A Tracking Performance Comparison of the Conventional Data Transition Tracking Loop (DTTL) with the Linear Data Transition Tracking Loop (LDTTL)

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The tracking (mean-square timing-error) performance of a linearized data-transition tracking loop (LDTTL) is presented. The difference between the LDTTL and the conventional DTTL is that the hard-limiter nonlinearity in the conventional DTTL's inphase arm is replaced by a linear device, as suggested by the maximum a posteriori (MAP) open-loop estimate of symbol timing at low symbol signal-to-noise ratio (SNR). The application of such a configuration is for error-correction-coded communication systems where current coding technology suggests symbol SNR values perhaps as low as −6 dB. The mean-square tracking jitter performances of the LDTTL and the conventional DTTL are compared over a wide range of symbol SNRs, with the former offering a noticeable advantage at low symbol SNRs.

I. Introduction

The data-transition tracking loop (DTTL) [1,2] was introduced in the late 1960s as an efficient symbol synchronization means for tracking a nonreturn-to-zero (NRZ) data signal received in additive white Gaussian noise (AWGN). The scheme as originally proposed is an inphase–quadrature (I-Q) structure where the I arm produces a signal representing the polarity of a data transition (i.e., a comparison of hard (± 1) decisions on two successive symbols) and the Q-arm output is a signal whose absolute value is proportional to the timing error between the received signal epoch and the receiver's estimate of it. The result of the product of the I and Q signals is an error signal that is proportional to this timing error, independent of the direction of the transition.

Years later, it was demonstrated that the closed-loop structure of the DTTL could be motivated by the maximum a posteriori (MAP) open-loop estimate of symbol timing based on an observation of, say, K symbols at high symbol signal-to-noise ratio (SNR), which at the time was in fact the SNR region of

[1] Communications Architectures and Research Section.

The research described in this publication was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

interest. As the design of communication systems became more and more power efficient through the use of error-correction coding, a greater and greater demand was placed on the symbol synchronizer, which now had to operate in a low symbol SNR region, with values based on today's coding technology perhaps as low as −6 dB. Since in this very low symbol SNR region the DTTL scheme as originally proposed would no longer be the one motivated by MAP estimation theory, it is also likely that its tracking capability would be seriously degraded in this region of operation. Furthermore, one would anticipate that a modification of the DTTL structure in accordance with the low SNR MAP timing estimate wherein the I-arm hard decisions are replaced by soft decisions (the hard limiter is replaced by a linear device) should outperform the conventional DTTL.

This article examines such a modified DTTL, herein called the linear DTTL (LDTTL) and compares its tracking performance as characterized by the mean-square timing error to that of the conventional scheme, which has been well established in the past and is documented in [1,2]. Assuming an NRZ data input, it is shown that, depending on the Q-arm window width, the LDTTL can achieve a gain of as much as 4 dB in loop SNR at a symbol SNR of −6 dB relative to that of the conventional DTTL with a hard-decision I-arm transition detector.

II. Performance Analysis of the LDTTL

Consider the linear DTTL illustrated in Fig. 1.[2] The input signal is assumed to be an NRZ signal, i.e., a random pulse train with binary polarities, which is mathematically characterized as

$$s(t,\varepsilon)=\sqrt{S}\sum_{n=-\infty}^{\infty}d_n p\left(t-nT-\varepsilon\right) \tag{1}$$

where S is the signal power, $p(t)$ is a unit amplitude rectangular pulse of duration T seconds, $\{d_n\}$ is an independent and identically distributed (i.i.d.) ±1 sequence with d_n representing the polarity of the nth data symbol, and ε is the unknown symbol timing epoch, which is assumed to be uniformly

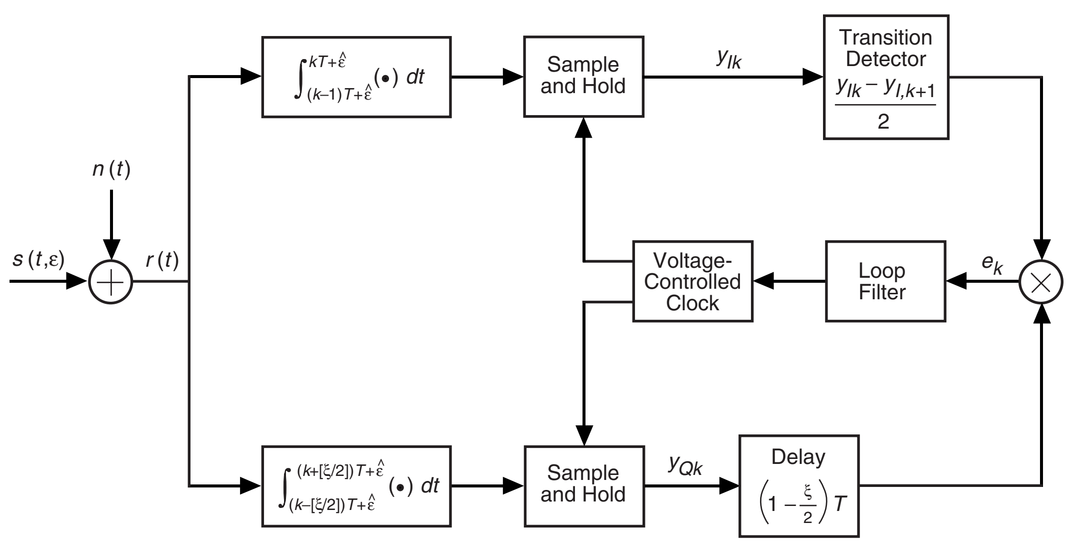

Fig. 1. The linear data-transition tracking loop.

[2] The conventional (nonlinear) DTTL has the identical configuration except that a hard limiter is included in the I arm prior to the transition detector.

distributed in the interval $-T/2 \le \varepsilon \le T/2$. The additive noise $n(t)$ is a white Gaussian process with single-sided power spectral density (p.s.d.) N_0 watts/hertz. The local clock produces a timing reference for the I and Q integrate-and-dump (I&D) filters that depends on its estimate $\hat{\varepsilon}$ of ε. Letting $0 \le \xi \le 1$ denote the "window width" of the Q-arm I&D, then the outputs of these filters corresponding to the kth symbol interval respectively are given by

$$
\begin{aligned}
y_{Ik} &= K_1 \int_{(k-1)T+\hat{\varepsilon}}^{kT+\hat{\varepsilon}} r(t)dt = \overbrace{K_1 \int_{(k-1)T+\hat{\varepsilon}}^{kT+\hat{\varepsilon}} s(t,\varepsilon)dt}^{c_k} + \overbrace{K_1 \int_{(k-1)T+\hat{\varepsilon}}^{kT+\hat{\varepsilon}} n(t)dt}^{\nu_k} \\
y_{Qk} &= K_2 \int_{(k-[\xi/2])T+\hat{\varepsilon}}^{(k+[\xi/2])T+\hat{\varepsilon}} r(t)dt = \overbrace{K_2 \int_{(k-[\xi/2])T+\hat{\varepsilon}}^{(k+[\xi/2])T+\hat{\varepsilon}} s(t,\varepsilon)dt}^{b_k} + \overbrace{K_2 \int_{(k-[\xi/2])T+\hat{\varepsilon}}^{(k+[\xi/2])T+\hat{\varepsilon}} n(t)dt}^{\mu_k}
\end{aligned}
\tag{2}
$$

Since μ_k and ν_k are not independent, it is convenient, as was done in [1,2], to express them in terms of a new set of variables:

$$
\begin{aligned}
\nu_k &= N_k + M_k \\
\mu_k &= N'_{k+1} + M'_k
\end{aligned}
\tag{3}
$$

where

$$
\begin{aligned}
N_k &= K_1 \int_{(k-1)T+\hat{\varepsilon}}^{(k-[1/2])T+\hat{\varepsilon}} n(t)dt \\
M_k &= K_1 \int_{(k-[1/2])T+\hat{\varepsilon}}^{kT+\hat{\varepsilon}} n(t)dt \\
N'_k &= K_2 \int_{(k-1)T+\hat{\varepsilon}}^{(k-1+[\xi/2])T+\hat{\varepsilon}} n(t)dt \\
M'_k &= K_2 \int_{(k-[\xi/2])T+\hat{\varepsilon}}^{kT+\hat{\varepsilon}} n(t)dt
\end{aligned}
\tag{4}
$$

with the following properties:

N_k, M_n are mutually independent for all k, n

N'_k, M'_n are mutually independent for all k, n

N'_k, M_n and M'_k, N_n are mutually independent for all k, n

N'_k, N'_n and M_k, M_n are mutually independent for all $k \ne n$

Furthermore, all M_k, M'_k, N_k, N'_k and their sums are Gaussian random variables with zero mean and variances

$$\sigma^2_{M_k} = \sigma^2_{N_k} = K_1^2 N_0 T/4$$
$$\sigma^2_{M'_k} = \sigma^2_{N'_k} = K_2^2 \xi N_0 T/4 \tag{5}$$

Taking the difference of two successive soft decisions y_{Ik} and $y_{I,k+1}$ and multiplying the average of the result by the quadrature I&D output y_{Qk} (delayed by $(1-\xi/2)T$) gives the loop-error signal,

$$e(t) = e_k = \left(b_k + M'_k + N'_{k+1}\right)\left[\frac{(c_k + M_k + N_k) - (c_{k+1} + M_{k+1} + N_{k+1})}{2}\right],$$
$$(k+1)\,T + \hat{\varepsilon} \le t \le (k+2)\,T + \hat{\varepsilon} \tag{6}$$

which is a piecewise constant (over intervals of T seconds) random process.

A. S-Curve Performance

The S-curve is by definition the statistical average of the error signal of Eq. (6) over the signal and noise probability distributions, i.e.,

$$g(\lambda) \triangleq E_{n,s}\left\{\left(b_k + M'_k + N'_{k+1}\right)\left[\frac{(c_k + M_k + N_k) - (c_{k+1} + M_{k+1} + N_{k+1})}{2}\right]\right\} \tag{7}$$

Letting $\lambda \triangleq (\varepsilon - \hat{\varepsilon})/T$ denote the normalized timing error $(-1/2 \le \lambda \le 1/2)$, then for the assumed NRZ signal of Eq. (1) the signal components of the I&D outputs become

$$b_k = \begin{cases} K_2\sqrt{S}T\left[d_{k-1}\left(\dfrac{\xi}{2}+\lambda\right) + d_k\left(\dfrac{\xi}{2}-\lambda\right)\right], & 0 \le \lambda \le \dfrac{\xi}{2} \\ K_2\sqrt{S}T d_{k-1}\xi, & \dfrac{\xi}{2} \le \lambda \le \dfrac{1}{2} \end{cases}$$
$$c_k = K_1\sqrt{S}T\left[d_{k-2}\lambda + d_{k-1}(1-\lambda)\right], \quad 0 \le \lambda \le \frac{1}{2} \tag{8}$$

Substituting Eq. (8) into Eq. (7) and performing the necessary averaging over the noise and the data symbols d_{k-2}, d_{k-1} and d_k give the desired result, namely,

$$g_n(\lambda) \triangleq \frac{g(\lambda)}{K_1K_2ST^2} = \begin{cases} \lambda\left(1-\dfrac{\xi}{4}\right) - \dfrac{3}{2}\lambda^2, & 0 \le \lambda \le \dfrac{\xi}{2} \\ \dfrac{\xi}{2}(1-2\lambda), & \dfrac{\xi}{2} \le \lambda \le \dfrac{1}{2} \end{cases} \tag{9}$$

By comparison, the corresponding result to Eq. (9) for the DTTL is [1,2]

$$g_n(\lambda) \triangleq \frac{g(\lambda)}{K_2\sqrt{S}T} = \begin{cases} \lambda \mathrm{erf}\left(\sqrt{R_s}(1-2\lambda)\right) - \frac{1}{8}(\xi - 2\lambda)\left[\mathrm{erf}\left(\sqrt{R_s}\right) - \mathrm{erf}\left(\sqrt{R_s}(1-2\lambda)\right)\right], & 0 \le \lambda \le \frac{\xi}{2} \\ \frac{\xi}{2}\mathrm{erf}\left(\sqrt{R_s}(1-2\lambda)\right), & \frac{\xi}{2} \le \lambda \le \frac{1}{2} \end{cases} \tag{10}$$

where $R_s \triangleq ST/N_0$ denotes the symbol SNR. Without belaboring the analysis, it is also straightforward to show that for $-(1/2) \le \lambda \le 0$, $g(\lambda) = -g(-\lambda)$, i.e., the S-curve is an odd function of the normalized timing error.

Note from Eq. (9) that the normalized S-curve for the LDTTL is independent of SNR whereas that for the conventional DTTL, Eq. (10), is highly dependent on SNR. Figure 2 is an illustration of Eq. (9) for various values of window width ξ.

The slope of the normalized S-curve at the origin ($\lambda = 0$) will be of interest in computing the mean-square timing-jitter performance. Taking the derivative of Eq. (9) with respect to λ and evaluating the result at $\lambda = 0$ gives for the LDTTL

$$K_g \triangleq \frac{dg(\lambda)}{d\lambda}\Big|_{\lambda=0} = K_1 K_2 S T^2 \left(1 - \frac{\xi}{4}\right) \tag{11}$$

whereas the corresponding result for the DTTL, based on the derivative of Eq. (10), is

$$K_g \triangleq \frac{dg(\lambda)}{d\lambda}\Big|_{\lambda=0} = K_2\sqrt{S}T\left[\mathrm{erf}\left(\sqrt{R_s}\right) - \frac{\xi}{2}\sqrt{\frac{R_s}{\pi}}\exp(-R_s)\right] \tag{12}$$

which clearly degrades with decreasing R_s.

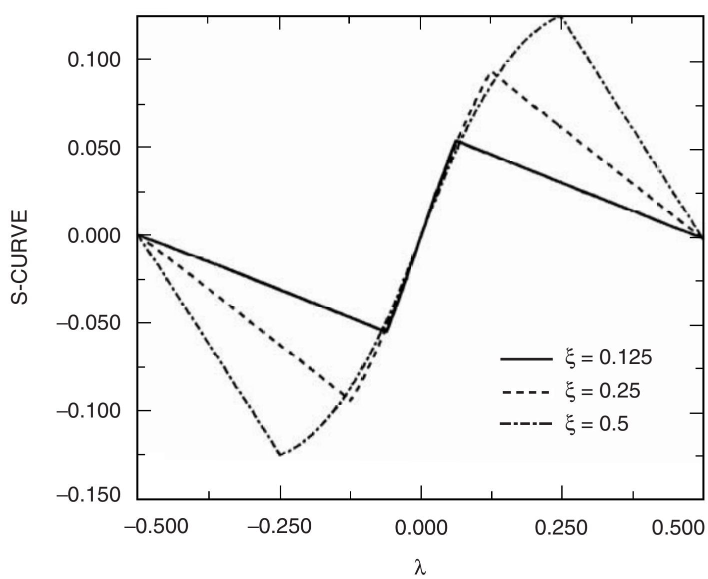

Fig. 2. Normalized S-curves for linear DTTL.

B. Noise Performance

The stochastic differential equation that characterizes the operation of the DTTL or the LDTTL is [1,2]

$$\dot{\lambda} = -KF(p)\big[g(\lambda) + n_\lambda(t)\big] \tag{13}$$

where K is the total loop gain, $F(p)$ is the transfer function of the loop filter with p denoting the Heaviside operator, and $n_\lambda(t)$ is the equivalent additive noise which characterizes the variation of the loop error signal around its mean (the S-curve). Because of the I&D and sample-and-hold operations in the I and Q arms of the loops, $n_\lambda(t)$ is a piecewise (over intervals of T seconds) constant random process. In particular,

$$n_\lambda(t) = e_k - E_{n,s}\{e_k\} = e_k - g(\lambda), \quad (k+1)T + \hat{\varepsilon} \le t \le (k+2)T + \hat{\varepsilon} \tag{14}$$

with a covariance function that is piecewise linear between the sample values:

$$\begin{aligned} R_n(\tau)\,|_{\tau=mT} &= E\left\{n_\lambda(t)n_\lambda(t+\tau)\right\}|_{\tau=mT} \\ &= E_{n,s}\left\{\left(e_k - E_{n,s}\left\{e_k\right\}\right)\left(e_{k+m} - E_{n,s}\left\{e_{k+m}\right\}\right)\right\} \\ &= E_{n,s}\left\{e_k e_{k+m}\right\} - g^2\left(\lambda\right) \triangleq R\left(m,\lambda\right), \quad m = 0, \pm 1, \pm 2, \cdots \end{aligned} \tag{15}$$

As is customary in the analysis of loops of this type, for loop bandwidths that are small compared to the reciprocal of the symbol time interval, $n_\lambda(t)$ can be approximated by a delta-correlated process with equivalent flat (with respect to frequency) p.s.d.

$$N_0' \triangleq 2\int_{-\infty}^{\infty} R_n(\tau)d\tau = 2T\left[R(0,\lambda) + 2\sum_{m=1}^{\infty} R(m,\lambda)\right] \tag{16}$$

Furthermore, for large loop SNR,[3] it is customary to consider only the value of the equivalent power spectral density at $\lambda = 0$, namely,

$$N_0' = 2T\left[R(0,0) + 2\sum_{m=1}^{\infty} R(m,0)\right] = 2T\left[E_{n,s}\left\{e_k^2\,|_{\lambda=0}\right\} + 2\sum_{m=1}^{\infty} E_{n,s}\left\{e_k e_{k+m}\,|_{\lambda=0}\right\}\right] \tag{17}$$

With a good deal of effort, the following results can be obtained from Eq. (6):

[3] Note that this assumption does not require that the symbol SNR be large. Large loop SNR simply implies that the loop operates in the so-called linear region, i.e., where the mean-squared value of the timing error is small and the probability density function (p.d.f.) of the timing error is Gaussian distributed.

$$E_{n,s}\left\{e_k^2\left|_{\lambda=0}\right.\right\} = \frac{1}{4}\left[E_s\left\{b_k^2\left(c_{k+1}-c_k\right)^2\left|_{\lambda=0}\right.\right\} + E_s\left\{b_k^2\left|_{\lambda=0}\right.\right\}E_n\left\{\left(N_{k+1}+M_{k+1}\right)^2+\left(N_k+M_k\right)^2\right\}\right.$$

$$+E_s\left\{\left(c_{k+1}-c_k\right)^2\left|_{\lambda=0}\right.\right\}E_n\left\{\left(N'_{k+1}+M'_k\right)^2\right\}$$

$$\left.+E_n\left\{\left(N'_{k+1}+M'_k\right)^2\left(N_k+M_k-N_{k+1}-M_{k+1}\right)^2\right\}\right] \tag{18a}$$

$$E_{n,s}\left\{e_k e_{k+1}\left|_{\lambda=0}\right.\right\} =$$

$$\frac{1}{4}\left[E_s\left\{b_k b_{k+1}\left(c_{k+1}-c_k\right)\left(c_{k+2}-c_{k+1}\right)\left|_{\lambda=0}\right.\right\} - E_s\left\{b_k b_{k+1}\left|_{\lambda=0}\right.\right\}E_n\left\{\left(N_{k+1}+M_{k+1}\right)^2\right\}\right] \tag{18b}$$

$$E_{n,s}\left\{e_k e_{k+m}\left|_{\lambda=0}\right.\right\} = 0, \quad m \neq 0,1 \tag{18c}$$

Averaging Eqs. (18a) through (18c) over the noise and signal (data sequence) and then using Eq. (5), we obtain after considerable evaluation the desired results, namely,

$$R(0,0) \triangleq E_{n,s}\left\{e_k^2\left|_{\lambda=0}\right.\right\} = \left(K_1K_2ST^2\right)^2\left[\frac{\xi}{4R_s}\left(1+\frac{\xi}{2}+\frac{1}{2R_s}\right)\right] \tag{19a}$$

$$R(1,0) \triangleq E_{n,s}\left\{e_k e_{k+1}\left|_{\lambda=0}\right.\right\} = -\left(K_1K_2ST^2\right)^2\frac{\xi^2}{32R_s} \tag{19b}$$

$$R(m,0) \triangleq E_{n,s}\left\{e_k e_{k+m}\left|_{\lambda=0}\right.\right\} = 0, \quad m \neq 0,1 \tag{19c}$$

Combining Eqs. (19a) through (19c), the equivalent power spectral density is then

$$N'_0 = T\left(K_1K_2ST^2\right)^2\left[\frac{\xi}{2R_s}\left(1+\frac{\xi}{4}+\frac{1}{2R_s}\right)\right] \tag{20}$$

The equivalent quantity for the conventional DTTL can be obtained from the results in [1,2] to be

$$N'_0 = T\left(K_2\sqrt{S}T\right)^2\left[\frac{\xi}{2R_s}\left[1+\frac{\xi R_s}{2}-\frac{\xi}{2}\left[\frac{1}{\sqrt{\pi}}\exp\left(-R_s\right)+\sqrt{R_s}\operatorname{erf}\sqrt{R_s}\right]^2\right]\right] \tag{21}$$

III. Mean-Square Timing-Error Performance

The mean-square timing jitter σ_λ^2 of either the LDTTL or the DTTL is readily computed for a first-order loop filter ($F(p) = 1$) and large loop SNR conditions. In particular, linearizing the S-curve to $g(\lambda) = K_g\lambda$ and defining the single-sided loop bandwidth by B_L, we obtain

$$\sigma_\lambda^2 = \frac{N'_0 B_L}{K_g^2} \tag{22}$$

where K_g is obtained from either Eq. (11) or Eq. (12) and N_0' from either Eq. (20) or Eq. (21). Making the appropriate substitutions in Eq. (22) gives the results

$$\sigma_\lambda^2 = \frac{\xi\left[1+\dfrac{\xi}{4}+\dfrac{1}{2R_s}\right]}{2\rho\left(1-\dfrac{\xi}{4}\right)^2} \quad \text{(LDTTL)}$$

$$\sigma_\lambda^2 = \frac{\xi\left[1+\dfrac{\xi R_s}{2}-\dfrac{\xi}{2}\left[\dfrac{1}{\sqrt{\pi}}\exp(-R_s)+\sqrt{R_s}\,\mathrm{erf}\sqrt{R_s}\right]^2\right]}{2\rho\left[\mathrm{erf}\left(\sqrt{R_s}\right)-\dfrac{\xi}{2}\sqrt{\dfrac{R_s}{\pi}}\exp(-R_s)\right]^2} \quad \text{(DTTL)} \tag{23}$$

where $\rho \triangleq S/N_0B_L$ is the so-called phase-locked loop SNR. Figure 3 is a plot of the ratio of $\sigma_\lambda^2|_{\mathrm{LDTTL}}$ to $\sigma_\lambda^2|_{\mathrm{DTTL}}$ in decibels as a function of R_s in decibels with quadrature-arm normalized window width ξ as a parameter. The numerical results clearly illustrate the performance advantage of the LDTTL at low symbol SNRs. In fact, in the limit of sufficiently small SNR, the ratio of the variances approaches the limit

$$\lim_{R_s\to 0}\frac{\sigma_\lambda^2|_{\mathrm{LDTTL}}}{\sigma_\lambda^2|_{\mathrm{DTTL}}} = \frac{\left(2-\dfrac{\xi}{2}\right)^2}{2\pi\left(1-\dfrac{\xi}{4}\right)^2\left(1-\dfrac{\xi}{2\pi}\right)} \tag{24}$$

which for $\xi \to 0$ (the theoretical value suggested by the MAP estimation of symbol synchronization approach) becomes

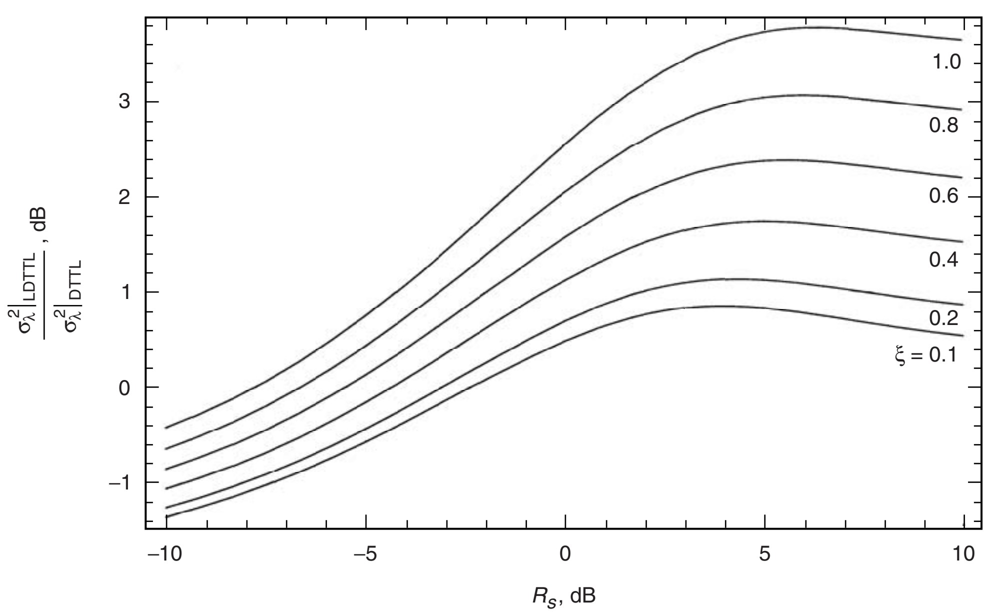

Fig. 3. The ratio of variances of the linear DTTL to the conventional DTTL versus symbol SNR with window width as a parameter.

$$\lim_{\substack{R_s \to 0 \\ \xi \to 0}} \frac{\sigma_\lambda^2 |_{\text{LDTTL}}}{\sigma_\lambda^2 |_{\text{DTTL}}} = \frac{2}{\pi} \tag{25}$$

The fact that this ratio approaches a finite limit is not surprising in view of a similar behavior for other synchronization loops motived by the MAP estimation approach. For example, when comparing the conventional Costas loop (motivated by the low SNR approximation to the MAP estimation of carrier phase) to the polarity-type Costas loop (motivated by the high SNR approximation to the MAP estimation of carrier phase), the ratio of variances of the phase error is given by [3]

$$\frac{\sigma_\phi^2 |_{\text{Conventional}}}{\sigma_\phi^2 |_{\text{Polarity-Type}}} = \frac{\text{erf}^2\left(\sqrt{R_s}\,\right)}{2R_s/\left(1+2R_s\right)} \tag{26}$$

which for sufficiently small SNR becomes

$$\lim_{R_s \to 0} \frac{\sigma_\phi^2 |_{\text{Conventional}}}{\sigma_\phi^2 |_{\text{Polarity-Type}}} = \frac{2}{\pi} \tag{27}$$

For large symbol SNR, the ratio of the variances in Eq. (23) approaches

$$\lim_{R_s \to \infty} \frac{\sigma_\lambda^2 |_{\text{LDTTL}}}{\sigma_\lambda^2 |_{\text{DTTL}}} = \frac{1+\dfrac{\xi}{4}}{\left(1-\dfrac{\xi}{4}\right)^2} \tag{28}$$

which for small window widths results in a small penalty for removing the I-arm hard limiter.

IV. Conclusion

The implementation of a closed-loop symbol synchronizer motivated by MAP estimation of timing error for NRZ signals has a structure whose I-arm nonlinearity is dependent on the symbol SNR. For large symbol SNR, the nonlinearity should be hard limited, which results in the conventional implementation of the DTTL. For small symbol SNR, the nonlinearity should be replaced by a linear device, resulting in the linear DTTL (LDTTL). Comparing the mean-square tracking-jitter performances of the two implementations clearly indicates an advantage of the LDTTL over the conventional DTTL at small symbol SNR. At large symbol SNR, the disadvantage of the LDTTL over the DTTL can be reduced by employing a small window width in the quadrature arm.

References

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