable for describing the CPM representation of OQPSK is defined as follows. The first bit defines whether the coming input bit, i.e., a_i, corresponds to an even interval (I) or odd interval (Q). In particular, we choose a "1" if the incoming bit will be assigned to the I channel (even bit interval) and a "0" if the incoming bit will be assigned to the Q channel (odd bit interval). Note that successive input bits will be alternately assigned to even (I) and odd (Q) intervals. Therefore, states starting with a 0 can only transition to states starting with a 1, and vice versa. The second and third bits of a trellis state correspond to the current phase state, which is represented by the current I bit and Q bit, respectively. Specifically, assuming a conventional Gray code mapping, the phase states $\pi/4, 3\pi/4, 5\pi/4, 7\pi/4$ are assigned the bit mappings (in the form of "IQ") $00, 10, 11, 01$. Note that at most one bit (I or Q) of the phase state can change during each state transition, i.e., the phase change is constrained to be $\pi/2$ rad, $-\pi/2$ rad, or 0 in each bit interval. For example, the phase state $00\,(\pi/4)$ can transition only to phase states $10\,(3\pi/4)$ and $01\,(7\pi/4)$, or remain in phase state $00\,(\pi/4)$, depending on whether the incoming bit is an I bit or a Q bit, and whether its value is 1 or 0. The corresponding phase changes are $\pi/2$ rad, $-\pi/2$ rad, and 0, respectively, which imply that the corresponding α_i's are $+1$, -1, and 0, respectively. Figure 2 is the eight-state trellis diagram illustrating the transitions from state to state in accordance with the above. The branches of the trellis are labeled with the value of α_i that results in the transition due to the corresponding phase state change.

It can be easily verified that the value of α_i on each branch is in accordance with Eq. (6). As an example, assume that we are currently at state 101. Thus, the current phase is $7\pi/4$ and the incoming bit will be assigned to the I channel (an even bit interval). Since the next incoming bit will occur in an odd bit interval, the allowable transitions are to states whose first bit is a 0. Also, since the current incoming bit is an I bit, then the Q bit (the third bit of the current state) cannot change. Thus, the two allowable states to which we can transition are 001 and 011, which is readily visible in Fig. 2. Suppose that, of these two choices, we transition to state 011, which implies that the corresponding input bit (the middle bit of the terminating state) is a 1 and the terminating phase is $5\pi/4$. Therefore, the phase change is $\pi/2$, which implies that $\alpha_i = -1$, as indicated by the corresponding branch label in Fig. 2. Alternatively, arbitrarily assuming the binary-to-antipodal mapping $0 \to 1,\ 1 \to -1$, we have that i is even (thus $(-1)^{i+1} = -1$), $a_i = -1$, and, corresponding to the current phase state 01, $a_{i-2} = 1, a_{i-1} = -1$, which when substituted in Eq. (6) gives $\alpha_i = -1$.

We see from the trellis diagram of OQPSK in Fig. 2 that, in any given bit (half-symbol) interval, the α_i's can assume only one of two equiprobable values, namely, 0 and $+1$ or 0 and -1, with the further restriction that a $+1$ cannot be followed by a -1, or vice versa. Thus, in reality, the modulation scheme is a binary CPM but one whose data alphabet can potentially vary (between two choices) from bit interval to bit interval. Another way of characterizing the variation rule for the data alphabet that is also readily seen from Fig. 2 is as follows: if the previous output α is 0, then the data alphabet for the current output α is switched relative to that available for the previous α, i.e., if it was (0,+1) for the previous transmission, it becomes $(0, -1)$ for the current transmission, and vice versa. On the other hand, if the previous α is a $+1$ or a -1, then the data alphabet for the current α remains the same as that available for the previous α, e.g., if it was (0,+1) for the previous transmission, it is again (0,+1) for the current transmission.

To improve bandwidth efficiency, SOQPSK introduces pulse shaping into the above CPM representation. In particular, MIL-SOQPSK uses a rectangular frequency pulse, i.e.,

$$g(t) = \begin{cases} \dfrac{1}{2T_b}, & 0 \le t \le T_b \\ 0, & \text{otherwise} \end{cases} \tag{7}$$

Equivalently, the phase pulse, $q(t)$, varies linearly with time over the bit interval and as such is still characterized by a full-response precoded CPM as in Fig. 1. In view of the fact that a rectangular pulse

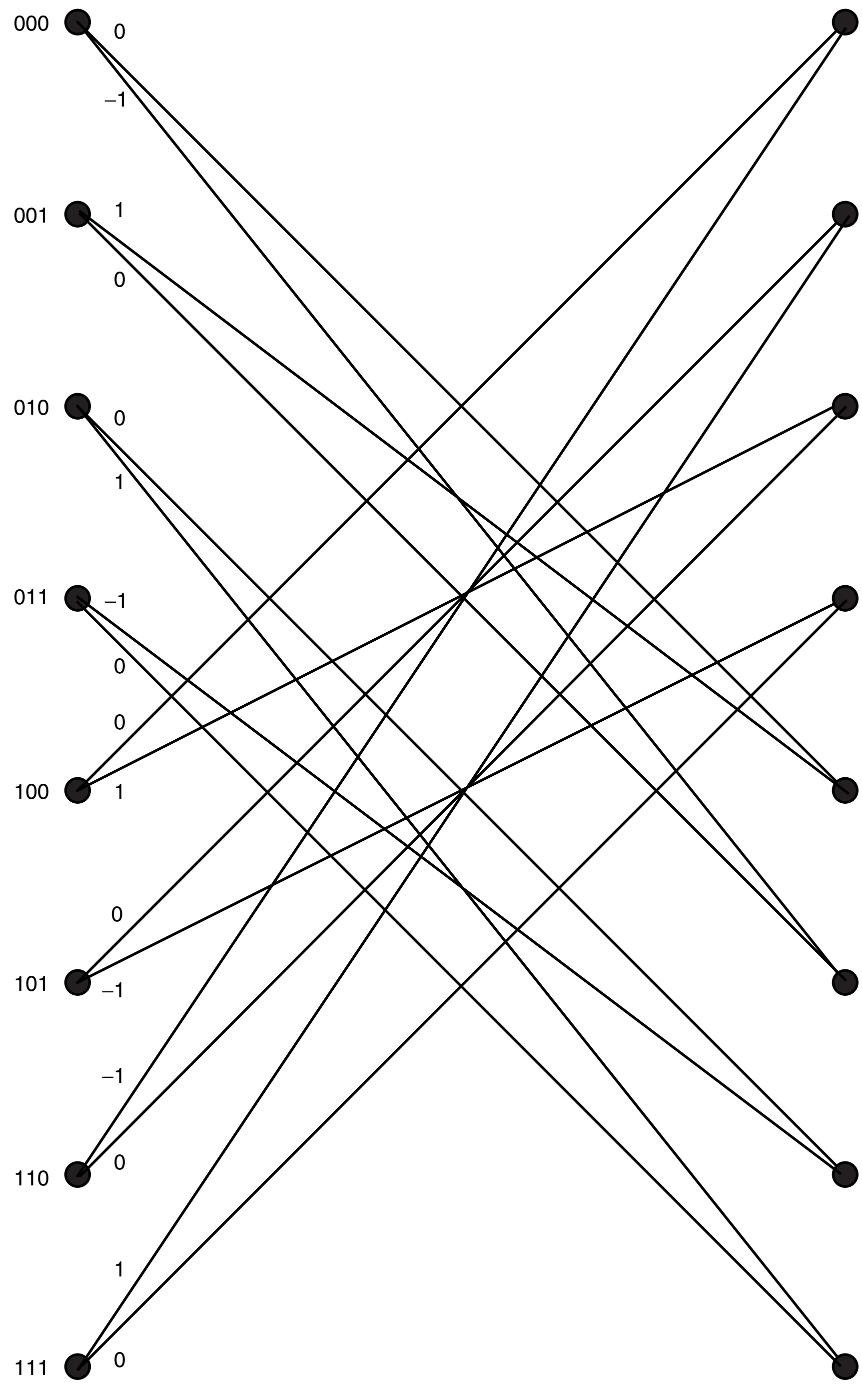


Fig. 2. 8-state trellis for precoder.

of duration equal to the bit period is used for $g(t)$, one might think that SOQPSK resembles minimum-shift keying (MSK); however, we remind the reader that for the latter the data alphabet is binary and fixed at $-1, +1$ whereas for the former it varies between $0, -1$ and $0, +1$. Thus, whereas in a given bit interval the phase for MSK is always linearly varying with either a positive or negative slope, the phase for SOQPSK can either vary linearly or remain stationary (have plateaus), the latter occurring during bit intervals where $\alpha_i = 0$. The phase trellis illustrating this behavior is shown in Fig. 3 and can be seen to time vary (alternate) between two different sets of transitions. Since for conventional OQPSK itself the phase trellis would only have plateaus (no linear variations), then in this sense MIL-STD SOQPSK can be viewed as a hybrid of OQPSK and MSK.

III. A Time-Invariant Trellis Representation Based on Pairs of Bits

Suppose now that we consider transitions between phase states corresponding to a pair of input bits. Without loss of generality, assume that the first bit of the input pair is always an I bit. Then, the trellis between the four phase states $\pi/4\,(00)$, $3\pi/4\,(10)$, $\pi/4\,(11)$, and $7\pi/4\,(01)$ can be easily derived from the eight-state trellis diagram of OQPSK shown in Fig. 2. Figure 4 illustrates such a trellis, where each branch is now labeled with a pair of output α values, say α_i, α_{i+1}. The corresponding two input bits are the same as the two bits representing the terminating phase state. For convenience, we have drawn the trellis in expanded form with each transition interval (now 2 bits in duration) showing the transitions leaving from one of the four states. One can associate a pair of waveforms that corresponds to each transition, namely, $s_I(t) = \cos\left[\phi(t, \alpha_i, \alpha_{i+1}) + \phi_0\right]$ and $s_Q(t) = \sin\left[\phi(t, \alpha_i, \alpha_{i+1}) + \phi_0\right]$, representing what would be time-synchronously transmitted as symbols (of 2-bit duration) on the I and Q channels. Here ϕ_0 is the initial phase indicated by the starting phase state of each transition, and from Eqs. (2) and (7),

$$\phi\left(t, \alpha_i, \alpha_{i+1}\right) = \begin{cases} \dfrac{\pi\alpha_i}{2T_b}t, & 0 \le t \le T_b \\[2ex] \dfrac{\pi\alpha_i}{2} + \dfrac{\pi\alpha_{i+1}}{2T_b}\left(t - T_b\right), & T_b \le t \le 2T_b \end{cases} \tag{8}$$

There are eight possible waveforms that can result for $s_I\left(t\right)$, denoted by $s_i(t);\ i = 0, 1, \cdots, 7$, and they are illustrated in Fig. 5. Similarly, there are eight possible waveforms that can result for $s_Q(t)$, denoted by $s_i'(t);\ i = 0, 1, \cdots, 7$, and they are illustrated in Fig. 6. What remains is to define a convenient labeling for these waveforms so that their assignment to the trellis branch transitions can be described by a simple mapping function dependent only on the $\boldsymbol{\alpha}$ sequence. Since this sequence is related to both the I and Q input bits by the precoder of Eq. (6), the notion of *cross-correlated mapping* becomes apparent.

The first observation is that if we are currently at state $\pi/4$ or $-\pi/4$, then the I-channel signals $s_I\left(t\right)$ for the four transitions leaving that state have the same set of signals, i.e., $s_0\left(t\right), s_1\left(t\right), s_2\left(t\right), s_3\left(t\right)$. Similarly, if we are currently at state $3\pi/4$ or $-3\pi/4$, then the I-channel signals $s_I\left(t\right)$ for the four transitions leaving that state also have the same set of signals, i.e., $s_4\left(t\right) = -s_0\left(t\right), s_5\left(t\right) = -s_1\left(t\right), s_6\left(t\right) = -s_2\left(t\right), s_7\left(t\right) = -s_3\left(t\right)$. Analogously, if we are currently at state $\pi/4$ or $3\pi/4$, then the Q-channel signals $s_Q\left(t\right)$ for the four transitions leaving that state have the same set of signals, i.e., $s_0'\left(t\right), s_1'\left(t\right), s_2'\left(t\right), s_3'\left(t\right)$. Similarly, if we are currently at state $-\pi/4$ or $-3\pi/4$, then the Q-channel signals $s_Q\left(t\right)$ for the four transitions leaving that state also have the same set of signals, i.e., $s_4'\left(t\right) = -s_0'\left(t\right), s_5'\left(t\right) = -s_1'\left(t\right), s_6'\left(t\right) = -s_2'\left(t\right), s_7'\left(t\right) = -s_3'\left(t\right)$. The second observation is that if the first of the two values of α, namely, α_i along the transition, is a zero, then the state either remains the same or switches to its negative counterpart. For example, if the originating state is $\pi/4$ and $\alpha_i = 0$, then the final state is either $\pi/4$ or $-\pi/4$, depending on the value of α_{i+1}. Similarly, if the first of the two values of α, namely, α_i along the transition, is $+1$ or -1, then the state switches to the other value or its negative counterpart. For example, if the originating state is $\pi/4$ and $\alpha_i = \pm 1$, then the final state is either $3\pi/4$ or $-3\pi/4$, depending on the value of α_{i+1}. These properties can immediately be observed from the trellis diagram in Fig. 4.

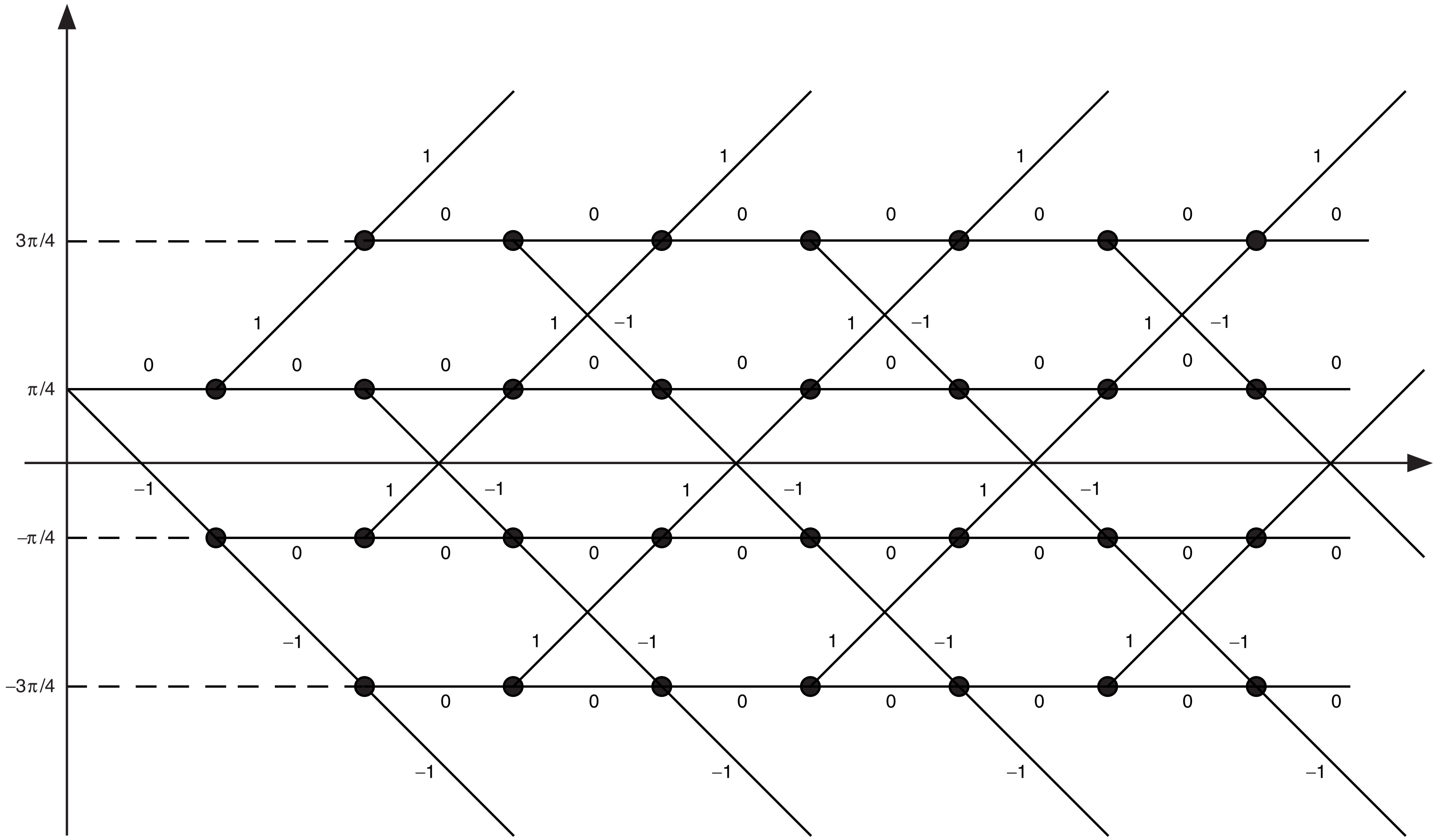


Fig. 3. Phase trellis diagram for MIL-STD SOQPSK (branches are labeled with values of α_i).

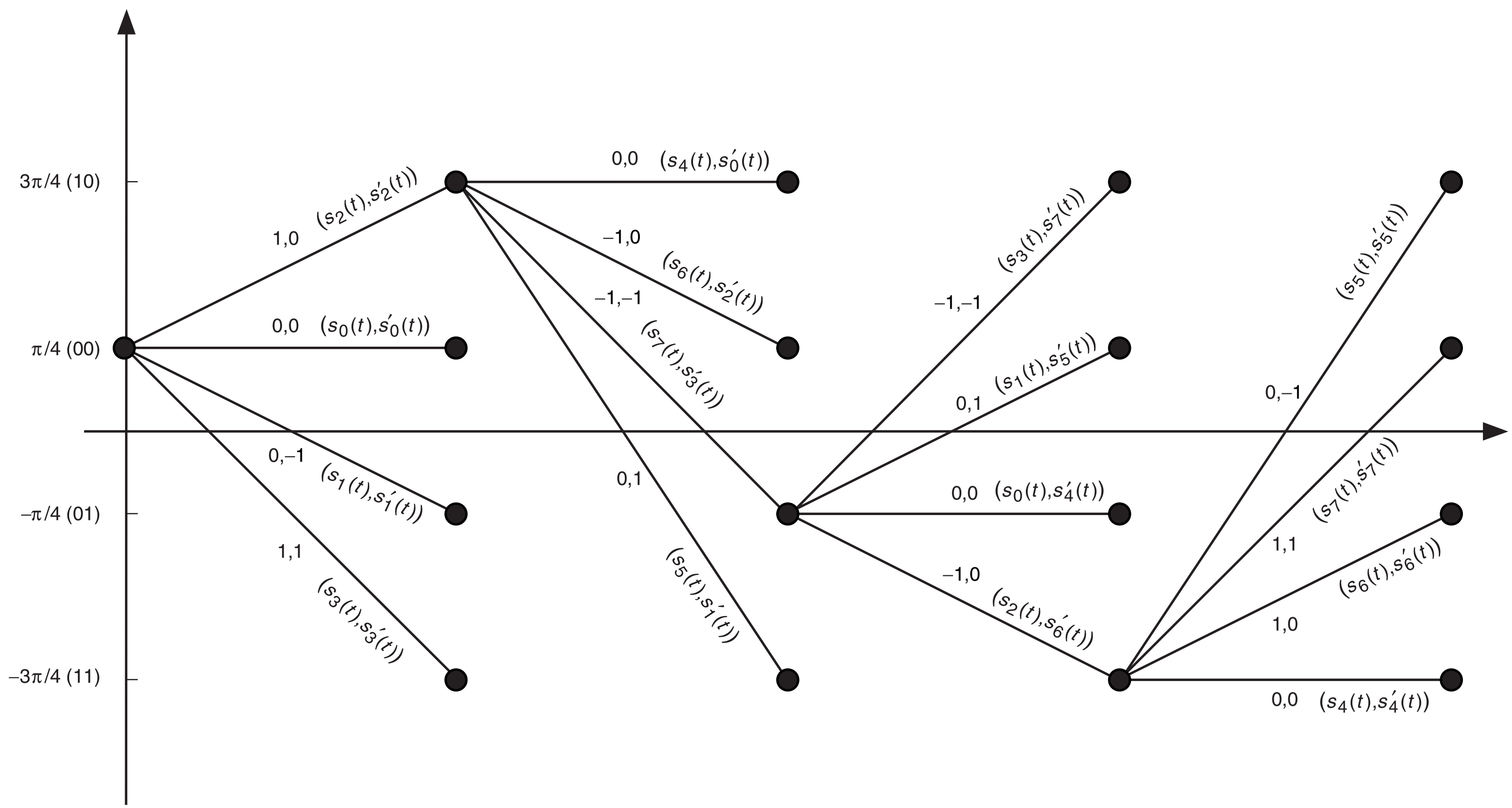


Fig. 4. Expanded (branches leaving each state) time-invariant phase trellis equivalent to Fig. 3.

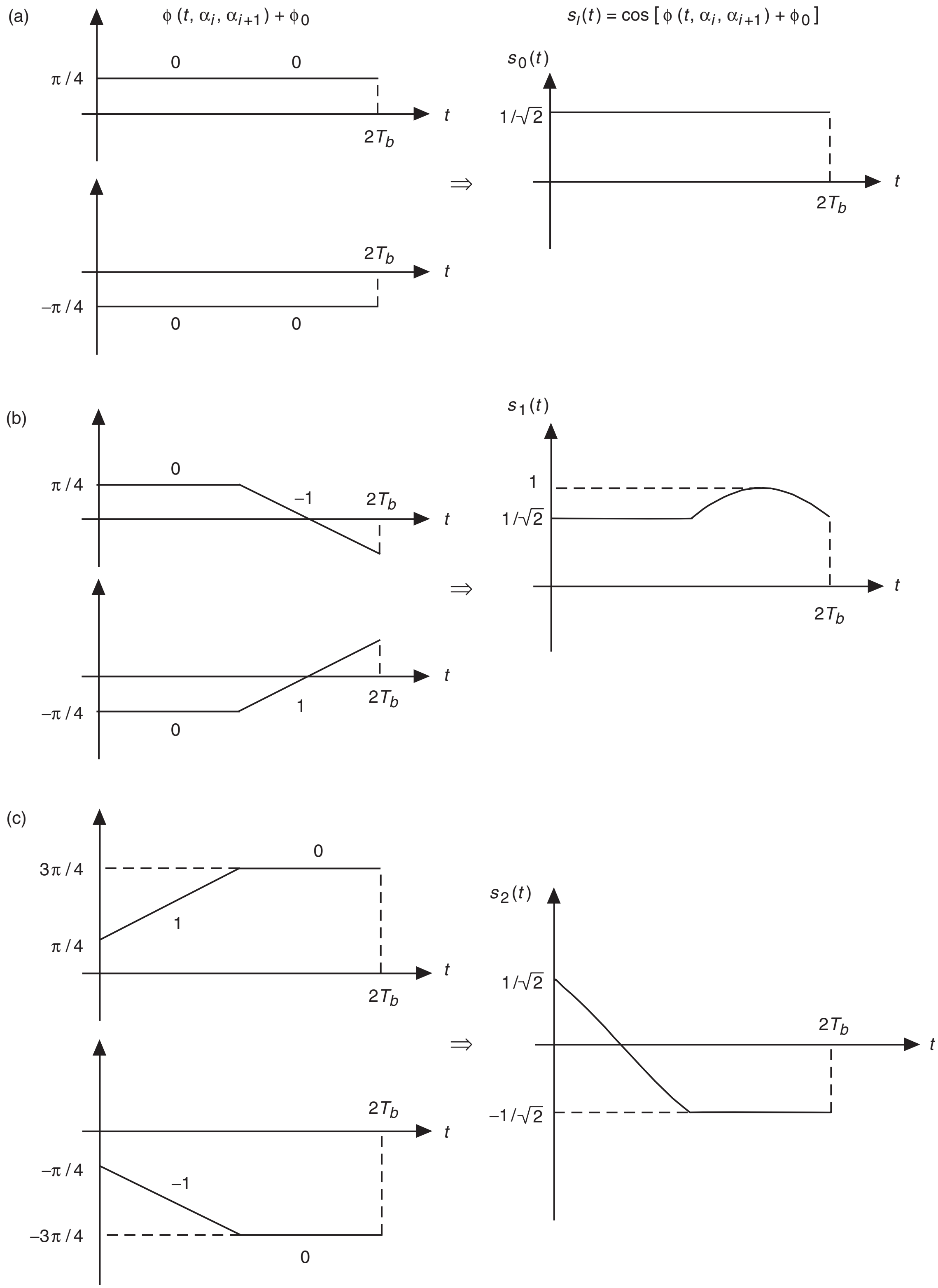


Fig. 5. Two-bit phase sequences and corresponding in-phase (I) waveforms: (a) $s_0(t)$**, (b)** $s_1(t)$**, (c)** $s_2(t)$**, (d)** $s_3(t)$**, (e)** $s_4(t)$**, (f)** $s_5(t)$**, (g)** $s_6(t)$**, and (h)** $s_7(t)$**.**

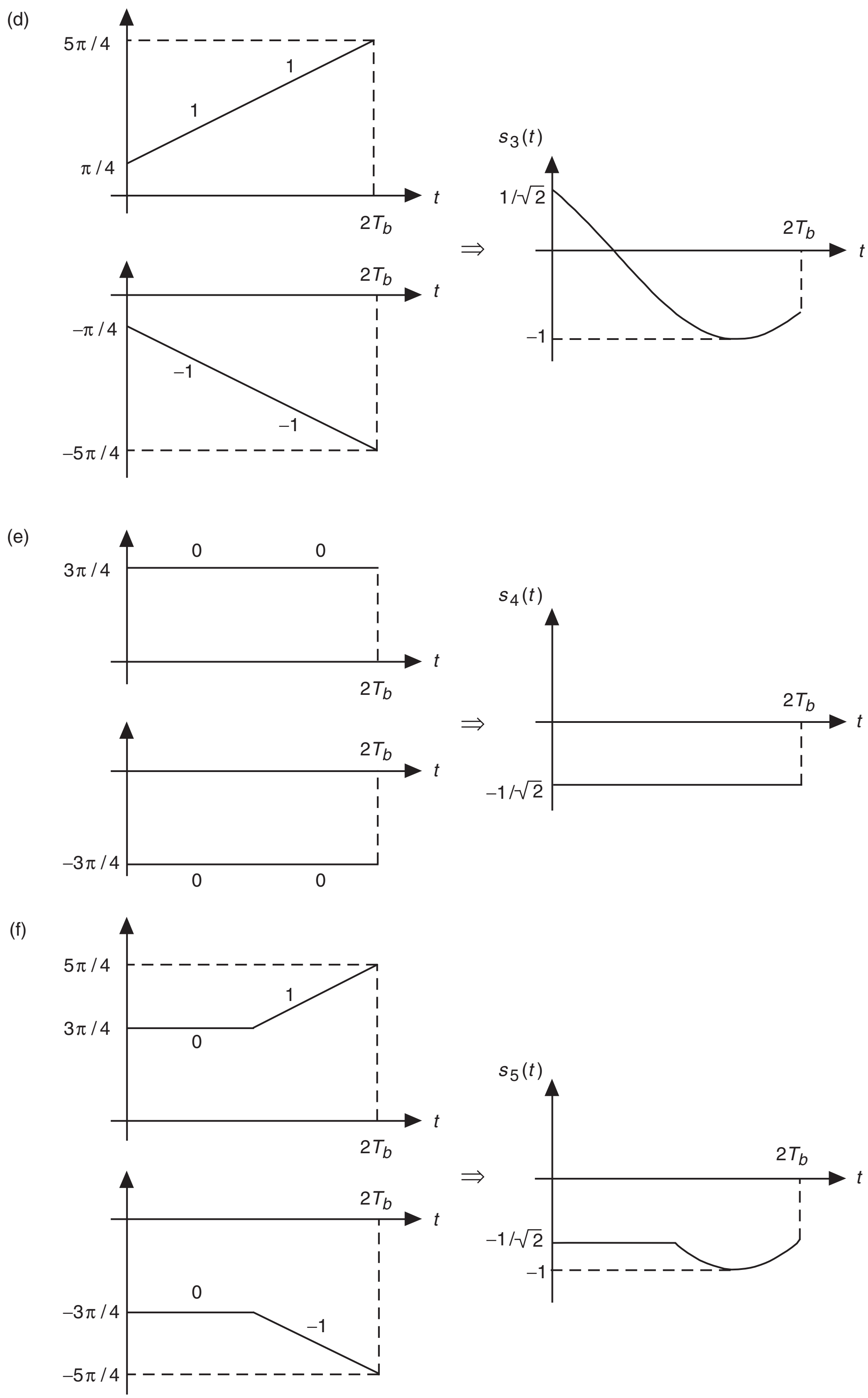

(d)
5π / 4
1
1
π / 4
t
2T_b
$s_3(t)$
1/√2
2T_b
t
⇒
2T_b
t
–π / 4
–1
–1
–5π / 4
–1
(e)
0
0
3π / 4
t
2T_b
$s_4(t)$
2T_b
t
⇒
2T_b
t
–1/√2
–3π / 4
0
0
(f)
5π / 4
1
3π / 4
0
t
2T_b
$s_5(t)$
2T_b
t
⇒
2T_b
t
–1/√2
–1
0
–3π / 4
–1
–5π / 4

Fig. 5 (contd).

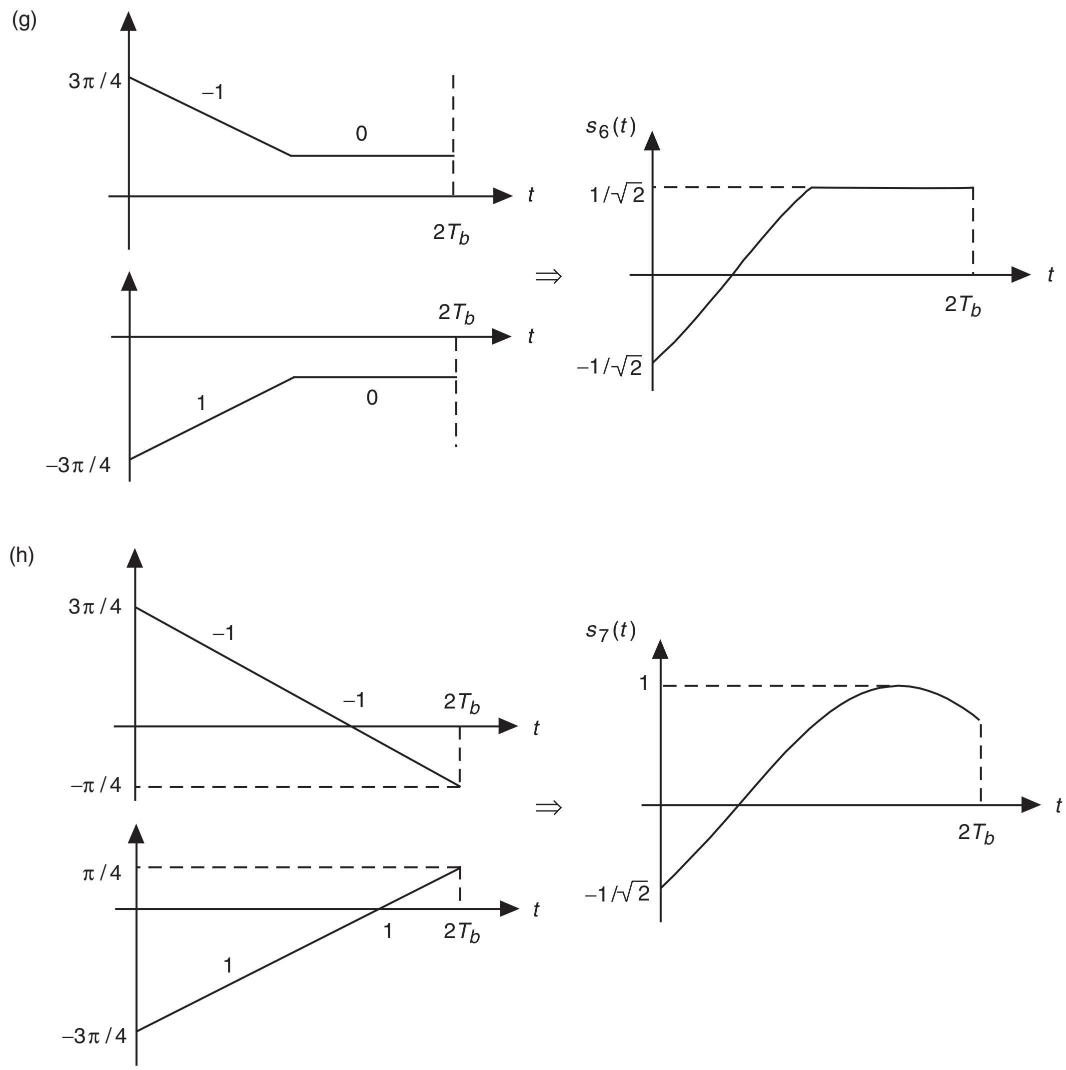


Fig. 5 (contd).

IV. Transmitter Implementation

Based upon the foregoing trellis representation of MIL-STD SOQPSK and the labeling of the waveforms illustrated in Figs. 5 and 6, it is possible to express the indices of the specific waveforms transmitted for $s_I(t)$ and $s_Q(t)$ in a given symbol (two-bit) interval, say $iT_b \le t \le (i+2)T_b$ (i even), in terms of two α values in this interval and the phase state at the beginning of the interval (which itself depends on the previous values of α). Specifically, corresponding to α_i and α_{i+1} in the above interval and ϕ_i at the start of this interval, we have $s_I(t) = s_n(t)$, where n has the binary-coded decimal (BCD) representation

$$n = k_I \times 2^2 + |\alpha_i| \times 2^1 + |\alpha_{i+1}| \times 2^0 \tag{9}$$

with

$$k_I = \begin{cases} 0, & \text{if } \phi_i = \pm\pi/4 \\ 1, & \text{if } \phi_i = \pm 3\pi/4 \end{cases} \tag{10}$$

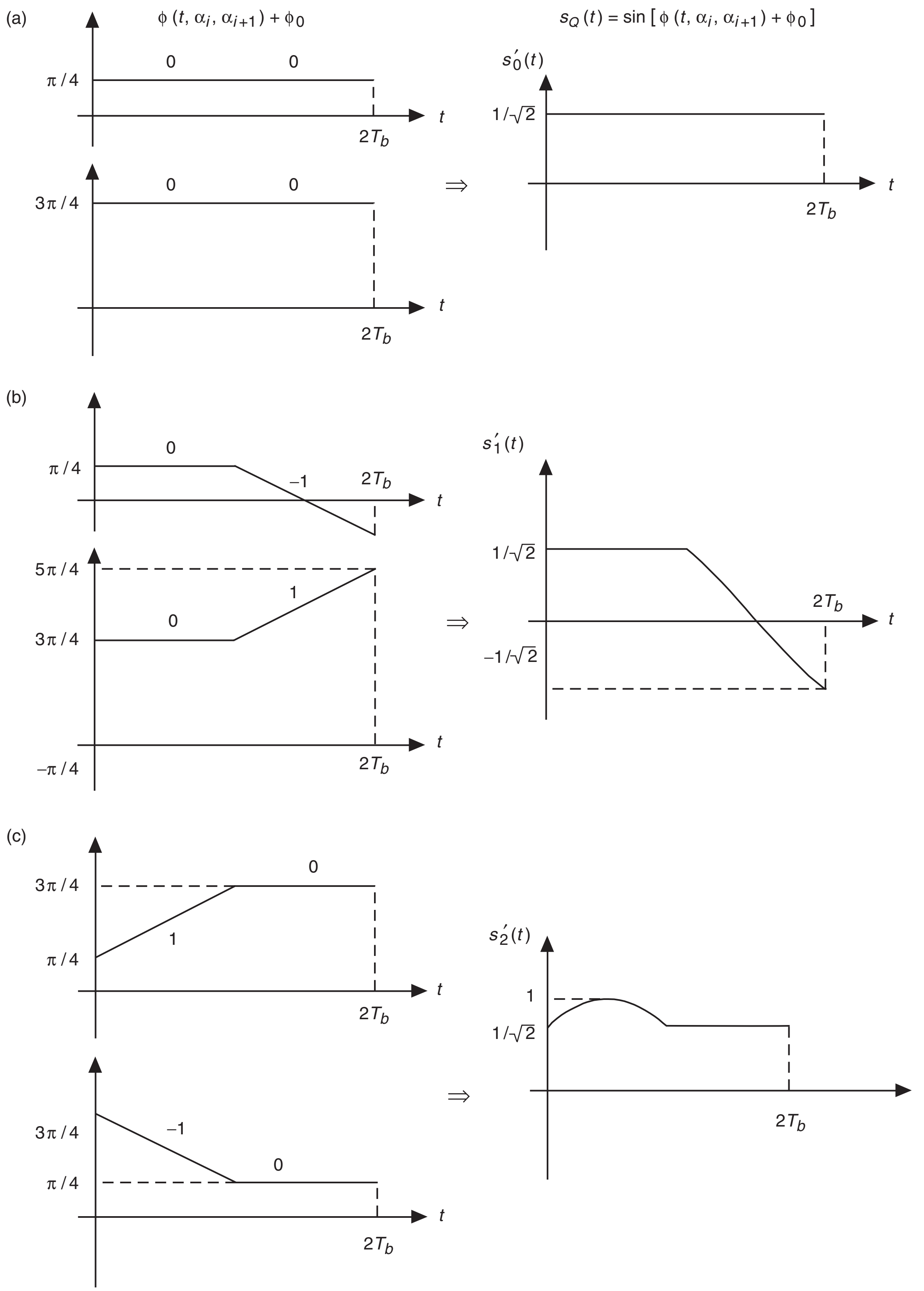
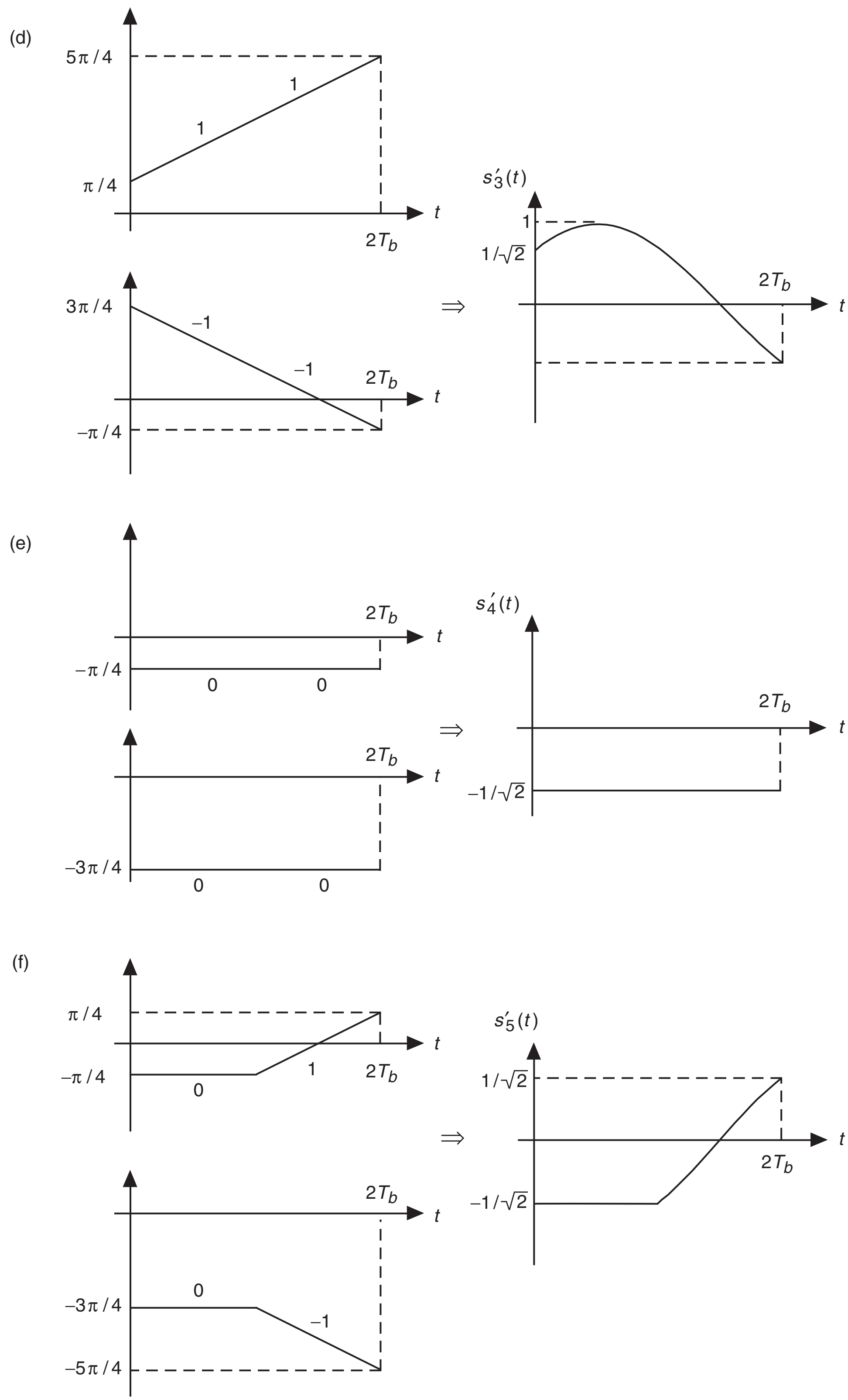


Fig. 6. Two-bit phase sequences and corresponding quadrature-phase (Q) waveforms: (a) $s'_0(t)$, (b) $s'_1(t)$, (c) $s'_2(t)$, (d) $s'_3(t)$, (e) $s'_4(t)$, (f) $s'_5(t)$, (g) $s'_6(t)$, and (h) $s'_7(t)$.



(d)
5π / 4
1
1
π / 4
t
2T_b
3π / 4
–1
–1
2T_b
t
–π / 4
⇒
$s_3'(t)$
1
1 / √2
2T_b
t
(e)
2T_b
t
–π / 4
0
0
2T_b
t
–3π / 4
0
0
⇒
$s_4'(t)$
2T_b
t
–1 / √2
(f)
π / 4
t
–π / 4
0
1
2T_b
2T_b
t
0
–3π / 4
–1
–5π / 4
⇒
$s_5'(t)$
1 / √2
t
2T_b
–1 / √2

Fig. 6 (contd).

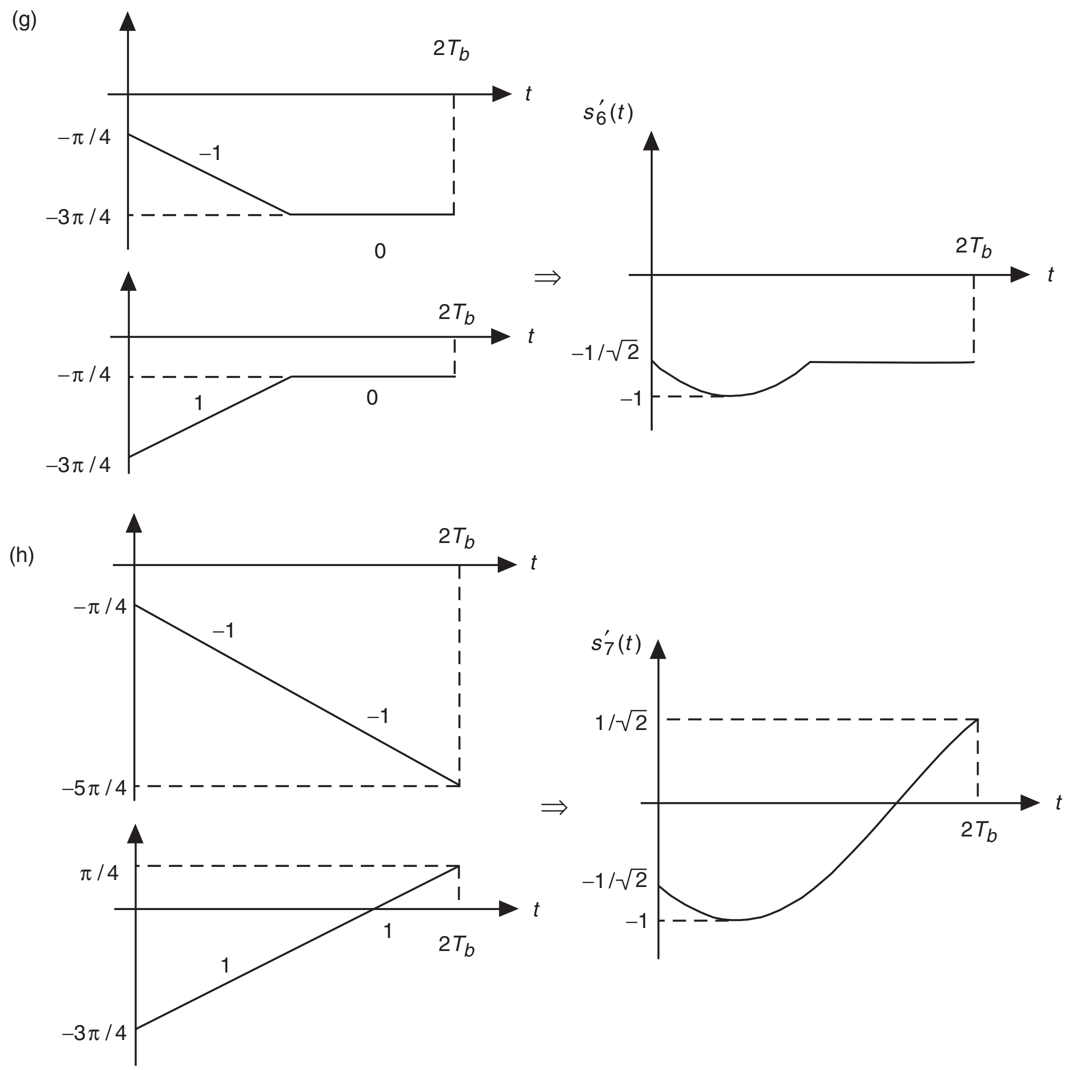


Fig. 6 (contd).

Similarly, we have $s_Q(t) = s'_n(t)$ where n has the BCD representation

$$n = k_Q \times 2^2 + |\alpha_i| \times 2^1 + |\alpha_{i+1}| \times 2^0 \tag{11}$$

with

$$k_Q = \begin{cases} 0, & \text{if } \phi_i = \pi/4, 3\pi/4 \\ 1, & \text{if } \phi_i = -\pi/4, -3\pi/4 \end{cases} \tag{12}$$

A block diagram of the equivalent XTCQM transmitter for MIL-STD SOQPSK based on the above considerations is presented in Fig. 7.

As an example of the above, suppose that the input data sequence is as follows: $a_{-2} = 1, a_{-1} = 1, a_0 = 1, a_1 = -1, a_2 = 1, a_3 = 1, a_4 = -1, a_5 = 1, a_6 = -1, a_7 = -1, a_8 = -1, a_9 = 1, a_{10} = -1, a_{11} = 1$. Then, in accordance with Eq. (6), the precoded data sequence is $\alpha_0 = 0, \alpha_1 = -1, \alpha_2 = 0, \alpha_3 = 1, \alpha_4 = 1, \alpha_5 = 0, \alpha_6 = 0, \alpha_7 = 1, \alpha_8 = 0, \alpha_9 = -1, \alpha_{10} = 0, \alpha_{11} = 0$. The $\boldsymbol{\alpha}$ sequence also can be

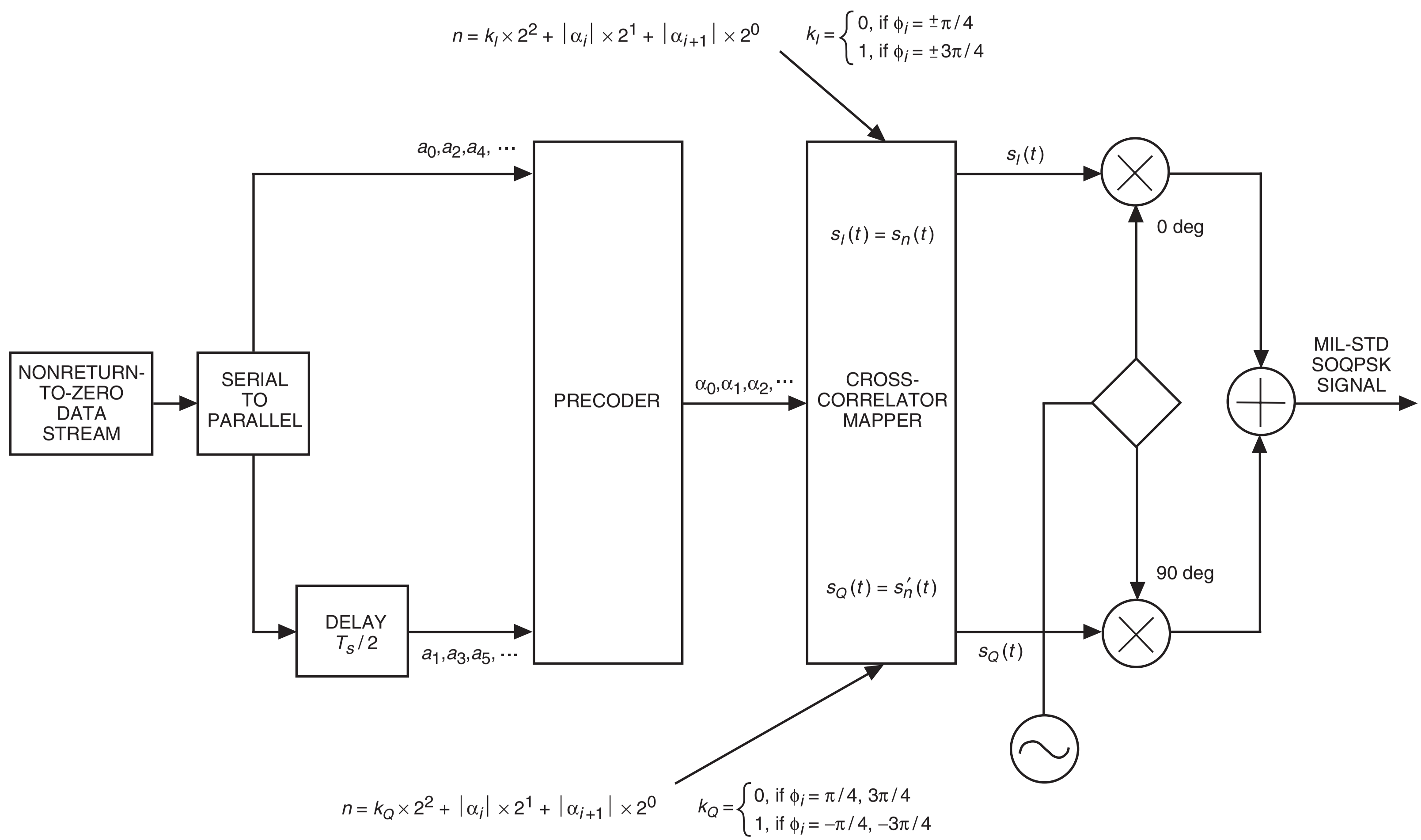


Fig. 7. XTCQM equivalent transmitter for MIL-STD SOQPSK.

easily obtained from the phase trellis in Fig. 4. Specifically, arbitrarily assuming the binary-to-antipodal mapping $0 \rightarrow 1,\ 1 \rightarrow -1$, we have the initial phase $\phi_0 = \pi/4\,(a_{-2}a_{-1} = 00)$ at $t = 0$, and the binary representation of the remaining data sequence (in pairs) is $a_0a_1 = 01, a_2a_3 = 00, a_4a_5 = 10, a_6a_7 = 11, a_8a_9 = 10, a_{10}a_{11} = 10$. Thus, it is readily seen from Fig. 4 that the corresponding $\boldsymbol{\alpha}$ sequence is in accordance with the above. In addition, the corresponding transmitted symbol sequence on the I channel would be $s_1\,(t)\,, s_1\,(t)\,, s_2\,(t)\,, s_5\,(t)\,, s_5\,(t)\,, s_4\,(t)$, whereas the corresponding transmitted symbol sequence on the Q channel would be $s_1'\,(t)\,, s_5'\,(t)\,, s_2'\,(t)\,, s_1'\,(t)\,, s_5'\,(t)\,, s_0'\,(t)$.

V. Conclusion

Based on the full-response CPM representation of offset QPSK and MIL-STD shaped offset QPSK (SOQPSK) modulations, we have shown that the latter can be represented in the form of a cross-correlated trellis-coded quadrature modulation (XTCQM). Such a representation is analogous to a similar form previously obtained for Feher-patented QPSK (FQPSK) and, as was the case there, will be particularly useful when concatenating this modulation with an outer turbo code coupled with iterative decoding.

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