In-Beam Phase Referencing with the Deep Space Network Array

W. Majid[1] and D. Bagri[1]

The Deep Space Network Array of the future provides an intriguing possibility of using the techniques of in-beam phase referencing to determine the position of spacecraft with accuracy at the level of 0.1 nrad. In this article, we discuss the prospects for carrying out such measurements at both 8.4 GHz (X-band) and 32 GHz (Ka-band). Our study suggests that at X-band in-beam calibration may be available as an astrometric tool over 20 to 30 percent of the sky. We point out that this estimate depends strongly on the number density of compact sources at the 1-mJy flux level. The prospects at Ka-band, on the other hand, are not very hopeful. We also discuss issues related to phase cycle ambiguity resolution and potential techniques to resolve them.

I. Introduction

In-beam phase referencing is a powerful technique for determining the relative angular position of a spacecraft with respect to a calibrator observed simultaneously in the same primary antenna beam as the spacecraft. With very small angular separation and zero temporal separation, the accuracy achieved is greatly enhanced by minimizing calibration errors caused by instrumental, electronic, atmospheric, geometric variation, and model errors. An improved sensitivity provided by the large collection area of the Deep Space Network (DSN) Array[2] allows for the detection of faint radio sources. The large field of view for small antennas coupled with the increased sensitivity offered by the DSN Array considerably increases the probability of finding an in-beam calibrator.

In Section II of this article, we review estimates of radio source count density at both 8.4 GHz (X-band) and 32 GHz (Ka-band). Using these estimates, we then obtain the probability of finding an in-beam calibrator in the primary beam of the array. In Section III, we discuss obstacles to the resolution of phase ambiguity that pose limits to the accuracy and robustness of this technique by imposing stringent conditions on a priori model errors. In Section IV, we suggest potential techniques for resolving these difficulties using a calibrator bootstrapping technique. Finally, we conclude with a summary of our findings and discuss future DSN measurements to verify the proposed techniques.

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[2] D. S. Bagri, J. I. Statman, and M. S. Gatti, "Proposed Array-Based Deep Space Network for NASA," *Proceedings of the IEEE*, Special Issue on "Technical Advances in Deep Space Communications and Tracking," submitted for publication.

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II. Source Count Estimates at X- and Ka-Bands

Integrated source counts are known to deviate sharply from a uniform distribution in Euclidean space, which predicts $N(S) \propto S^{-1.5}$, where N is the integrated number of sources with flux larger than S. A number of surveys have been carried out at various frequencies over the years to estimate the integrated radio source count down to levels of a few microjanskys. A recent review [4] estimated integrated source counts at both X- and Ka-bands for sources with flux densities in the range of 1 to 1000 mJy. The estimate at X-band was partially obtained from extrapolation of previous surveys at lower frequencies and higher flux densities (>100 mJy) in conjunction with more recent surveys of microjansky population at X-band. As pointed out in the study, the accuracy of this estimate suffers from a clear lack of data in the range of 1 to 50 mJy, which is of primary interest to the DSN Array. The Ka-band estimate was derived by a simple scaling of the X-band estimate using a mean spectral index of −0.65 [1,2].

We have recently learned of a survey carried out by Henkel and Partridge (2005) [3] that has made an attempt to fill the gap for source count estimates in the range of 1 to 50 mJy at 8.5 GHz. They have carried out two blind surveys using the Very Large Array in its "D" configuration (the smallest configuration with a maximum baseline of ∼1 km) covering a total area of about 1.5 deg^2, yielding a total of 31 source detections. However, their source count estimates are derived from a combination of their second survey with a total of 19 source detections and 16 additional sources from archival data of 4 narrow patches of sky.

These estimates are consistently higher than those obtained from the extrapolated data of Majid and Bagri (2006) [4] (Fig. 1). The difference is notable in the range of 10 to 30 mJy, while at 1 mJy the difference is only a factor of two, consistent with the errors in both studies. We note that at around 50 mJy the two estimates are once again in agreement within the stated errors of the studies. These discrepancies may be due to either incorrect assumptions about the spectral indices of sources with flux in the range of the study or perhaps lack of sufficient data in the 2005 survey. In particular we note that the Henkel and Partridge 2005 estimates relied on as few as 2 sources in their 45-mJy bin and 5 sources

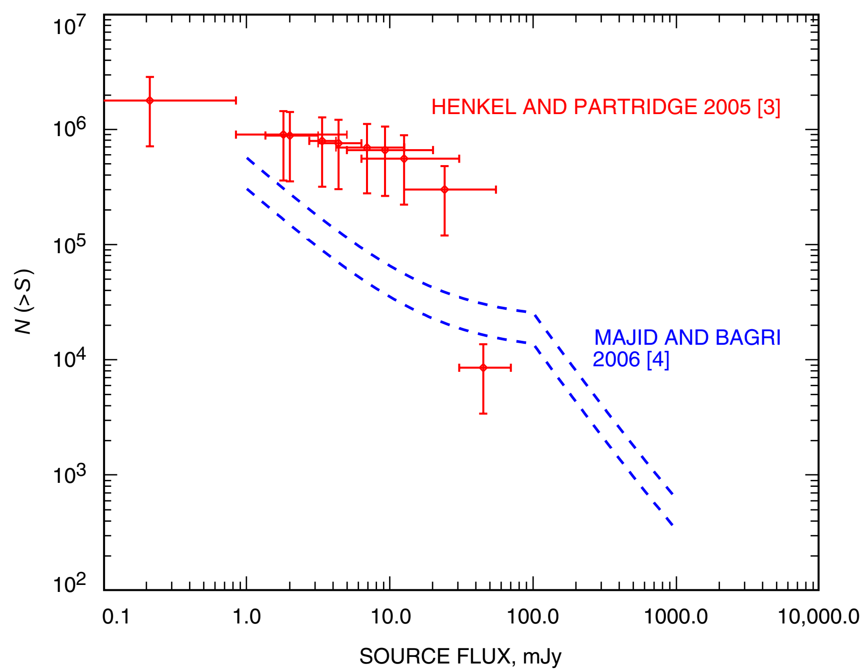

Fig. 1. Integrated radio source count estimate (over all the sky) at X-band from two studies: Henkel and Partridge 2005 [3] and Majid and Bagri 2006 [4].

in each of their 12- and 4-mJy bins. Since this region of flux density is of great relevance to the future DSN Array, further studies with improved statistics of source counts are needed to better understand and quantify the availability of suitable calibration sources for the array.

III. Viability of In-Beam Calibration at X- and Ka-Bands

In addition to requiring a knowledge of the number of sources, we also need the primary beamwidth at the desired operating frequency, as well as a knowledge of the minimum flux detection limit, in order to ascertain whether it is possible to carry out routine in-beam calibration. In this section, we briefly provide estimates for each of these parameters.

A. Fraction of Suitable Sources

As pointed out in [4], source count estimates cover a broad range of radio source types without taking into account the suitability of sources as astrometric calibrators. The fraction of these sources compact enough to be used on intercontinental baselines as reference calibrators is currently not well-known. We are carrying out a study to directly measure the compactness fraction of radio sources above 10 mJy using recently obtained very long baseline array (VLBA) data. Since the number of in-beam calibrators is determined by source population at the $\sim$1-mJy level, the compactness fraction of this population needs to be determined empirically since a new class of source population may be present in the weaker sample. For the purpose of this study, however, we estimate this fraction to be $\sim$30 percent (see [4]).

B. Primary Beamwidth

To carry out in-beam phase referencing with the DSN Array, a reference calibrator is needed within the primary beam of the array. The current design concept of the DSN Array[3] calls for arraying a large number of 12-m-diameter antennas. At operating frequencies of 8.4 GHz (X-band) and 32 GHz (Ka-band), the primary beam size of a circular aperture as characterized by the half-power beamwidth (HPBW) is given by $\theta_{\mathrm{HPBW}} \sim \lambda/D$, where λ is the operating wavelength and D is the diameter of the antenna. The beamwidth is then estimated to be 10 arcmin at X-band and 2.5 arcmin at Ka-band.

C. Coherence Time Limits Calibrators to 1 mJy

While the number of sources increases as the detection limit is lowered to observe weaker sources, the integration time, τ, to detect a source increases inversely with the square of the flux density following the radiometer equation: $\tau \propto S^{-2}$, where S is the source flux density.

We have used the radiometer equation to estimate the integration time needed to achieve a 10σ detection (a signal-to-noise ratio (SNR) of 10 is equivalent to a phase root-mean-square (rms) error of 6 deg) as a function of source flux density at both X- and Ka-bands with a 100-element subset of the DSN Array. While the conceptual design of the DSN Array calls for a large number of antennas distributed in each of the three longitudes currently covered by the DSN, we are assuming here that at any given time about 100 antennas may be available for astrometry. We have further assumed a bandwidth of 100 MHz at X-band and 500 MHz at Ka-band (these numbers reflect the protected bandpass for spacecraft telecommunication). In addition, the system temperature is assumed to be 30 K at X-band and 60 K at Ka-band.

Figure 2 shows the integration time (right axis) required as a function of source flux density for each band. We have also shown on the left axis of Fig. 2 the probability of having one source in the primary beam of the array (assuming 12-m antennas) as a function of source flux density. At X-band, Fig. 2(a), it appears that in-beam phase referencing may be possible over a good fraction of the sky. Sources with

[3] Ibid.

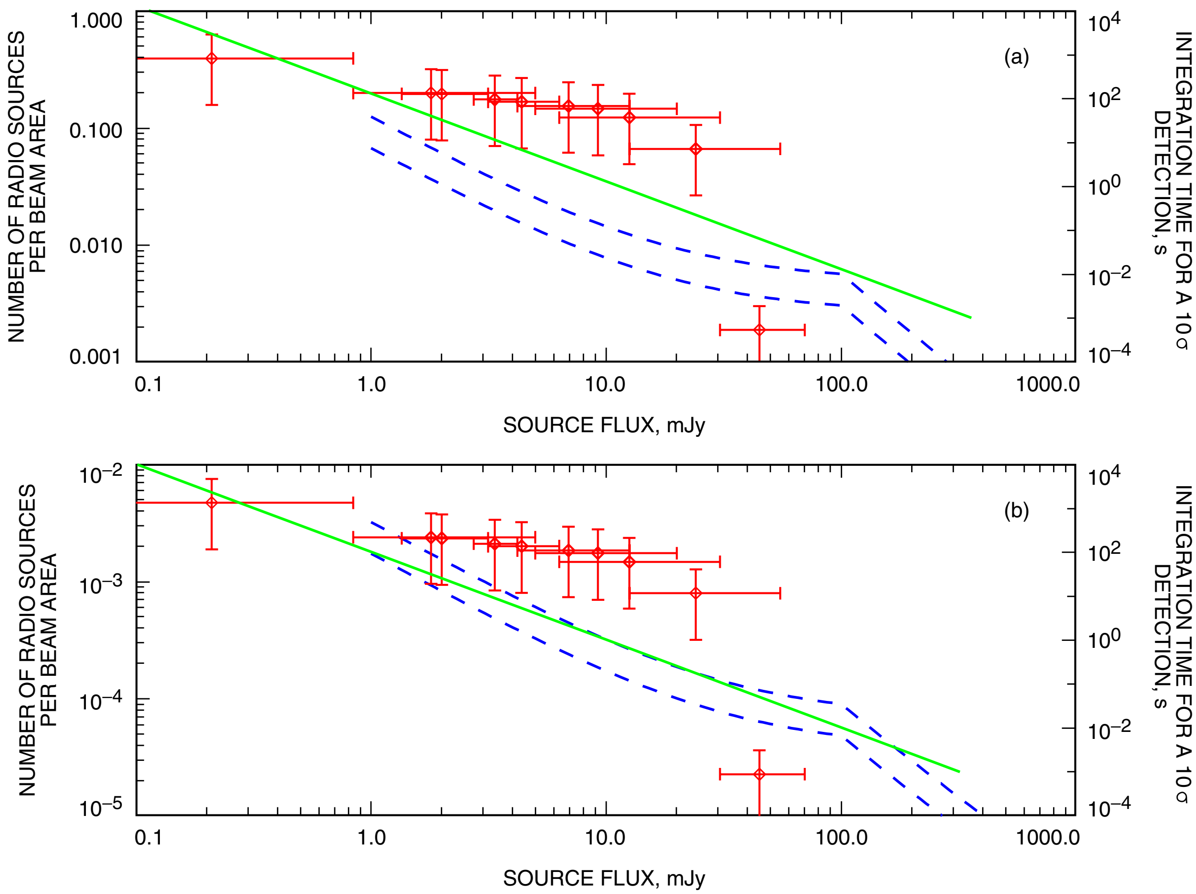

Fig. 2. Integrated time (green curve, right ordinate) needed for a 10σ detection versus source flux density at (a) X-band and (b) Ka-band. Each figure also shows the number of radio sources expected per primary beam area (left ordinate) at both frequency bands using the two studies mentioned in the text (red points are from Henkel and Partridge 2005 [3], and dashed blue lines are extrapolated 1σ limits from Majid and Bagri 2006 [4]). We have assumed that only 30% of the radio sources are suitable and compact enough as calibrators. The primary beam area is calculated assuming a 12-m antenna.

flux densities of 1 mJy can be detected with high detection SNR in ∼100-s integration time. At this flux density range, we expect to have a suitable in-beam calibrator in 20 to 30 percent of the sky.

At Ka-band, Fig. 2(b), the picture is dismal. Even at the 100-μJy level, there is only a 1 percent probability of finding a calibrator in the primary beam. The integration time needed for the detection of a 100-μJy calibrator is of the order of 10^4s, which is far above the expected coherence time (∼100 s) at this frequency regime. We therefore conclude that at Ka-band there is not a sufficient number of sources present to allow for in-beam phase referencing in any substantial portion of the sky.

IV. Resolving Phase Cycle Ambiguities

Using the interferometer phase directly to determine the delay provides great precision that scales with the inverse of radio frequency. However, without an a priori model that is better than one full radio frequency cycle (120 ps at X-band and 35 ps at Ka-band), phase cycle ambiguities need to be resolved. The VLBA accomplishes this feat by using information on its shorter baseline (larger fringe width ambiguity) to step-wise connect phase at longer baselines. With the single intercontinental baseline geometry of the currently envisaged DSN Array for very long baseline interferometry (VLBI), other methods are needed to resolve phase ambiguity.

The traditional technique for resolving the phase ambiguity on a single baseline is to use the group delay $\partial\phi/\partial\nu$ (the rate of change of phase with frequency) over a spanned bandwidth $\Delta\nu$. The cycle ambiguity, while still present, is much larger by the ratio of the radio frequency of observation, ν, to the spanned bandwidth, $\Delta\nu$. At X-band the group delay ambiguity is roughly 10 ns (for an assumed bandwidth of 100 MHz), while at Ka-band the number is 2 ns (assuming a bandwidth of 500 MHz). A 10-ns ambiguity is equivalent to a source position error of roughly 400 nrad ($\sim$0.1 arcsec) over an intercontinental baseline. A 10-ns ambiguity is also equivalent to a baseline uncertainty of 3 meters. Current VLBI models are well enough constrained that the a priori delay model is well within the group delay ambiguities, thus enabling group delay measurements to determine angular position or delay with great accuracy (though not good enough precision). Therefore, a group delay measurement over a single baseline could be used to resolve the phase ambiguity; however, the rms error in group delay is inversely proportional to the spanned bandwidth as opposed to the radio frequency in the case of phase delay rms:

$$\sigma(\tau_{\mathrm{group}}) \sim \frac{1}{2\pi \ \mathrm{SNR} \ \Delta\nu} \tag{1}$$

To determine group delay within a fraction of the phase cycle (120 ps at X-band), a factor of $\nu/\Delta\nu$ larger phase measurement accuracy is required, which necessitates a factor of $\nu/\Delta\nu$ larger SNR as compared with a phase delay measurement of similar precision. At X-band, therefore, the typical minimum SNR for calibrators required for determining the angular position of the spacecraft with a group delay measurment is $\gtrsim$100.

Requiring a large detection SNR limits the measurements to calibrators with large flux densities, which in turn substantially lowers the number of available astrometric calibrators. This method of resolving the phase cycle ambiguity clearly will not allow the use of routine in-beam calibration, except in fortuitous cases where a very strong calibrator happens to be in the same primary beam as the spacecraft.

A. Using Two Calibrators

In order to determine the phase cycle ambiguity, we propose using two calibrators: a relatively strong primary calibrator and the secondary in-beam calibrator. In this approach, the relative position of the spacecraft is first determined with respect to the strong primary calibrator, using the group delay method as is currently done at the DSN.[4] This measurement can provide an a priori relative position that is good at the level of 1 nrad as long as the calibrator is within 5 to 10 deg of the spacecraft. In the next step, the newly derived a priori position of the spacecraft is used to determine the relative position of the spacecraft and the secondary in-beam calibrator with much better precision, using the phase referencing technique of in-beam calibration. Knowledge of the a priori spacecraft position is used to resolve the phase ambiguity, allowing the power of the precision of the phase delay measurement to be brought to bear. Such a bootstrapping technique, while a little inefficient due to slewing of the antenna to point at the strong calibrator, has the potential to allow in-beam phase referencing to be carried out when a suitable calibrator (even a faint one) happens to be in the same primary beam of the antenna as the spacecraft.

This approach requires the presence of a strong calibrator within 5 to 10 deg of the target spacecraft. Here we determine whether this is a reasonable expectation based on source count studies. Source count surveys at X-band suggest 0.08 sources per square degree with flux greater than 300 mJy, which translates to roughly a total number of 3300 radio sources. Assuming a compactness fraction of 30 percent, we are left with about 1000 sources. This total number translates into an expected 1.9 sources within 5 deg of any point in the sky, which would make this approach feasible, as demonstrated by the success of the delta differential one-way range campaigns over the years.[5]

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[4] J. S. Border, "2004 Error Budget for DSN Delta Differential One-way Range Measurements," JPL Interoffice Memorandum 335-04-04-D (internal document), Jet Propulsion Laboratory, Pasadena, California, 2004.

[5] Ibid.

An additional piece of information that must be obtained is an a priori position of the in-beam calibrator to within a fraction of the fringe width. This can be done well in advance, perhaps weeks to months before the actual spacecraft pass depending on the stability of the in-beam calibrator structure. The VLBA is an ideal instrument to make such measurements. The integration time needed for detection of a 1-mJy source at X-band is ~300 s with an SNR of 15 and a typical bandwidth of 500 MHz. A high detection SNR is needed to allow for sufficient signal to determine the angular position to a small fraction of the fringe phase. In order to achieve sufficient u-v coverage, a source might be observed over several hours and possibly multiple epochs.

B. One Shorter Baseline per Intercontinental Baseline

Another potential approach for resolving phase ambiguity is to use the VLBA model by first resolving phase over shorter baselines. This approach can be carried out at the cost of distributing the DSN Array into two sub-clusters over a 200- to 300-km baseline with a main cluster at the center. While this approach is perhaps not realistic given the cost implications, we have included a brief discussion here.

The fringe width is of the order λ/B, where B is the length of the baseline. Over an intercontinental baseline, the fringe width is around 4.5 nrad (at X-band). As we have discussed above, resolving the ambiguity requires an a priori knowledge of the relative position at a fraction of the fringe width. This requirement is relaxed if the baseline is shorter. For example, over a 300-km baseline, the fringe width is 120 nrad, and it is feasible to imagine knowing the a priori relative position of the source and calibrator to within a fraction of 120 nrad.

However, in order to detect sources at the millijansky level, the baseline sensitivity needs to be maintained for the shorter baseline by placing a sufficient number of antennas in the sub-cluster. This can be achieved by various configurations. Since the long baseline determines the component of the delay along the projected direction of the baseline to the source, only one short baseline is needed colinear with the long baseline.

V. Conclusions

In-beam phase referencing is a powerful technique in astrometry that may have important implications in the era of the future DSN Array. Our study suggests that at X-band the potential for carrying out in-beam phase referencing measurements is promising. Using current source count estimates and source compactness ratio estimates, we determined that there is a ~30 percent chance of finding an in-beam calibrator in the primary beam of the 12-m antenna being considered for the DSN Array. One of the crucial assumptions in this estimate is the compactness fraction of sources at the 1-mJy flux density scale. At present this fraction is not well-known. We have pointed out that we are carrying out a VLBA-based study that measures the compactness fraction at the 10-mJy level. However, there are suggestions that at another order-of-magnitude-weaker flux scales other source populations may dominate the radio sky. We therefore recommend follow up studies at the VLBA to empirically determine this fraction at the millijansky scale.

At Ka-band the prospects of in-beam phase referencing are not very good. At this frequency band, the beam area is smaller by a factor of 15 with respect to the beam size at X-band, while at the same time the source count is also decreasing compared to X-band, and the system sensitivity is reduced due to increased system temperature of the antennas. Even an array composed of 6-m antennas would not improve the use of in-beam calibration in more than 1 percent of the sky.

We have also provided a recipe for overcoming the phase cycle ambiguity problem that is inherent in an in-beam phase referencing measurement that involves a weak calibrator, as it almost always would, except in fortuitous cases when a strong calibrator happens to be in the same beam as in the target source. Our approach relies on using a two-step calibration scheme, where first a "rough" position of the target (spacecraft) is determined using a stronger calibrator (primary calibrator) several degrees away from the target. The newly determined position of the target is then used to resolve the phase cycle ambiguity in a second measurement, using the in-beam or secondary calibrator. We suggest the principles behind this approach can be put to test at the current DSN, using a pair of 34-m beam-waveguide antennas. Such a measurement would also allow a direct comparison of the accuracy achieved using the standard nodding calibration scheme to that achieved with in-beam calibration.

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