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A Proposed Frequency Synthesis Approach to Accurately Measure the Angular Position of a Spacecraft

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This article describes an approach for measuring the angular position of a spacecraft with reference to a nearby calibration source (quasar) with an accuracy of a few tenths of a nanoradian using a very long baseline interferometer of two antennas that measures the interferometer phase with a modest accuracy. It employs (1) radio frequency phase to determine the spacecraft position with high precision and (2) multiple delay measurements using either frequency tones or telemetry signals at different frequency spacings to resolve ambiguity of the location of the fringe (cycle) containing the direction of the spacecraft.

I. Introduction

The relative angular position of a spacecraft with respect to a known radio source (typically a quasar) currently is determined by making phase measurements using very long baseline interferometry (VLBI) (see, for example, Thornton and Border [1]). The phase measurements, in current practice, are made at a pair of frequencies within a band, known as differential one-way ranging (DOR) tone frequencies. The phase measurements are used to estimate delay in time of arrival (group delay) of the signals between the two antennas. The difference of the delay estimates between the spacecraft and calibration source is used to determine the angular position offset. The accuracy of the delay estimates depends on the frequency separation between the tones and the accuracy of the phase measurements. Therefore, accuracy of the phase measurements limits the accuracy of the angular position estimate since tone separation at a band is constrained by spectrum allocation regulatory requirements. Also, if the frequency separation between the tones is too large, then there may be phase ambiguity of one or more cycles, causing ambiguity in the apparent delay. This can be solved by making measurements at several frequency separations.

On the other hand, the angular separation can also be determined by subtracting the interferometric phases of the spacecraft and reference source, rather than their group delays. Very high precision is obtained even at moderate accuracy of the phase measurements but for the fringe (cycle) ambiguity, caused by uncertainty in the position of the interferometer interference fringe containing the spacecraft with respect to the location of the fringe containing the calibration source.

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One way to resolve the “fringe ambiguity” is to make measurements at many interferometric baselines simultaneously, as is possible by using the very long baseline array (VLBA). In this approach, the final resolution is provided by the longest baseline, and shorter baselines help in resolving fringe ambiguity. The same result (of making measurements with different baselines) can be obtained with one pair of antennas if we use Earth rotation to change the projected baseline length. We will have to depend on the rotation of the Earth to substantially change the baseline length, which will require a long duration for making measurements. Also, because a spacecraft is a moving target with time, we will have to depend on some form of modeling to account for the spacecraft motion. Further, for VLBI with a baseline on the order of 10^4 km, if we use Earth rotation to provide a substantially shorter projected baseline, then one of the antennas will have to be looking at a very low elevation angle, which would increase uncertainty in path length due to the troposphere and ionosphere.

Another way, proposed in this article, is to use a single baseline while combining the radio frequency (RF) phase and group delay techniques so as to synthesize phase measurements over a wide range of effective frequencies. In the following, we provide an outline of a practical approach to achieve high angular position accuracy, with only one pair of (effective) antennas and a short observation with relatively moderate accuracy phase measurements (as compared with what we currently need with the delta-DOR technique). We call this “frequency synthesis.” The motivation for developing this approach was the possibility of relaxing the phase stability requirements for the Deep Space Network (DSN) Array [2]. The approach is described in the next section.

II. Frequency Synthesis Approach

We have 1-MHz ranging tones and about 40-MHz DOR tones at 8.4 GHz (X-band). Since we have 40-MHz separation for DOR tones for the 50-MHz frequency allocation at X-band, it is reasonable to assume that we can have about 400-MHz DOR tone separation at 32 GHz (Ka-band) for the 500-MHz frequency allocation. It means that if we make simultaneous measurements at the X- and Ka-bands we can have measurements at frequency separations of 2 MHz, 40 MHz, and 400 MHz, and at the same time use RF phase measurements at 8.4 GHz and 32 GHz. The largest ratio between adjacent frequencies in this set is (8.4 GHz)/(400 MHz) = 21. Therefore, we can cover the full range of 1 MHz to 32 GHz with a value less than 21 for the ratio of frequencies (or bandwidths) between any two steps.

The maximum value of 21 for the ratio of frequencies used for resolving the fringe ambiguity means that the phase measurement at any frequency with a few degrees of accuracy would be adequate. The problem of connecting phases from measurements at different frequencies/bands can be resolved using calibration on a nearby quasar. Changing the requirement to allow reduced accuracy of phase measurements implies we can have relaxed requirements on the instrumental phase stability and lower signal-to-noise ratio (SNR) for the phase measurements.

In light of the lower SNR requirement, it is possible to reduce the integration time and/or to use a weaker source for calibration. This allows use of a source closer to the spacecraft because there are more sources for calibration if we lower the requirement on source strength for the calibrator. This reduces the error contribution due to calibration of atmospheric variations and instrumental phase stability between the phase measurements for the spacecraft and the calibration source.

III. Error Budget Analysis for the Frequency Synthesis Approach

The values of a delta-DOR error budget[2,3] and related parameters for the current practice of using only group delay for making angular position measurements are given in Fig. 1 and Table 1. Here we

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[2] J. Border, personal communication, Jet Propulsion Laboratory, Pasadena, California, May 2004.

[3] Also see J. Border, “2004 Error Budget for DSN Delta Differential One-way Ranging Measurements,” JPL Interoffice Memorandum 335-04-04-D (internal document), Jet Propulsion Laboratory, Pasadena, California, July 9, 2004.

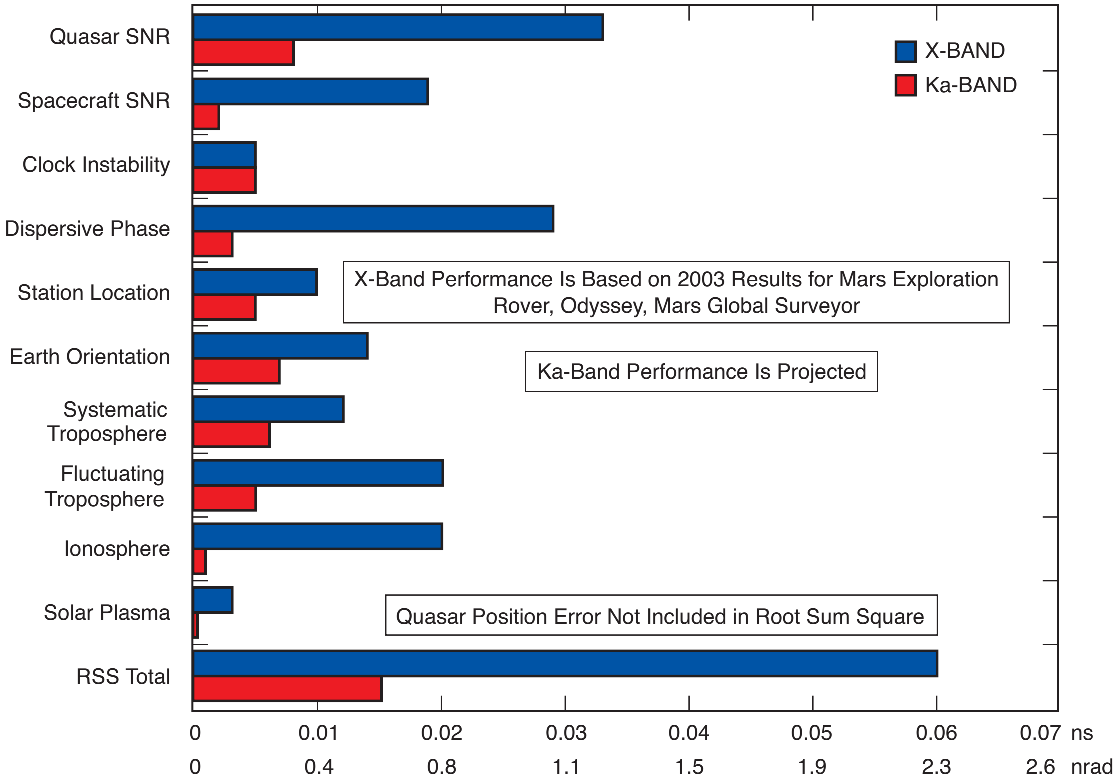

Fig. 1. DSN Delta-DOR error budget.

Table 1. DSN delta-DOR error budget assumptions.[a]

Component	X-band assumptions	Ka-band improvements
Quasar SNR	34 m/34 m; 0.8 Jy; 38-MHz bandwidth; 32 min	(1/4) × flux; 2 × T_{sys}; 10× bandwidth; 10× (no. of bits)
Spacecraft SNR	20 dB-Hz; 16 min; 38-MHz bandwidth	10× bandwidth
Clock instability	1×10^{-14} s/s at 10 min	No change
Dispersive phase	0.2-deg ripple; 38-MHz bandwidth	10× bandwidth
Station location	3-cm baseline; 6-deg angular separation	×2 improvement in baseline knowledge
Earth orientation parameters (EOP)	4-cm real-time knowledge; 6-deg angular separation	×2 improvement in EOP calibration
Systematic troposphere	0.5-cm zenith delay; 20-deg elevation; 5-deg elevation difference	×2 improvement in systematic troposphere calibration
Fluctuating troposphere	20-deg elevation; 6-deg angular separation	×4 improvement in fluctuating troposphere calibration
Ionosphere	20-deg elevation; 6-deg angular separation	×15 improvement (square of Ka-/X-band frequency ratio)
Solar plasma	90-deg Sun–Earth–Probe angle	×15 improvement (square of Ka-/X-band frequency ratio)

[a] From J. Border, personal communication, Jet Propulsion Laboratory, Pasadena, California, May 2004.

calculate the minimum detectable signal, sensitivity, phase measurement accuracy, and expected signal strength required to achieve the desired phase measurement accuracy:

- Minimum detectable signal = ΔT (for a system temperature of $T_{sys} = 50$ K at Ka-band, assuming one antenna at elevation = 20 deg and a second antenna at a high elevation, bandwidth (BW) = 1 MHz, and integration = 1 minute) = 6.5 mK
- Antenna temperature (for a two-antenna interferometer using the equivalent of 34-m antennas with 55 percent efficiency) = 356 mK/Jy
- $\Delta\ \emptyset = 3\sigma$ phase accuracy required to resolve ambiguity (using a 400-MHz spacing delay measurement to 8400-MHz RF fringe phase) = 360 deg$/(21 * 2 * 3) \sim 3$ deg
- SNR required (for $\Delta\ \emptyset$= 3 deg) = 19
- Antenna signal required = ΔT (with BW = 4 MHz, integration = 2 min)$*$SNR = $\left[6.5/(4 * 2)^{1/2}\right] * 19 = 44$ mK
- Calibrator flux density required (for $\text{BW } = 4 \text{ MHz}, \text{integration } = 2 \text{ min}; 3\sigma) = 0.12$ Jy

For the approach outlined in this article, the required integration time and calibration source flux density needed for the phase measurement accuracy are each about an order of magnitude less than what is currently used (Table 1).

For the lower flux density requirement on the calibration source, we can reduce the angular separation between the spacecraft and calibrator. If we look at the error budget from Fig. 1 and Table 1, it is obvious that the measurement errors can be drastically reduced with reduced integration and separation between the spacecraft and calibration source. We probably can reduce the overall errors by at least an order of magnitude or more by using calibration sources of about 0.1 to 0.15 Jy and using an integration time of a couple of minutes instead of the currently used ~1 Jy calibrator and a 15- to 30-minute integration. In fact, the 1- to 2-minute integration on each calibrator and spacecraft can be split into many smaller durations, with cycling between the calibrator and spacecraft many times more frequently, thus reducing any variations caused by instrumental instability and propagation variations, etc. However, this has to be optimized with the time spent on slewing the telescope and setup time for observation on any source.

The highest frequency ratio for delay measurements for bootstrapping to resolve fringe ambiguity in the above analysis is 21. Therefore, if we make the phase measurements with root-mean-square (rms) measurement accuracy of 3 deg, we can resolve fringe ambiguities with 3σ confidence. It means we can easily use RF phase for calculating spacecraft position accurately. Now, if the measurement accuracy for the RF phase at an X-band frequency is, say, 20 deg (essentially dominated by differential variations between the spacecraft and calibrator due to time variations caused by the local oscillator and propagation path), we can still get a position accuracy of about 0.2 nrad with an interferometer with a 10^4 km baseline having about 3.5-nrad fringe spacing.

With broadband telemetry signals, it will be possible to use much larger bandwidths on both the spacecraft and calibrator for delay determination and RF phase measurements. Also, with the DSN Array, we will be able to use a much larger aperture than the 34-m equivalent for this application. These factors will allow use of still shorter integration times and weaker calibration sources. This will further reduce the contributions to the error budget due to atmospheric and instrumental variations and uncertainty in the interferometer baseline and Earth orientation, etc.

IV. Comments on Positions of Reference Sources and Some Other Sources of Errors

Once the DSN Array is built, it will have enough antennas and other assets that at least one, and possibly a few antennas, at each complex—and the needed inter-complex network bandwidth to transport data for cross-correlation to make VLBI measurements—should be available most of the time. This would allow us to do the following:

- Refine the position of the VLBI catalog sources (we will call these the primary calibration source) to the required accuracy (perhaps 0.1 nrad)
- Keep track of the Earth orientation/polar motion more accurately, allowing a smaller error contribution from these factors to the overall measurements
- Locate and measure the accurate positions of compact VLBI calibration sources to the desired accuracy near the track of spacecraft whose accurate positions have to be measured (we will call these sources the secondary calibration sources)

Then the secondary calibration sources can be used for making measurements of spacecraft position accurately using the frequency synthesis approach described here. In fact, accurate position of the secondary calibration sources also can be determined using the frequency synthesis approach and using primary calibration sources for reference.

V. Advantages and Limitations of the Frequency Synthesis Approach

The frequency synthesis approach uses RF phase measurements at 8.4 GHz and 32 GHz for precise position and bootstrapping from delay measurements with spans of bandwidths from 1 to 400 MHz to resolve fringe (cycle) ambiguities and find relative position of a spacecraft with reference to a calibrator (quasar). This approach

- Needs only one baseline (although multiple baselines may help, if available)
- Can use broadband (telemetry) signals
- Or can use an RF carrier-plus-broadband signal, or DOR tones in a delta-DOR observing mode, or a combination of these

The bootstrapping allows phase measurements with moderate accuracy to resolve fringe ambiguities. This allows the phase measurements to be in error by many degrees of phase. This simplifies the requirement on the phase stability of the hardware and reduces the required SNR of the measurements. It means we can use shorter integration times and/or a weak close-by calibration source, thus reducing errors due to variations in atmosphere and instrument drift, etc.

Since we should be able to achieve a few tenths of a nanoradian accuracy with RF phase at X-band, we need accurate ($\sim$0.1 nrad for a few tenths of a radian spacecraft position determination) position of calibration sources only at X-band, and not at Ka-band. This may be easier to achieve because source changes are more rapid at Ka-band than at X-band. However, source structure at sub-nanoradian position accuracy may be an issue.

In situations when there is a large (telemetry) signal bandwidth, there is no need of DOR tones, but DOR tones will be required when there is only limited signal bandwidth. However, in those situations, is it necessary to have very high accuracy position measurements or is it possible to relax the requirement to a few nanoradians? (Also, see the discussion below in connection with the availability of DOR tones with only 40-MHz spacing).

It is required that (1) the basic electronics of the array be reasonably stable (phase stability of $\lesssim$1 deg; at present for delta-DOR work we need almost an order of magnitude better stability) and (2) the drift of the reference clock at the two sites be a small fraction of an RF cycle over the time between the measurements on the spacecraft and calibrator. For the array phasing to work satisfactorily, we will have to make the basic electronics stable enough to work at Ka-band, and therefore it should have adequate stability at X-band over short durations of a few minutes. Also, with reasonable efforts, stability of the reference clock should not be a problem as VLBI routinely does long integrations on weak sources at X-band (it is also required for the Doppler measurements, and, further, it is only one reference clock per station about which we have to worry).

With an inter-continental ($\sim$10,000-km) baseline, a few tenths of a nanoradian differential measurement should be possible with just bootstrapping (removing fringe ambiguities) only to RF measurements at X-band and not requiring RF fringe phase at Ka-band. It means that if we are satisfied with an accuracy of a few tenths of a nanoradian we may not need a very accurate catalog of calibration sources at Ka-band.

A question arises: what will happen to the spacecraft position accuracy if we relax the phase stability requirement for the DSN Array and if we have tone spacing of only 40 MHz for a measurement and do not get 400-MHz tone spacing or wideband telemetry data? Our present committed accuracy for position measurements is 5 nrad (one sigma). With 40-MHz tone spacing, for a 10^4 km baseline, fringe spacing is about 750 nrad, and therefore a 1-deg phase error will provide an uncertainty of about 2 nrad. Therefore, it should be possible to achieve accuracy of 5 nrad with tone spacing of 40 MHz, but we will have to minimize errors from other sources. This means we may have to increase the effective antenna gain to system temperature ratio (G/T) over what we currently use (a 34-m antenna equivalent) for VLBI measurements in order to allow use of weaker calibration sources that may be closer to the spacecraft and also to allow use of shorter integration times. This will reduce error contributions from other sources, such as tropospheric variations between spacecraft and calibrator, baseline and Earth orientation errors, etc.

On the other hand, if we increase the allowed error to about 8 nrad, for instance, when there are only 40-MHz tones available, and insist on high accuracy only when either 400-MHz spacing tones or wide bandwidth telemetry signals are present, then we may be able to relax the instrumental stability to, say, about 2 deg.

Other concerns are the amount of data to be transported for correlating signals from two antennas, to make phase measurements, and the software development effort required for implementing the scheme. These are addressed in the following:

- The amount of data depends on whether tones or broadband telemetry signals are used for phase measurements, but irrespective of how it is done, I do not think the amount of data to be transported is going to be a problem, because what is required is only a few minutes of data (although it may be wide bandwidth if we want to use broadband telemetry signals).
- We will have to put some effort into developing the required software. However, to estimate the software effort, it may be worth planning and running a test on two known quasars located nearby each other in the sky in order to develop a methodology for eliminating fringe ambiguities before trying to develop software.

VI. Conclusion

We have outlined the frequency synthesis scheme to do high accuracy spacecraft position determination using moderate accuracy phase measurements made with two-antenna VLBI. The approach uses both RF phase and group delay measurements and requires broadband telemetry signals from spacecraft with an approximately 400-MHz bandwidth (or something close to it) or DOR tones with approximately 400-MHz spacing in addition to the approximate 40-MHz tones available currently.

Acknowledgment

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References

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[2] D. S. Bagri, J. I. Statman, and M. S. Gatti, "Array-Based Deep Space Network for NASA," IEEE Aerospace Conference, Big Sky, Montana, March 5–12, 2005.