A Rectangular Constellation-Based Blind Equalization Technique

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Blind equalization techniques have been developed over the past three decades that estimate and equalize communication channels without requiring training sequences and without channel knowledge. This article presents a new blind equalization algorithm with particular relevance to large-order, square or rectangular constellations. Comparisons with standard blind equalization methods are also presented.

I. Introduction

In 1999, we developed a new algorithm particularly suitable for the blind equalization of large-order, rectangular signal constellations, e.g., quadrature amplitude modulation (QAM), distorted by multipath channels and carrier offsets [1]. Since this algorithm was never fully documented, we present in this article a summary of the algorithm, beginning in this section with an overview of blind equalization. In particular, blind equalization provides for the recovery of unknown signals via a finite-dimensional, linear projection of a channel output data vector. In component form, we have

$$z_n = \sum_{i=0}^{L-1} w_i \, y_{n-i} \tag{1}$$

where the w_i denote the L projection (blind equalizer) coefficients and the z_n are the complex output samples from the blind adaptive equalizer. The y_n denote complex (baseband) samples from the unknown communication channel, which can in turn be expressed as a convolution of the sampled channel impulse response, f_i, with an unknown sequence of independent and identically distributed (iid) complex source symbols, a_n:

$$y_n = \sum_{i} f_i a_{n-i} \tag{2}$$

The blind adaptive filter coefficients w_i are derived using only the available channel output data, without knowledge of either the transmitted signal waveform or the linear channel.

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Approaches to blind equalization can be broadly categorized into four different classes: (1) property restoral techniques wherein the projection coefficients are derived via the maximization (or minimization) of various cost criteria [2–7]; (b) direct channel coefficient estimation techniques that utilize higher-order statistics (i.e., cumulants) [8]; (c) maximum-likelihood estimation techniques wherein both the channel and the unknown signal are estimated simultaneously [9]; and (d) direct channel coefficient estimation techniques that exploit cyclostationary properties of the transmitted waveform [10]. The first class of techniques is perhaps the broadest since it can incorporate a wide range of cost criteria. Furthermore, this class of techniques is the oldest and consequently has been studied extensively. The majority of this analysis has dealt with the convergence of time-recursive, gradient-based algorithms for achieving the maximum/minimum of the cost functions, such as the Godard (or constant modulus) algorithm [4,5,11]. Our new algorithm as presented in the following section falls in this first class.

II. Blind Equalization Algorithm for Rectangular Constellations

Here we present a new blind equalization technique based on the first class of techniques, which involves the maximization (or minimization) of a cost function. In particular, we start with a new blind equalization cost function that arises from a uniformly most powerful (UMP) scale invariant hypothesis test between factored (rectangular) generalized Gaussian distributions as discovered in [6]:

$$O_{s\,rect}^2 = \frac{\left\{ \frac{1}{N} \sum_{k=1}^{N} |z_k|^2 \right\}^{1/2}}{\left\{ \frac{1}{N} \sum_{k=1}^{N} \left(|z_{xk}|^s + |z_{yk}|^s \right) \right\}^{1/s}} \tag{3}$$

where the z_n are the complex equalizer output time samples as defined in Eq. (1) (z_{xn} and z_{yn} denote the real and imaginary parts, respectively, of z_n). As $s \to \infty$, $O_{s\,rect}^2$ approximates the limiting cost function [6]:

$$O_{s\,rect}^2 \overset{s\to\infty}{\to} = \frac{\sqrt{\frac{1}{N} \sum_{i=1}^{N} |z_i|^2}}{\max \left\{ |z_{xi}|\,, |z_{yi}| \right\}_{i=1}^{N}} \tag{4}$$

In implementing a gradient update algorithm for maximizing Eq. (3) (either batch or time recursive), we need to compute the complex gradient of $O_{s\,rect}^2$ with respect to the complex equalizer taps. Carrying out this differentiation yields

$$\frac{\partial O_{s\,rect}^2}{\partial w_k} = \frac{1}{N} \sum_{i=1}^{N} \Psi_{rect}(z_i) y_{i-k}^* \tag{5}$$

where

$$\Psi_{rect}(z_i) = \frac{O_{s\,rect}^2}{\left\langle |z|^2 \right\rangle_N} \left\{ z_i - \frac{\left\langle |z|^2 \right\rangle_N}{\left\langle |z_x|^s + |z_y|^s \right\rangle_N} \left(|z_{xi}|^{s-2} z_{xi} + j\, |z_{yi}|^{s-2} z_{yi} \right) \right\} \tag{6}$$

and $\langle \cdot \rangle_N$ denotes an N-sample time average. Using Eqs. (5) and (6), we can now iteratively compute the optimal equalizer taps that maximize Eq. (3) via

$$w_k \leftarrow w_k + \mu \frac{\partial O_{s\,rect}^2}{\partial w_k} \tag{7}$$

as follows:

(1) Start with a batch collection of N data samples, y_n, and initial equalizer weights.

(2) Given the initial equalizer weights, compute the equalizer outputs, z_n.

(3) Compute a new set of equalizer weights from Eqs. (5) through (7) and repeat until convergence.

Alternatively, we can derive a time-recursive algorithm by replacing the time average in Eq. (5) with the instantaneous estimate

$$\frac{\partial O_{s\,rect}^2}{\partial w_k} = \frac{1}{N}\sum_{i=1}^{N} \Psi_{rect}(z_i) y_{i-k}^* \approx \Psi_{rect}(z_n) y_{n-k}^* \tag{8}$$

and replacing the time-average operations $\langle\cdot\rangle_N$ in Eq. (6) with simple one-pole, lowpass recursive estimates.

Equations (7) and (8) comprise the time-recursive algorithm for updating the equalizer weights:

$$w_k(n+1) = w_k(n) + \mu \frac{O_{s\,rect}^2}{\left\langle |z|^2 \right\rangle_N} \left\{ z_n - \frac{\left\langle |z|^2 \right\rangle_N}{\langle |z_x|^s + |z_y|^s \rangle_N} \left(|z_{xn}|^{s-2} z_{xn} + j\,|z_{yn}|^{s-2} z_{yn} \right) \right\} y_{n-k}^* \tag{9}$$

where $O_{s\,rect}^2 = \left\{ (1/N)\sum_{k=1}^{N} |z_k|^2 \right\}^{1/2} \div \left\{ (1/N)\sum_{k=1}^{N} \left(|z_{xk}|^s + |z_{yk}|^s \right) \right\}^{1/s}$ and again the time averages $\langle\cdot\rangle_N$ are computed with simple one-pole, lowpass recursive estimates.

Alternatively, we can simplify the computations in Eq. (9) considerably by replacing the $\langle\cdot\rangle_N$ with true statistical expectations under the assumption of perfect equalization, i.e., when $z = a$. Under this assumption, Eq. (9) reduces to

$$w_k(n+1) = w_k(n) - \beta_{rect} \left\{ \left(|z_{xn}|^{s-2} z_{xn} + j\,|z_{yn}|^{s-2} z_{yn} \right) - R_{Orect} \cdot z_n \right\} y_{n-k}^* \tag{10}$$

where we define the normalized step size as $\beta_{rect} \hat{=} \mu \cdot \left\{ E|a|^2 \right\}^{1/2} / \left\{ E\left(|a_x|^s + |a_y|^s \right) \right\}^{1/s} / E\left(|a_x|^s + |a_y|^s \right)$ and the constant as $R_{Orect} \hat{=} 2E|a_x|^s / E|a|^2$ (a_x and a_y denote the real and imaginary parts of the complex source symbols, a). As in the case of the constant modulus algorithm (CMA) [4,5], this constant controls the scale of the equalized constellation at convergence. Equation (10) represents our new rectangular constellation-based blind equalization (RECBEQ) algorithm. It converges rapidly for input rectangular constellations distorted by multipath.

III. Asymptotic Performance Analysis

Previously [12], we have analyzed the asymptotic[2] performance of RECBEQ in terms of the achievable, average intersymbol interference, $\overline{ISI}$, near convergence [3]:

[2] As $N \to \infty$ in Eq. (3) or, equivalently, the steady-state performance of Eq. (10).

$$\overline{ISI} \equiv E\left\{\frac{\sum\limits_{n\neq n_{\max}} |c_n|^2}{|c_{n_{\max}}|^2}\right\} \tag{11}$$

where c_n denotes the combined channel/equalizer impulse response coefficients, i.e.,

$$c_n \equiv \sum_{i=0}^{L-1} w_i\, f_{n-i} \tag{12}$$

Ideally, we want the equalizer to remove all ISI, in which case $\overline{ISI} = 0$. In [12], it was shown that

$$\overline{ISI} \approx \frac{L-1}{N} \cdot A \tag{13}$$

where

$$A = \frac{\gamma_1\, E\left\{|a_x|^{2s-2} + |a_y|^{2s-2}\right\} - 1}{\left\{1 - \gamma_2(s-1)E\left[|a_x|^{s-2}\right]\right\}^2} \tag{14}$$

and $\gamma_1 \equiv E\left[|a|^2\right] / \left\{E\left[|a_x|^s + |a_y|^s\right]\right\}^2$ and $\gamma_2 \equiv E\left[|a|^2\right] / E\left[|a_x|^s + |a_y|^s\right]$. A represents an asymptotic figure of merit for blind equalizers—the smaller the A, the better. It can be derived for a wide class of cost functions [12], including the important constant modulus cost function O_s^2:

$$O_s^2(z) = \frac{\frac{1}{N}\sum\limits_{k=1}^{N} |z_k|^2}{\left\{\frac{1}{N}\sum\limits_{k=1}^{N} |z_k|^s\right\}^{2/s}} \tag{15}$$

which forms the basis for CMA [5,6].

Plots of A versus s are presented in Fig. 1 for both $O^2_{s\,rect}$ and O_s^2 (labeled as $O_s^{r=2}$), corresponding to 16-QAM and 64-QAM. Although A goes to zero as $s \to \infty$ in all cases, the superior performance of $O^2_{s\,rect}$ over O_s^2 for these constellations is clearly observed. Thus, based on this asymptotic precision analysis, we would recommend the utilization of $O^2_{s\,rect}$ for blindly restoring square constellations (whereas O_s^2 should be used to restore circularly symmetric constellations [12]).

IV. Extensions to Residual Frequency Offsets

When residual carrier offsets are present, the system must also include a data-directed phase-locked loop (PLL). A conventional system architecture [13], incorporating the CMA blind equalizer and data-aided PLL, is depicted in Fig. 2. The equalizer output is phase corrected ("de-rotated") by the PLL output, which is driven by symbol decisions based on the phase-corrected equalizer output. The viability of this architecture is based on the fact that the CMA blind equalizer algorithm is not affected by phase rotations of the input signal constellation and thus phase correction can occur after the CMA equalizer.

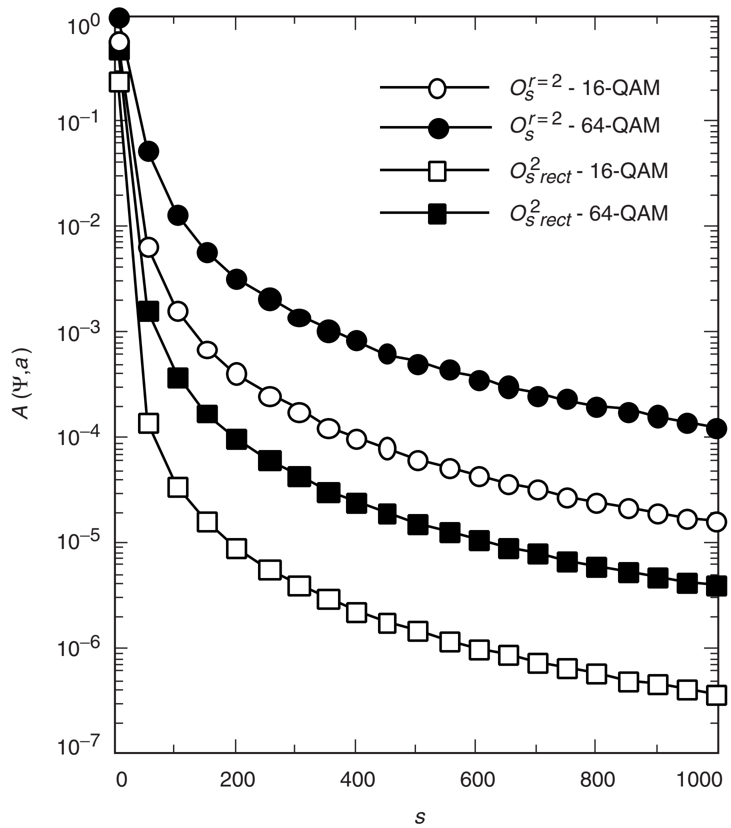

Fig. 1. Asymptotic figure of merit for 16-QAM and 64-QAM symbol constellations.

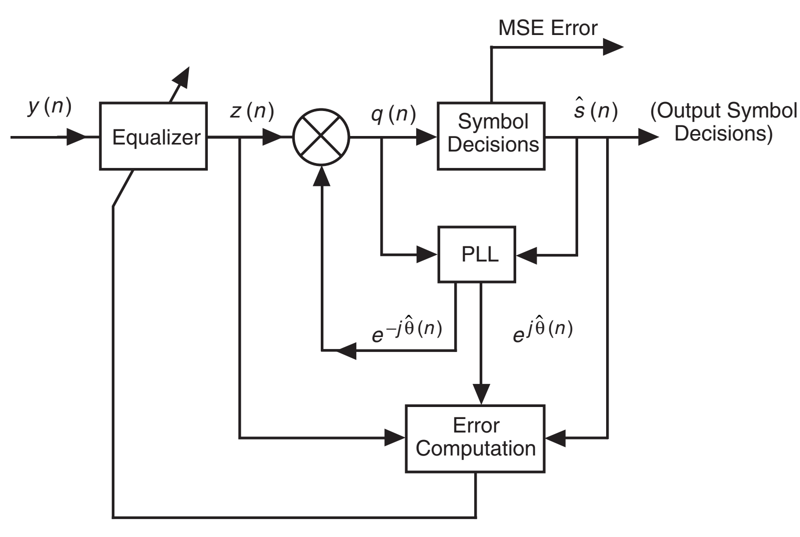

Fig. 2. Conventional architecture for joint blind equalization via CMA and carrier recovery.

Such is not the case with RECBEQ, which is sensitive to the phase orientation of the signal constellation. Through extensive testing, we have discovered that the RECBEQ algorithm, Eq. (10), can acquire a rotating rectangular constellation but without the same level of precision that would be achieved if the constellation were static. This led us to develop the modified architecture depicted in Fig. 3. Here the *input* to the RECBEQ equalizer is phase corrected by the PLL output, which again is driven by symbol decisions based directly on the equalizer output. In this way, RECBEQ initially equalizes the rotating constellation to such an extent that the PLL can lock up and finalize the joint equalization/carrier recovery process.

V. Simulation Results

To demonstrate its extremely rapid convergence for input QAM constellations, we present in Fig. 4 a simulated convergence curve produced by the RECBEQ algorithm[3] as configured in Fig. 3. For this example, the input constellation was 16-QAM, transmitted at 2400 symbols/s, with a 1-Hz carrier offset, and the multipath channel was based on that used in [3]. In addition, in Fig. 4 corresponding learning curves are presented for CMA and a variant of CMA [14], which is also insensitive to phase rotations of the input signal constellation. As is seen, RECBEQ yields the fastest convergence (within $20 \times 512 = 10,240$ symbols) as compared with the phase insensitive techniques, clearly illustrating the potential of the new blind equalization technique. Note that the convergence curves (mean-squared "slicing" error in decibels versus symbols processed) for all algorithms exhibit a dual plateau effect. At the first plateau (mean-squared error (MSE) ~ -12 dB), the equalizers have started to settle down, but the data-directed PLL has not yet locked on to the channel. Once the PLL has converged, the MSE approaches the second and final plateau (MSE ~ -30 dB). At this point, we can switch to decision-directed equalization if desired. Other higher-order constellations (e.g., 256-QAM) have been tested and clearly exhibit the superior performance of the RECBEQ algorithm.

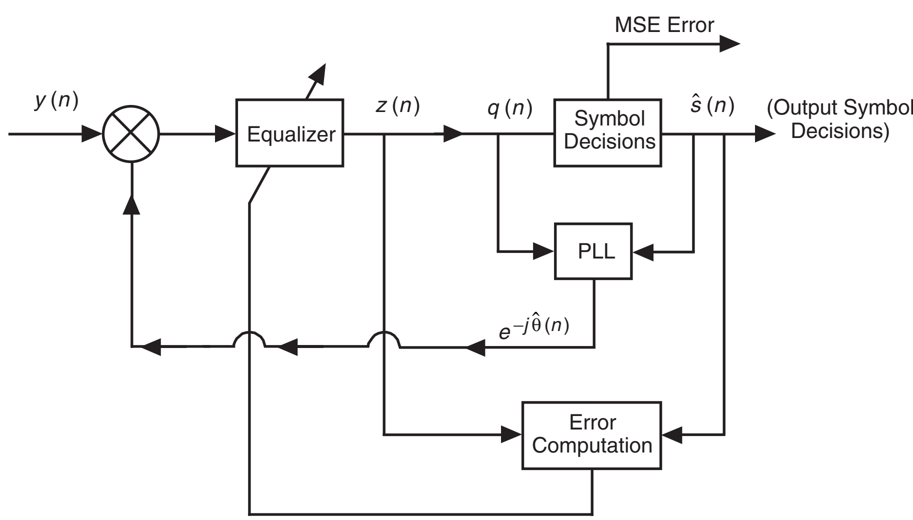

Fig. 3. New architecture for joint blind equalization via RECBEQ and carrier recovery.

[3] With $s = 8$ in Eq. (10).

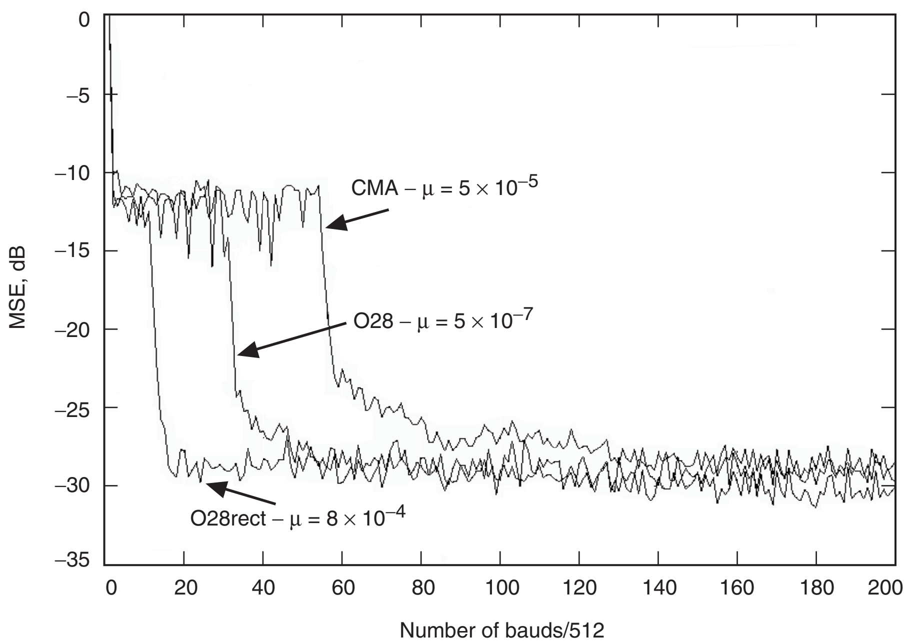

Fig. 4. Equalizer convergence curves for 16-QAM constellations corresponding to RECBEQ (O28rect), CMA, and a variant of CMA (O28) [14].

VI. Conclusions

In this article, a new blind equalization algorithm (RECBEQ) has been presented which is particularly well-suited for blindly equalizing rectangular constellations. The algorithm works with static or rotating constellations. In the latter case, a data-directed PLL must be incorporated with the equalizer contained inside the PLL. Note that in general the extra delay associated with the RECBEQ tapped delay line will limit the capture range of the PLL [15], but this can be compensated by carrier sweep acquisition. We have also presented an asymptotic performance analysis for the new equalizer. These results provide some guidance as to the expected performance of the new algorithm for a given symbol constellation and in fact were supported by simulation results presented for a 16-QAM constellation. Finally, we note that the time domain equalization algorithm presented here may be readily extended to blind adaptive arrays [16].

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