New Empirically Derived Solar Radiation Pressure Model for Global Positioning System Satellites During Eclipse Seasons

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We describe variants of the new empirical solar radiation pressure models for Global Positioning System (GPS) satellites, introduced by Bar-Sever and Kuang [1]. The new variants differ from the previous models only during eclipse seasons (low β angle). We assess the performance of the models during eclipse seasons.

I. Introduction

In [1], Bar-Sever and Kuang introduced a set of solar pressure models for Global Positioning System (GPS) satellites based on in-orbit tracking data, and analyzed the performance of the models for satellites outside eclipse seasons. For the purpose of this analysis, eclipse periods are defined, rather crudely, as spanning β angles between -14.5 deg and $+14.5$ deg, where β is the angle between the Earth–Sun line and the satellite orbital plane.

Eclipsing satellites are much more difficult to model because of their complex attitude and the intermittent exposure to solar flux. Block IIA satellites, in particular, exhibit a highly non-linear attitude behavior during eclipse seasons.

This article describes new variants of the models described by Bar-Sever and Kuang that extend the solar pressure models of [1] into eclipse season and analyzes their performances during eclipse seasons. The variants will have the same designations as the original models, but with the addition of the suffix "e."

II. Model Parameterization and Combination of Solutions

The solar radiation pressure (components in m/s^2) is expanded as a Fourier series in ε, the Earth–spacecraft–Sun angle:

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[1] Tracking Systems and Applications Section.

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$$F_x = s\,10^{-5}(\mathrm{AU}/r)^2/m(S\_X_1 \sin\varepsilon + S\_X_2 \sin 2\varepsilon + S\_X_3 \sin 3\varepsilon + S\_X_5 \sin 5\varepsilon + S\_X_7 \sin 7\varepsilon) \quad (1)$$

$$F_y = C\_Y_0 + 10^{-5}(\mathrm{AU}/r)^2/m(C\_Y_1 \cos\varepsilon + C\_Y_2 \cos 2\varepsilon) \quad (2)$$

$$F_z = s\,10^{-5}(\mathrm{AU}/r)^2/m(C\_Z_1 \cos\varepsilon + C\_Z_3 \cos 3\varepsilon + C\_Z_5 \cos 5\varepsilon) \quad (3)$$

where s is a dimensionless scaling factor, nominally unity, AU is the astronomical unit (1.4959787066×10^8 km), r is the distance between the spacecraft and the Sun in kilometers, and m is the spacecraft mass in kilograms. The dimension of the Fourier expansion inside the parentheses is, therefore, 10^{-5} newtons ($1\ \mathrm{N} = 1\ \mathrm{kg} \times m/s^2$).

The parameters C_Y_1 and S_X_2 were found to depend on the β angle as follows:

$$F(\beta) = A + B \sin\beta + C/\sin\beta + D\cos\beta \quad (4)$$

In all our models the solar pressure is set to zero during umbra, including the Y-bias, and during penumbra crossing the full solar pressure model is scaled by the fraction of the Sun disk that is "seen" by the satellite.

A. The GSPM.04ae Model

Outside eclipse season, this model is identical to GSPM.04a. For eclipsing satellites, we simply extend that model into the -14.5 deg $< \beta < +14.5$ deg regime. However, to avoid the singularity of C_Y_1 near $\beta = 0$, we remove the singular term of $1/\sin\beta$ when $|\beta|$ is less than 1 deg. Table 1 describes the parameters of this model.

B. The GSPM.04be Model

For Block IIA satellites, which exhibit a highly non-linear attitude behavior, we continue to use the GSPM.II.97 "split" model for eclipsing satellites. Block IIR satellites have much simpler attitude behavior, and for them we simply extend the above model into the -14.5 deg $< \beta < +14.5$ deg regime

Table 1. Values of combined solar radiation pressure parameters for the GSPM.04ae model, in 10^{-5} newtons.

Parameters	Block IIA[a]	Block IIR[a]
S_X_1	-8.982	10.931
S_X_2	-0.0219	0.1279
S_X_3	0.0151	0.2767
S_X_5	0.1040	-0.2045
S_X_7	0.0038	0.0568
C_Z_1	-8.6044	-11.6408
C_Z_3	0.0158	0.0627
C_Z_5	0.0553	0.0674
C_Y_2	0.01729	-0.0067
C_Y_1[b]	$0.0091 + 0.0539 \sin\beta + 0.0265/\sin\beta$	$0.0010 - 0.0199 \sin\beta - 0.0107/\sin\beta$

[a] Note that there are different body-fixed coordinates for Block IIA and Block IIR.

[b] For $|b| < 0.0174533$ rad (1 deg), eliminate the $1/\sin\beta$ term.

by replacing β with a value of $+14.5$ deg if β is positive, and with a value of -14.5 deg if β is negative. Table 2 describes the parameters of this model for non-eclipsing satellites.

III. Performance Assessment for Eclipsing Satellites

Figure 1 illustrates the performances of these models for fitting 10-day arcs. The complicated "ae" model is far superior to the "be" model for Block II/IIA satellites. Eclipsing Block IIR satellites are overall much better modeled than Block II/IIA satellites, and the differences in performance between the "ae" and "be" models are small.

Table 2. Values of combined solar radiation pressure parameters for the GSPM.04be model for non-eclipsing satellites, in 10^{-5} newtons.

Parameters	Block IIA[a]	Block IIR[a]
S_X_1	-8.982	10.9310
S_X_2	$-0.0509 + 0.0002 \sin\beta$ $+0.0002/\sin\beta + 0.0407 \cos\beta$	$0.0172 + 0.0022 \sin\beta$ $-0.0016/\sin\beta + 0.1477 \cos\beta$
S_X_3	0.0045	0.2476
S_X_5	0.1060	-0.2283
S_X_7	0.0028	-0.0140
C_Z_1	-8.6044	-11.6411
C_Z_3	0.0225	0.0583
C_Z_5	0.0543	0.0571
C_Y_2	0.0175	-0.0064
C_Y_1	$0.0271 + 0.0459 \sin\beta$ $+0.0302/\sin\beta - 0.0252 \cos\beta$	$-0.0195 - 0.0172 \sin\beta$ $-0.0119/\sin\beta + 0.0272 \cos\beta$

[a] Note that Eqs. (1) through (3) are defined with different body-fixed coordinates for Block IIA and Block IIR.

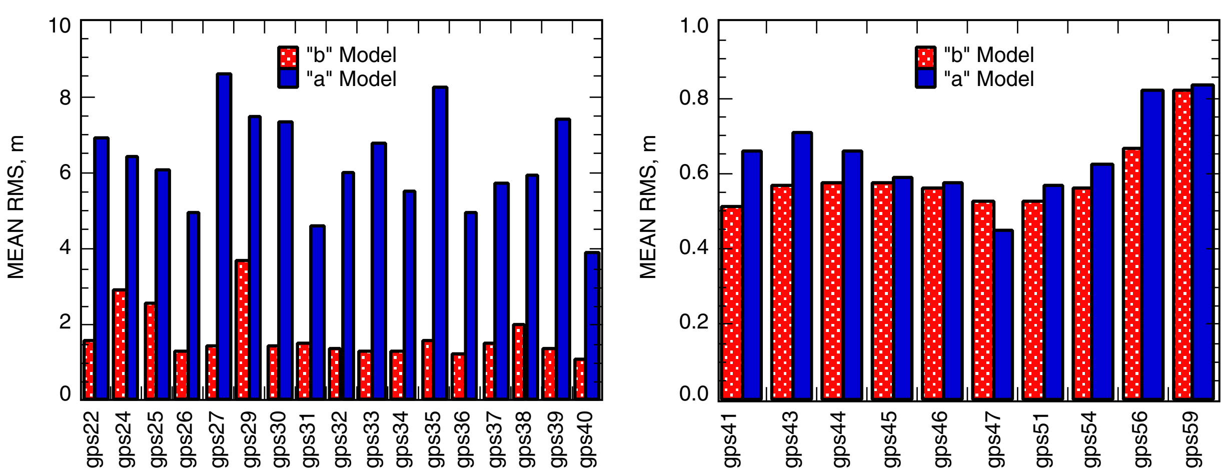

Fig. 1. Average data fit rms to 10-day-arc truth orbits for eclipsing (a) Block IIA satellites and (b) Block IIR satellites. The rms fits for Block IIA satellites are 1.83 m for the "a" model and 6.78 m for the "b" model. The rms fit for Block IIR satellites is 0.59 m for both the "a" and "b" models.

IV. Conclusions

Solar pressure models for eclipsing satellites are necessarily more complex than for non-eclipsing satellites. The new models presented here perform nearly as well as the non-eclipsing models.

Acknowledgment

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Reference

[1] Y. E. Bar-Sever and D. Kuang, "New Empirically Derived Solar Radiation Pressure Model for Global Positioning System Satellites," *The Interplanetary Network Progress Report*, vol. 42-159, Jet Propulsion Laboratory, Pasadena, California, pp. 1–11, November 15, 2004.
http://ipnpr/progress_report/42-159/159I.pdf