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Noise Temperature of Cascaded Mirrors Having Resistive and Spillover Losses

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This article presents the derivation of a new formula for calculating the noise temperature of mirrors in cascade in a beam-waveguide (BWG) system. Resistive and spillover losses of each mirror are taken into account. No simplifying assumptions are made, and loss factors are not assumed to be unity. Comparison of noise temperatures calculated by the new formula and a simpler approximate formula (used by other authors in the past) shows negligible differences when used to calculate noise temperatures of cascaded low-loss mirrors such as those in current Deep Space Network BWG antenna systems.

I. Introduction

It was previously shown in an article by Veruttipong [1] that an approximate formula can be used to calculate the noise temperature of a cascade of similar mirrors having both resistive and spillover losses. However, the limitations of accuracy and errors of this approximate noise temperature formula are not known.

Since lowering noise temperature is important for improving the performance of low-noise beam-waveguide (BWG) antennas for the Deep Space Network (DSN), it is important that limiting accuracies of any approximate formulas used to calculate the noise temperature contributions be known and understood. Due to the limitations of the approximate formulas, it was decided that a rigorous formula needed to be derived from basic considerations with no simplifying assumptions. It is the purpose of this article to show the step-by-step derivations and officially document them for future reference. Section II of this article presents the derivation. Section III gives an example of calculated results obtained through the use of the new and old approximate formulas. Section IV presents concluding remarks.

II. Derivation

Figure 1 shows a basic block diagram modeling the resistive and spillover loss of a single mirror in a BWG antenna system. For a DSN BWG antenna system, the noise source temperature shown in Fig. 1 includes noise-temperature contributions from the sky and contributions due to subreflector and main-reflector resistive and spillover losses, as well as the BWG shroud wall loss down to the first mirror. The

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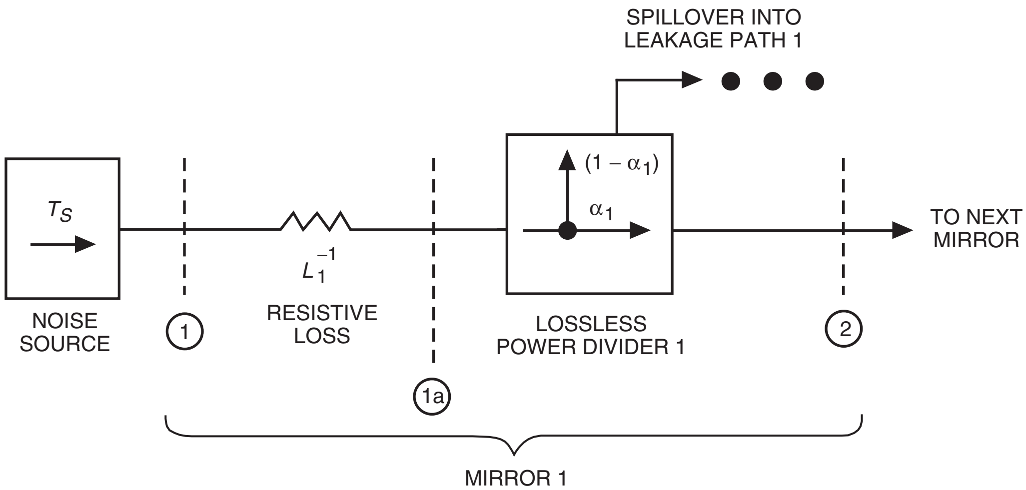

Fig. 1. Basic diagram of a single mirror having resistive and spillover losses.

34-m BWG antenna at DSS 13 was reported to have a source noise temperature at the input to the first mirror (port 1) of about 8 K [2] when the antenna was pointed to zenith sky at 8.45 GHz. This source temperature at port 1 passes through the resistive loss of the first mirror. Then, as shown in Fig. 1, the equivalent source temperature at output port 1a can be derived as

$$T'_{S1} = T_S L_1^{-1} + \left(1 - L_1^{-1}\right) T_P \tag{1}$$

where T_S is the source noise temperature at input port 1, L_1 is the power loss factor ($\geq$1) due to the resistive loss of mirror 1, and T_P is the physical temperature of mirror 1. In this article, unless otherwise stated, all noise and physical temperatures have units of kelvin. Then this equivalent source noise temperature at port 1a goes into a lossless power divider. The lossless power divider is just a theoretical model used to depict the power splitting that occurs in free space, where one output of the power divider goes to the main path to arrive at the next mirror while the other output is the power that spills past the mirror (due to beam spreading in free space). This spillover power goes along a leakage path, and a portion of it arrives at the receive horn aperture. Let α_1 represent the ratio of (output power going to the main path) to (the total input power at port 1a), and $(1 - \alpha_1)$ is the ratio of (spillover power going to the leakage path) to (the total input power at port 1a). Then the new equivalent source temperature in the main path at port 2 is

$$T''_s = T'_{s1}\alpha_1 = \underbrace{T_s L_1^{-1}\alpha_1}_{\substack{\text{attenuated} \\ \text{source NT}}} + \underbrace{\left(1 - L_1^{-1}\right) T_p \alpha_1}_{\substack{\text{mirror resistive} \\ \text{loss NT}}} \tag{2}$$

where the first term is the attenuated source temperature and the second term is the mirror-only noise temperature (NT) contribution due *only to the resistive loss* of mirror 1 after being attenuated by the power split.

Equation (2) does not include noise temperature due to absorption that occurs in the leakage path. The spillover output power (or noise temperature) travels along a leakage path that includes reflections off the shroud walls, dissipation inside the cavity-like enclosure behind the mirror, and reflections off the basement walls, floor, and ceiling before entering the receiver port via the receive horn aperture. If any reflected noise power from the spillover environment radiates back into the power divider shown in Fig. 1, it is assumed that none of it gets to the receiver via the main path.

It is clear that the leakage diagram shown in Fig. 1 is overly simplified. The leakage path out of the power divider as depicted in Fig. 1 is not a single path but could be multiple paths. Furthermore, all or just a fraction of the $(1-\alpha_1)$ spillover power ultimately arrives at the receive horn aperture, as depicted in Fig. 2. It should be stated that Figs. 1 and 2 are conceptual block diagrams only and are useful for describing the power splitting that occurs in free space and the combining of noise temperatures at the receive horn aperture. Due to the difficulty of modeling the leakage phenomenon accurately, a spillover noise temperature term purposely is not included in Eq. (2). However, as was shown by Veruttipong [1], the overall spillover contribution to system noise temperature can be calculated through use of a simple formula given as

$$T_{\text{spill}} = K_1 T_1 + K_2 T_2 \tag{3}$$

where K_1 is equal to the total effective spillover power ratio for mirrors 5 and 6 in the basement (pedestal room) and K_2 is the total effective spillover power ratio for the four mirrors above the basement ceiling. The symbols T_1 and T_2 are, respectively, the effective brightness temperatures of the leakage environment for the mirrors below and above the basement ceiling. Veruttipong's experiments [1] showed that, at DSS 13, which has no basement shroud, T_1 and T_2 were 300 K and 240 K, respectively. In Eq. (3), this author uses the symbols K_1 and K_2 in place of the symbols α_1 and α_2 used by Veruttipong in [1]. In this article, this author defines α_1 as the main path power ratio for mirror 1 [see the definition preceding Eq. (2)] while Veruttipong in [1] defines this same symbol to be the total effective spillover power ratio of mirrors 5 and 6.

If spillover actually takes place before the first mirror's resistive loss in Fig. 1, it is necessary only that a new power divider be inserted between the source at port 1 and the first mirror's resistive loss. Then replace T_s in Eq. (1) by $T_s\alpha_o$, where α_o is the ratio of power output in the main path to the total power input to the power divider. It can be seen that in Fig. 1 and in Eq. (2) that replacing T_s with $T_s\alpha_o$ will affect the attenuated source temperature, but α_o does not appear in the last part of Eq. (2). The overall spillover contribution as expressed in Eq. (3) may increase slightly due to α_o but is unrelated to the mirror noise-temperature contribution. Therefore, the existence of a hypothetical initial power splitter will not be considered.

For this article, it is of interest to derive a more rigorous formula for calculating the noise temperature due only to resistive losses of the mirrors but to include the attenuation factor caused by the spillover loss ratio. Therefore, begin the derivation by using only the last term in Eq. (2) expressed as

$$T_{m,1} = \left(1 - L_1^{-1}\right) \alpha_1 T_p \tag{4}$$

This noise temperature goes along the main path to the second mirror shown in Fig. 2. Using a procedure similar to that used in deriving Eq. (2), the noise temperature at the output of mirror 2 is

$$T_{m,2} = T_{m,1} L_2^{-1} \alpha_2 + \left(1 - L_2^{-1}\right) \alpha_2 T_p \tag{5}$$

Substitution of Eq. (4) gives

$$\begin{aligned} T_{m,2} &= \left[\left(1 - L_1^{-1}\right) \alpha_1 T_p\right] L_2^{-1} \alpha_2 + \left(1 - L_2^{-1}\right) \alpha_2 T_p \\ &= \left\{ \left[\left(1 - L_1^{-1}\right) \alpha_1\right] F_1 + \left(1 - L_2^{-1}\right) \alpha_2 F_2 \right\} T_p \end{aligned} \tag{6}$$

where $F_1 = L_2^{-1}\alpha_2$ and $F_2 = 1$.

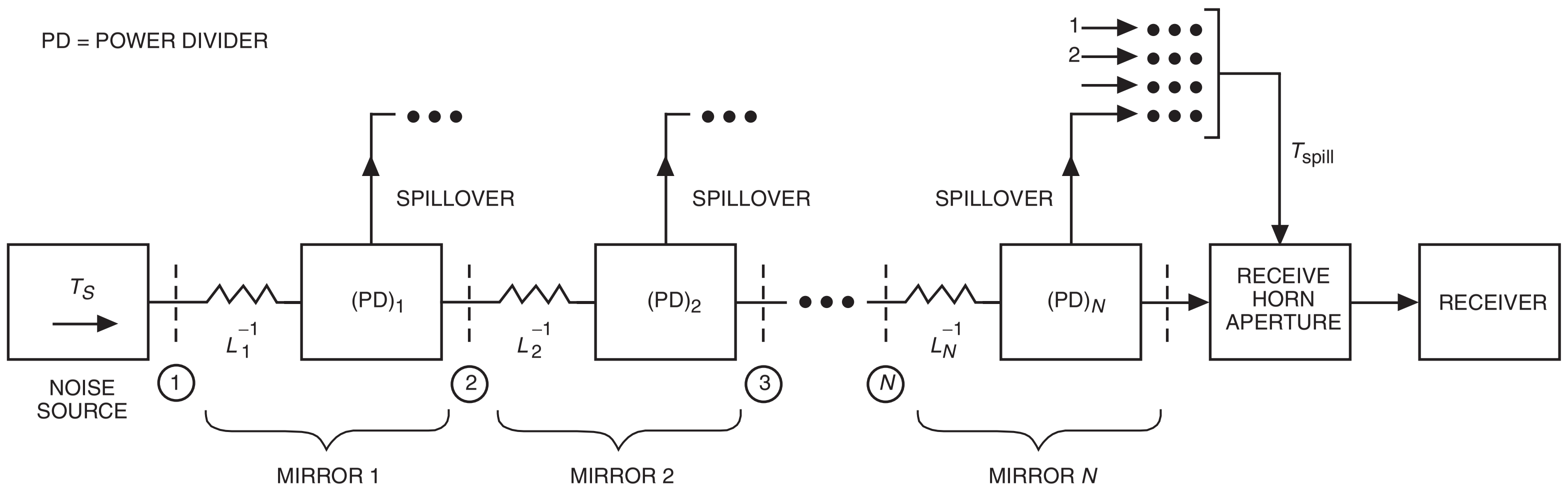

Fig. 2. Block diagram for the system of cascaded mirrors having resistive and spillover losses.

Extending this process to N number of mirrors in cascade, assuming that each mirror has spillover losses going into leakage paths as shown in Fig. 2, the total noise temperature at the output of N mirrors in the main path is

$$T_{m,N} = \sum_{i=1}^{N} \left(1 - L_i^{-1}\right) \alpha_i F_i T_p \tag{7}$$

where

$$F_i = \prod_{i+1}^{N} \left[L_{i+1}^{-1} \alpha_{i+1}\right] \tag{8}$$

The symbol $\prod$ denotes the product of the terms inside the bracket. When $i = N$, set $F_N = 1$ and ignore Eq. (8).

For example, take the case where $N = 4$ mirrors. Then from Eq. (7),

$$T_{m,4} = \left[\left(1 - L_1^{-1}\right) \alpha_1 F_1 + \left(1 - L_2^{-1}\right) \alpha_2 F_2 + \left(1 - L_3^{-1}\right) \alpha_3 F_3 + \left(1 - L_4^{-1}\right) \alpha_4 F_4\right] T_p$$

where from Eq. (8)

$$F_1 = L_2^{-1} \alpha_2 L_3^{-1} \alpha_3 L_4^{-1} \alpha_4$$

$$F_2 = L_3^{-1} \alpha_3 L_4^{-1} \alpha_4$$

$$F_3 = L_4^{-1} \alpha_4$$

and

$$F_4 = 1$$

For a mirror surface with resistive loss, L_i^{-1} is just equal to the power reflection coefficient of the mirror or

$$L_i^{-1} = |\Gamma_i|^2 \tag{9}$$

and

$$1 - L_i^{-1} = 1 - |\Gamma_i|^2 \tag{10}$$

where Γ_i is the voltage reflection coefficient as seen looking at the surface of the ith mirror.

A. Special Case

For the case where there are no spillover losses, or where $\alpha_i = 1$, substitution into Eqs. (7) and (8) will show that

$$T_{m,N} = \left(1 - \prod_{i=1}^{N} |\Gamma_i|^2\right) T_p \tag{11}$$

The derivation of Eq. (11) can most easily be seen by considering the example of $N = 4$ mirrors in the paragraph after Eq. (8). Make the substitutions of $\alpha_i = 1$ and $L_i^{-1} = |\Gamma_i|^2$, and it can be seen that

$$T_{m,4} = \left[\left(1 - |\Gamma_1|^2\right) F_1 + \left(1 - |\Gamma_2|^2\right) F_2 + \left(1 - |\Gamma_3|^2\right) F_3 + \left(1 - |\Gamma_4|^2\right) F_4\right] T_p$$

where from Eq. (8)

$$F_1 = |\Gamma_2|^2 |\Gamma_3|^2 |\Gamma_4|^2$$

$$F_2 = |\Gamma_3|^2 |\Gamma_4|^2$$

$$F_3 = |\Gamma_4|^2$$

and

$$F_4 = 1$$

Substitutions of these values into the above expression for $T_{m,4}$ result in

$$T_{m,4} = \left[\left(1 - |\Gamma_1|^2 |\Gamma_2|^2 |\Gamma_3|^2 |\Gamma_4|^2\right)\right] T_p$$

The extension of this example for 4 mirrors to N mirrors will result in the derivation of Eq. (11). Then for the special case where $|\Gamma_i|$ is identical for all mirrors, Eq. (11) becomes

$$T_{m,N} = \left(1 - |\Gamma_i|^{2N}\right) T_p \tag{12}$$

which was a formula previously presented by Otoshi in various articles, including [3,4].

B. General Case

In an article by Otoshi [5], it was shown that, for mirrors fabricated from high-conductivity metals such as aluminum, the noise temperature formula for circular polarization (accurate up to 89.5-deg incidence angle) is

$$\left(1 - |\Gamma_i|^2\right) = \frac{4R_{si}}{\eta_o} \bullet \frac{1}{2}\left(\cos\theta_i + \frac{1}{\cos\theta_i}\right) \tag{13}$$

where R_{si} is the metallic surface resistivity of the ith mirrors in ohms per square, η_o is the free space characteristic impedance in ohms, and θ_i is the ith mirror's angle of incidence in degrees. Surface resistivity is related to electrical conductivity and frequency by the simple formula derived in [3] and [5] as

$$R_s = 0.02\pi\sqrt{\frac{f_{\text{GHz}}}{10\sigma_n}} \tag{14}$$

where f_{GHz} is frequency in gigahertz and σ_n is the normalized metal surface electrical conductivity obtained by dividing the actual electrical conductivity in mhos/m by 10^7.

Substitutions of Eqs. (9) and (10) into Eqs. (7) and (8) and further substitution of Eq.(13) give

$$T_{m,N} = \frac{2T_p}{\eta_o}\sum_{i=1}^{N} R_{si}\alpha_i\left(\cos\theta_i + \frac{1}{\cos\theta_i}\right)F_i \tag{15}$$

where

$$F_i = \prod_{i=1}^{N}\left[\left|\Gamma_{i+1}\right|^2 \alpha_{i+1}\right] \tag{16}$$

and $|\Gamma_i|^2$ for a circularly polarized wave can be derived from Eq. (13) as

$$\left|\Gamma_i\right|^2 = 1 - \frac{2R_{si}}{\eta_o}\left(\cos\theta_i + \frac{1}{\cos\theta_i}\right) \tag{17}$$

Examination of Eq. (16) shows that if $|\Gamma_{i+1}|^2 \approx 1$, $\alpha_{i+1} \approx 1$, then $F_i \approx 1$, and then Eq. (15) will approach being the same as the approximate formula previously presented in [1] as

$$\left(T_{m,N}\right)_{approx} = \frac{2T_p}{\eta_o}\sum_{i=1}^{N} R_{si}\alpha_i\left(\cos\theta_i + \frac{1}{\cos\theta_i}\right) \tag{18}$$

It can be seen that the above approximate formula can be derived from the more rigorous formula given in Eq. (15) by assuming that $F_i = 1$ for all the mirrors. In the next section, it will be shown that making these assumptions for mirrors in current DSN BWG systems will result in negligible errors.

III. Example

For comparison of numerical values obtained from the rigorous and approximate formulas, the same values in the example of [1] for typical DSN BWG mirrors will be used. The numerical values are $\sigma_n = 2.3$ for aluminum, $f_{\text{GHz}} = 8.45$ GHz, $\eta_o = 120\pi$ ohms, and an incidence angle value of 30 deg for mirrors 1 and 2 and an incidence angle value of 45 deg for mirrors 3, 4, 5, and 6. Substitution of these values into Eqs. (14) and (17) results in

$$R_s = 0.038 \text{ ohms/square}$$

$$|\Gamma_1|^2 = |\Gamma_2|^2 = 0.99959$$

$$|\Gamma_3|^2 = |\Gamma_4|^2 = |\Gamma_5|^2 = |\Gamma_6|^2 = 0.99957$$

Furthermore, use the same typical value of $\alpha_i = 0.995$ (or 0.5 percent for the spillover ratio) for each BWG mirror as was used in the example in [1]. Substitution of the above values and $T_P = 290$ K into the rigorous formula given by Eq. (15) results in a calculated noise-temperature value for six DSN BWG mirrors at 8.45 GHz of

$$T_{m,6} = 0.723 \text{ K}$$

Substitution of these same values into the approximate formula given by Eq. (18) results in a calculated noise temperature value of

$$(T_{m,6})_{approx} = 0.733 \text{ K}$$

For this example, the rigorous formula gives an overall noise temperature of six mirrors that is only 0.01 K smaller than that from the approximate formula (that assumes $F_i = 1$). The error in using the approximate formula is only 1.35 percent.

IV. Concluding Remarks

The derivation of a more rigorous formula for calculating the noise temperature of cascaded mirrors has been presented. For current DSN BWG antenna systems where $N = 6$, the use of the approximate formula rather than the rigorous formula would result in negligible errors. For a large number of mirrors, >10, or for lossier mirror systems, consideration should be given to using the more rigorous formula.

The rigorous formula presented in this article applies only to noise temperature due to the resistive losses of the mirror and attenuation caused by the power dividers that were used to model and account for the effects of spillover losses. An expression for the overall spillover noise-temperature contribution (to the system noise temperature) also was presented, but calculations were not made because it was too difficult to accurately model all of the losses and reflections in the leakage paths. The actual spillover noise-temperature contributions, however, can be determined through the use of experimentally obtained data and the analytical method described by Veruttipong in [1].

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