Availability of Calibration Sources for Measuring Spacecraft Angular Position with Sub-Nanoradian Accuracy

W. Majid[1] and D. Bagri[1]

Precision measurements are now capable of determining the angular positions of spacecraft in the sky with accuracies of 2 to 5 nrad using compact radio sources having a flux density of at least a few hundred millijansky at 8.4 GHz for calibration. Further improvements in position measurement accuracy may be possible with the use of appropriate calibrators near the direction of the spacecraft even if the calibrators are much weaker (a few millijansky) in flux density. In this article, we discuss the calibrator flux density required to achieve sub-nanoradian astrometric accuracy and attempt to estimate the density of suitable calibrators, using existing source count surveys. We point out, however, that the fraction of these sources that is suitable for use as a calibrator is not well understood and requires further study.

I. Introduction

Current precision techniques for measurement of spacecraft angular position, based on group delay measurements, achieve an accuracy of a few (2 to 5) nrad.[2] This requires making measurements with 10- to 15-minute integration times on a calibration source (compact quasar) with a flux density of typically at least a few hundred millijansky (mJy). Therefore, a calibrator generally is 5 to 10 deg away from the direction of the spacecraft. The angular and temporal separation between the spacecraft and the reference source causes calibration errors that limit the accuracy of the measurements.

Further improvement in measurement accuracy to the sub-nanoradian level may be possible using combinations of group delay and phase delay or the recently proposed frequency synthesis approach [1]. The frequency synthesis approach requires using span bandwidths of a few hundred megahertz from broadband telemetry signals or using the differential one-way ranging (DOR) tones on the spacecraft. The improvement in accuracy is made possible if the calibration errors caused by angular and temporal separation between the spacecraft and the reference calibrator are kept low. The article on the frequency synthesis approach [1] indeed suggests using calibration sources within a few degrees from the direction of the spacecraft.

---

[1] Tracking Systems and Applications Section.

[2] J. S. Border, "2004 Error Budget for DSN Delta Differential One-way Range Measurements," JPL Interoffice Memorandum 335-04-04-D (internal document), Jet Propulsion Laboratory, Pasadena, California, 2004.

The research described in this publication was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

To use reference sources that are within a few degrees from the direction of the spacecraft requires lowering the minimum flux density detection limit of the measurements. In this article, we determine the required strength (flux density) of the calibration sources. Using data from previous surveys, we then estimate the number density of radio sources over the desired flux density range. Due to the intrinsic morphology of the sources, only a fraction of these sources at any flux density is suitable as reference calibrators with high astrometric value. In the last part of this article, we provide an estimate for the fraction of suitable sources. Finally, we suggest ways to determine this fraction with better confidence.

The organization of this article is as follows: In Section II, we estimate the source flux density needed to achieve sub-nanoradian accuracy. In Section III, we review a number of radio source surveys and estimate $\log N - \log S$ (the number density per unit of flux density) of radio sources at both 8.4 GHz (X-band) and 32 GHz (Ka-band). Section IV discusses source structure and uses two existing surveys to obtain a rough estimate of the fraction of useful sources with astrometric value as very long baseline interferometry (VLBI) calibrators. Section V provides a rough estimate of the number of available calibration sources at both X-band and Ka-band. Finally, we conclude by summarizing our results and suggest further work to improve estimates of the density of suitable calibration sources.

II. Calibration Source Strength Requirement

First we calculate the minimum detectable signal for calibration measurement, using the relation $\Delta T = T_{sys}/(B \cdot \tau)^{1/2}$. We assume a typical system temperature of $T_{sys} = 20$ K at X-band, a signal recording bandwidth of $B = 500$ MHz using a VLBI Mark 5 recording system (the next generation of wideband VLBI science receivers under development also will achieve a similar recording bandwidth), and an integration time of $\tau = 60$ s. With these assumptions, we obtain a minimum detectable signal of $\sim$0.3 mK (3σ). Moreover, the gain sensitivity of the interferometer using a pair of 70-m Deep Space Network (DSN) antennas is $\sim$1.6 K/Jy, which results in a minimum detectable flux density of 0.2 mJy (3σ).

For a VLBI measurement, the root-mean-square (rms) angular position error σ_θ is given by (e.g., see [2]):

$$\sigma_\theta = \frac{1}{(2\pi \times \mathrm{SNR} \times \mathrm{BW} \times \mathrm{LT})} \tag{1}$$

Here SNR is the signal-to-noise ratio for the measurement, BW is the spanned bandwidth, and LT is the light travel time over the interferometric baseline. Therefore, to achieve $\sim$1-nrad position accuracy, we could use sources with flux densities as low as 2 mJy for a 10^4-km baseline VLBI interferometer using a pair of 70-m antennas with $\Delta T = 0.3$ mK and a spanned bandwidth of 500 MHz. Alternatively, with a pair of 34-m antennas, we could use sources with a flux density of $\sim$10 mJy. On the other hand, if the current implementation of the wideband VLBI science receivers (WVSRs) is used for data acquisition and recording (with improved phase response over that of the Mark 4/5 VLBI recording system), we would have to increase the minimum source flux density by roughly a factor of 3.5 due to the somewhat limited recording rate of 80 Mb/s (as already stated, the next generation of WVSR currently under development is designed to sustain a recording rate of 1 Gb/s, similar to that of the Mark 5 system).

It may be possible to increase the calibration time somewhat, but it appears that we still require calibration sources with flux densities of several millijansky or stronger. With the DSN Large Array [3], having a larger equivalent collecting area at each complex, a signal bandwidth of up to 500 MHz, and a comparable span bandwidth of 400 MHz at Ka-band, it will be possible to use much weaker calibration sources, perhaps below 1 mJy. Therefore, to find suitable sources for calibration, we will examine the availability of the sources with flux densities >1 mJy at Ka-band and >3 mJy at X-band.

III. Radio Source Count at X- and Ka-Bands

The density of sources typically is described as $dN = N_0 S^\gamma dS$, where dN is the number of sources per unit solid angle in flux density interval dS at a flux density of S. For flat Euclidean space with uniform radio source distribution, the value of the slope, γ, is -2.5. The integrated source count, therefore, is expected to be $N \propto S^{\gamma+1}$ or in Euclidean space $N \propto S^{-1.5}$. Observations [4–6] carried out over the past three decades, however, indicate pronounced deviation from a single source population distributed randomly in a flat Euclidean space. At various source brightness scales, different populations contribute to the overall source count.

A. Review of Previous Surveys to Determine Extragalactic Radio Source Counts

For calibrator flux densities above a millijansky (sources of interest as discussed in Section II), the measured source count shows a more complex behavior that cannot be fitted with a simple exponential form. It has long been known, for instance, that at the highest flux densities, $\geq$10 Jy, the source count behaves as expected in a Euclidean geometry because of the fact that the bright sources contain many nearby objects. In the range of 1 to 10 Jy, the observed number of sources increases rapidly with a rate that actually exceeds the Euclidean value. However, the overall number of sources in this range is limited to a few hundred sources.

In the 100-mJy to 1-Jy range, the source count seems to reach a plateau relative to the Euclidean geometry, followed by a relative drop. Observations carried out at 2.7 GHz [4] are used to derive a best fit over this region that can be described as

$$N \sim 10^3 \, S_{2.7}^{-1.8} \text{ deg}^{-2} \tag{2}$$

where N is the number of sources above flux density S in millijansky. The expected error in N in each of these surveys appears to be at the level of 10 to 20 percent.

Below this region, over almost three decades of flux density observations, the source counts exhibit a steady drop relative to the Euclidean value that can be characterized by a slope γ of -1.1 [6]. This survey was carried out at 8.4 GHz with the Very Large Array (VLA). Figure 1, taken from Fig. 6 in [6], shows a $\log N - \log S$ plot covering over 6 decades of source flux density observations. The complex profile of the integrated source count is evident from this figure. Interestingly, there is very limited information on source density for sources with flux densities between 2 and 40 mJy.

For comparison, another survey of weaker sources, carried out with the VLA at 1.4 GHz [5] of a region containing the Hubble Deep Field, estimated a source count in the 40- to 1000-μJy range that is described as

$$N \sim 34.7 \, S_{1.4}^{-1.38} \text{ deg}^{-2} \tag{3}$$

The expected error is once more at the level of 10 to 20 percent.

We note that most source count surveys over the past decades were carried out at different frequency bands, and all are below 32 GHz. The source counts at Ka-band have to be extrapolated from the lower-frequency surveys.

B. Implications in the 1- to 1000-mJy Range

Non-thermal radio sources emit radiation that exhibits power law distributions. Their flux density is often specified in terms of a spectral index $S_\nu \propto \nu^\alpha$. Typical values of α for quasars and radio galaxies

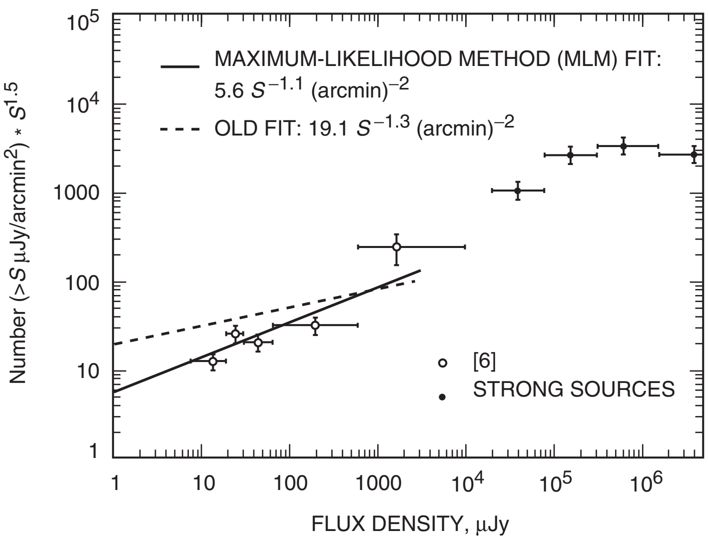

Fig. 1. A log $N -$ log S **plot in the range of a few μJy to a few Jy, taken from Fomalont et al. (Redrawn with permission from [6].)**

are in the range of $-0.8 < \alpha < -0.5$ [6,7]. For the weak sample of sources in the Fomalont et al. 2002 survey [6], the median value for α is reported to be -0.65. Adopting this value for α, we estimate the source count at both X- and Ka-bands using data from surveys carried out at lower frequency bands [8], with the assumption that at the flux densities of interest (10 to 1000 mJy) we are essentially looking at the same population of sources. Figure 2 shows the expected source distribution per squared degrees at X- and Ka-bands.

For convenience, in Table 1 we report the estimated number of sources at both frequency bands for different patches of sky at four different flux sensitivity scales. The values in this table are expected to be accurate at the level of 10 to 20 percent (in accordance with the errors in the parent surveys). However, we note that, since there are a number of radio source populations contributing to the entire source count, our simplistic adoption of one value for the spectral index of the entire population will introduce systematic errors in our estimates in Table 1 and Fig. 2. We estimate that these numbers could be off at most by factors of 1.5 to 2 given the range of spectral indices that might be present in the source population, e.g., see [7].

IV. Radio Source Structure

The available radio source count surveys serve only as an upper limit to the number of available sources for precision astrometry work. The suitability of a radio source as a calibrator depends on its intrinsic structure and variability (variability could cause variation in the centroid's position of the radio source). Sources of high astrometric value are those that have a very compact (∼nanoradian) structure [9].

Using VLBI techniques, maps of radio sources with milliarcsecond (mas)[3] resolution have been obtained over the past two decades. These detailed maps provide evidence for source structure in a variety of forms, from doubles to asymmetric sources with elongated features. Radio sources thus are divided based on their morphology into two groups—extended and compact objects.

[3] In the astrometric community, it is common to use units of nanoradians, while the imaging community prefers to use units of milliarcseconds. In this article, we use both units as appropriate since we refer to literature from both communities. The conversion factor to keep in mind is 1 mas = 5 nrad.

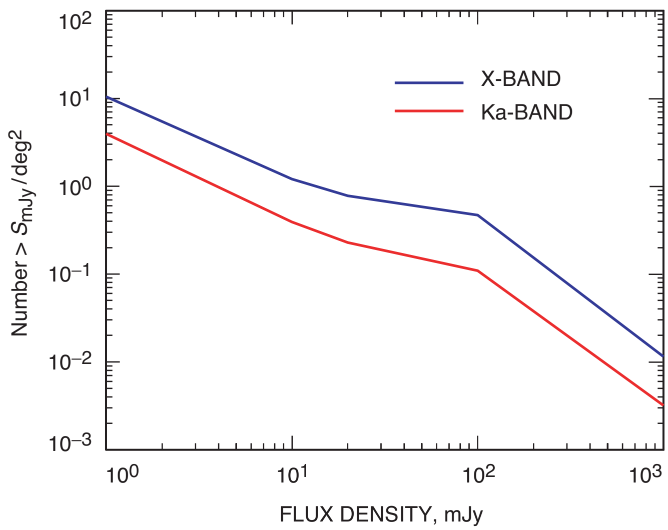

Fig. 2. Estimated source count per square degree at X- and Ka-bands in the range of 1 mJy to 1 Jy. The ordinate scale indicates the integrated number of sources with flux densities greater than *S* (mJy).

Table 1. Expected source count at X- and Ka-bands.

Sky patch	X-band				Ka-band			
	1 mJy	10 mJy	100 mJy	1 Jy	1 mJy	10 mJy	100 mJy	1 Jy
1×1 deg	10.51	1.21	0.47	0.012	3.97	0.39	0.11	0.003
2×2 deg	42.0	4.8	1.9	0.05	15.9	1.6	0.44	0.01
5×5 deg	263	30	12	0.3	100	10	3	0.1
All sky	430,000	50,000	19,000	470	160,000	16,000	4,500	130

Many surveys have been carried out to measure source structure and angular size of radio sources. However, the vast majority of these surveys have a limited angular resolution of a few arcseconds. Only a few surveys provide information on source structure with milliarcsecond resolution. We estimate the fraction of sources suitable for calibration from two recent surveys with the Very Large Baseline Array (VLBA).

A. Comparison with Two Surveys

To determine the fraction of sources with milliarcsecond angular size, we have looked for VLBI data sets. Unfortunately, there are very limited data on the angular extent of VLBI sources in a controlled statistically complete sample at low flux densities. Below, we look at two very different surveys—one covering a large portion of the sky with a flux detection limit around 100 mJy and the second focusing on a narrow part of the sky with a low flux detection limit of 2 mJy.

One such data set is a survey [10] of the National Optical Astronomy Observatory's (NOAO) Boötes field carried out using the National Radio Astronomy Observatory (NRAO) VLBA. The survey covers a 9 deg^2 field and was carried out at 5 GHz to obtain phase-referenced images of the 76 sources identified from the Faint Images of the Radio Sky at Twenty-Centimeters survey (FIRST) [11]. Sources were selected from the FIRST catalog with peak flux densities above 10 mJy at 5-arcsecond resolution (the

FIRST catalog was developed at a frequency of 1.4 GHz). Based on the radio source count surveys (see the previous section), we expect between 32 and 93 sources (for a spectral index range of −0.6 and −1.2, respectively) with a flux density above 10 mJy. The range of counts is based on our poor understanding of the spectral index for various populations of radio sources.

Of the 76 FIRST sources, this survey detected 30 with the VLBA with peak flux densities above $6\sigma \sim 2$ mJy at 2-mas resolution. Sources that were not detected presumably had compact flux densities below the detection limit of this survey. The detected sources are reported to have compactness ratios within the full range of the resolution. Most of the VLBA detections are reported to be unresolved or slightly resolved. Antenna separations in the VLBA spans from 240 to 8600 km. Of the 30 detected sources, 8 appear to have prominent structure (many in the form of two lobes) at the level of a few milliarcseconds and thus do not appear to be suitable as calibrators. The fraction of suitable sources in this sample is then estimated to be about 30 percent at a flux limit of 2 mJy.

The Very Long Baseline Array Calibrator Survey (VCS) [12] also can be used to get a rough estimate of the fraction of the overall radio sources that are suitable as reference calibrators, albeit at a higher flux detection limit of 100 mJy. The VCS1 (first phase of the VCS) has been carried out with the VLBA in dual-frequency 2.2 GHz (S-band) and X-band. Sources were selected from the Jodrell Bank–VLA Astrometric Survey [13] carried out at 5 GHz, with compact emission on scales of 10 to 30 mas and a flux density ≥200 mJy (at 5 GHz). The VCS1 survey has a declination limit of −30 deg. We estimate the sky coverage of the survey to be ∼31,000 deg^2, keeping in mind that the survey's sensitivity drops at lower declinations. Over the years, the VCS has been extended to the southern sky, but for the purpose of this discussion, we will use only the results of the first survey.

Although the selected sample is not statistically a complete set of compact sources, it is a representative sample and should at least serve as a rough estimate of the fraction of available sources at the survey's milliarcsecond resolution limit. The flux limit of the detected sources is around 70 mJy, with most of the sources having a flux of >100 mJy. We will use 100 mJy as the flux limit. Based on the source count model, we expect $15,000 \pm 2000$ sources above 100 mJy over the estimated sky coverage of the VCS1. The VCS1, on the other hand, contains 1332 sources. The fraction of suitable sources at the milliarcsecond level for reference calibrators is then estimated at the level of ∼10 percent.

V. Estimate of the Number of Suitable Sources

Using the rough estimates of the previous section, we expect to find ∼1 reference source with a flux density of ∼10 mJy at X-band within a 2×2 deg area. At Ka-band, the prospect of finding reference sources within the same patch of sky is slimmer. However, within a 5×5 deg patch, we expect to find between 1 and 2 reference sources at the 10-mJy level at Ka-band. It is important to note, however, that these numbers are not well understood statistically, and the error on these numbers is statistically not reliable.

Given the very limited data on the angular extent of VLBI sources, especially for any controlled sample at low flux densities, the estimates at 10 mJy for both X- and Ka-bands are poorly known. Clearly, more data need to be collected in statistically complete samples with flux density limits of 10 mJy to arrive at a meaningful value for the fraction of radio sources suitable for precision astrometry. Only then will it be possible to answer the question of the fraction of suitable sources with any level of meaningful statistical confidence. We believe such deep surveys can be carried out practically using the NRAO VLBA instrument at X-band, 22 GHz (K-band), and 43 GHz (Q-band), focusing on 2 to 3 representative fields covering perhaps 3×3 deg each. Such a survey should be able to better constrain the number of suitable sources at a few millijansky level at X- and Ka-bands for precision astrometry.

VI. Conclusions

Improvement in measurement accuracy of spacecraft angular position is now possible with new techniques and improvement in recording hardware. These improvements are possible, however, only if the calibration errors due to angular and temporal separation of spacecraft and the reference calibrators can be minimized. Calibration errors due to the angular separation between the spacecraft and the calibrators can be kept low by using reference sources that are within 1 to 2 deg of the spacecraft. We have shown that, to achieve this level of accuracy with techniques such as the frequency synthesis approach [1], reference calibrators with flux densities as low as a few millijansky need to be used. We have looked at a number of radio source surveys and estimated a $\log N - \log S$ number count of radio sources at both X- and Ka-bands. We have pointed out that, for VLBI measurements focused on determining the angular position of a spacecraft, the relevant number count is not the overall number of radio sources, but the number of radio sources compact enough at the milliarcsecond level to be of use as reference calibrators. Based on two different compact surveys, we estimate a fraction of 10 to 30 percent of the radio sources may be suitable for calibration purposes at the flux densities of interest. We have pointed out that further study is needed to arrive at an unbiased and statistically meaningful number for the fraction of available sources at the level of 10 mJy. At this flux detection sensitivity, we have suggested that it is possible to carry out measurements of the angular position of spacecraft with sub-nanoradian accuracy.

Acknowledgments

We thank Charles Naudet for helpful discussions and useful comments. We also thank the referee, Dayton Jones, for his careful reading of the manuscript and detailed suggestions that have improved the quality of this article.

References

[1] D. S. Bagri, "A Proposed Frequency Synthesis Approach to Accurately Measure Angular Position of a Spacecraft," *The Interplanetary Network Progress Report*, vol. 42-163, Jet Propulsion Laboratory, Pasadena, California, pp. 1–7, November 15, 2005. http://ipnpr/progress_report/42-163/163I.pdf

[2] A. R. Thompson, J. M. Moran, and G. W. Swenson, *Interferometry and Synthesis in Radio Astronomy*, New York: John Wiley & Sons, Inc., 2001.

[3] M. S. Gatti, "The Deep Space Network Large Array," *The Interplanetary Network Progress Report*, vol. 42-157, Jet Propulsion Laboratory, Pasadena, California, pp. 1–9, May 15, 2004. http://ipnpr/progress_report/42-157/157O.pdf

[4] J. V. Wall and J. A. Peacock, "Bright Extragalactic Radio Sources at 2.7 GHz. III—The All-Sky Catalogue," *Monthly Notices of the Royal Astronomical Society*, vol. 216, pp. 173–192, September 1, 1985.

[5] E. A. Richards, "The Nature of Radio Emission from Distant Galaxies: The 1.4 GHz Observations," *The Astrophysical Journal*, vol. 533, pp. 611–630, 2000.

[6] E. B. Fomalont, K. I. Kellermann, R. B. Partridge, R. A. Windhorst, and E. A. Richards, "The Microjansky Sky at 8.4 GHz," *The Astrophysical Journal*, vol. 123, pp. 2402–2416, May 2002.

[7] R. H. Donnelly, R. B. Partridge, and R. A. Windhorst, "6 Centimeter Radio Source Counts and Spectral Index Studies Down to 0.1 Millijansky," *The Astrophysical Journal*, vol. 321, pp. 94–112, October 1, 1987.

[8] R. Windhorst, D. Mathis, and L. Neuschaefer, "The Evolution of Weak Radio Galaxies at Radio and Optical Wavelengths," in *The Evolution of the Universe of Galaxies; Proceedings of the Edwin Hubble Centennial Symposium*, ed., R. G. Kron, Astronomical Society of the Pacific Series, vol. 10, pp. 389–403, 1990.

[9] P. Charlot, "Radio Source Structure in Astrometric and Geodetic Very Long Baseline Interferometry," *The Astrophysical Journal*, vol. 99, pp. 1309–1326, April 1990.

[10] J. M. Wrobel, G. B. Taylor, T. A. Rector, S. T. Myers, and C. D. Fassnacht, "Faint Radio Sources in the NOAO Bootes Field: VLBA Imaging and Optical Identifications," *The Astrophysical Journal*, vol. 130, pp. 923–935, September 2005.

[11] R. H. Becker, R. L. White, and D. J. Helfand, "The FIRST Survey: Faint Images of the Radio Sky at Twenty Centimeters," *The Astrophysical Journal*, vol. 450, pp. 559–577, 1995.

[12] A. J. Beasley, D. Gordron, A. B. Peck, L. Petrov, D. S. McMillan, E. B. Fomalont, and C. Ma, "The VLBA Calibrator Survey—VCS1," *The Astrophysical Journal Supplement Series*, vol. 141, pp. 13–21, 2002.

[13] A. R. Patnaik, I. W. A. Browne, P. N. Wilkinson, and J. M. Wrobel, "Interferometer Phase Calibration Sources. I—The Region $35° \leq \delta \leq 75°$ deg," *Monthly Notices of the Royal Astronomical Society*, vol. 254, pp. 655–676, February 15, 1992.