Estimating Accurate Relative Spacecraft Angular Position from Deep Space Network Very Long Baseline Interferometry Phases Using X-Band Telemetry or Differential One-Way Ranging Tones

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At present spacecraft angular position with the Deep Space Network (DSN) is determined using group delay estimates from very long baseline interferometry (VLBI) phase measurements employing differential one-way ranging (DOR) tones. Group delay measurements require high signal-to-noise ratio (SNR) to provide modest angular position accuracy. On the other hand, VLBI phases with modest SNR can be used to determine the position of a spacecraft with high accuracy, except for the interferometer interference fringe cycle ambiguity, which can be resolved using multiple baselines, requiring several antenna stations as is done, for example, using the Very Long Baseline Array (VLBA) (e.g, the VLBA has 10 antenna stations). As an alternative to this approach, here we propose estimating the position of a spacecraft to half-a-fringe-cycle accuracy using time variations between measured and calculated phases, using DSN VLBI baseline(s), as the Earth rotates (i.e., estimate position offset from the difference between observed and calculated phases for different spatial frequency (U, V) values). Combining the fringe location of the target with the phase information allows for estimate of spacecraft angular position to a high accuracy.

One of the advantages of this scheme, in addition to the possibility of achieving a fraction of a nanoradian measurement accuracy using DSN antennas for VLBI, is that it is possible to use telemetry signals with at least a 4 to 8 Msamples/s data rate (bandwidth greater than ∼8 to 16 MHz) to measure spacecraft angular position instead of using DOR tones, as is currently done. Using telemetry instead of DOR tones will eliminate the need for spacecraft coordination for angular position measurements and will minimize calibration errors due to instrumental dispersion effects.

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I. Introduction

Angular position estimates with the Deep Space Network (DSN) are made using very long baseline interferometry (VLBI) [1]. Phase measurements at widely separated frequencies are used to calculate group delay, and these delays are employed to estimate the angular position of a spacecraft with respect to a nearby calibration source. The accuracy of the angular position estimates is limited by the spanned bandwidth (BW) for the spacecraft signal because of spectrum management requirements. On the other hand, VLBI phase measurements can be used to determine the position of a compact radio source (including spacecraft) with high accuracy, except for the (interferometer interference) fringe cycle ambiguity. This can be resolved using multiple-interferometer baselines, e.g., as used by the Very Long Baseline array (VLBA), or by using group delay measurements employing wideband telemetry or widely separated ranging tones, as are used in the DSN.

To resolve the fringe ambiguity using group delay estimates, very high accuracy phase measurements are required because of limited ranging-tone separation (due to spectrum allocation). This requires high signal-to-noise ratio (SNR) for the phase measurements, and that necessitates using a strong calibration source and/or long integration on the calibrator. Using a strong calibration source generally makes it necessary to have a calibrator angularly far away from the target source because the density of sources in sky goes down for stronger sources. Long integration causes poor phase measurement accuracy due to instrumental variations with time and propagation path changes caused by atmospheric fluctuations for the signal arriving at the interferometer antennas. Large angular separations also cause calibration errors due to different atmospheric delays along the different lines of sight and lack of knowledge of the exact observing geometry between the source and the interferometer baseline. These errors are essentially proportional to the angular separation.

The effects due to long integration times on each of the sources and therefore large temporal separation between observations of the target and calibrator, required to provide adequate SNR for the measurements on the sources, can be ameliorated by interleaving many cycles of short-duration observations of the calibrator and target source. As atmospheric fluctuations have power law spectra for temporal or position separation, the effects of atmospheric fluctuations can be minimized by using short cycle times (once interferometer phases are calibrated, a long total integration time can be achieved by combining observations over many cycles). However, there are practical limits to minimizing cycle time because of limited antenna slew rates and settling times required before starting the measurements.

The differential path-length changes for signals arriving from the two sources at the antennas increase as the angular separation between the calibrator and target is increased. The calibration errors also increase with increasing angular separation between the calibrator and target due to lack of knowledge of the exact source-baseline geometry, caused by uncertainty in the Earth rotation parameters and positions of the antennas. Therefore, to keep these calibration errors small, it is desirable to keep the angular separation between the calibrator and target source small.

II. Concept

This article proposes to use variations (with time) between observed and calculated phases from an interferometer to estimate the offset from an assumed position of the target (i.e., estimate position offset from the difference between observed minus calculated phases for different spatial frequency (U, V) values [2]).

The initial position of the target could be off from the actual position by several fringe cycles. It can be estimated using group delays from moderate SNR VLBI phase measurements. The minimum length of observation (or spatial frequency coverage) required for resolving fringe cycle ambiguity depends on the accuracy of the phase measurements. To resolve fringe ambiguity to half-cycle accuracy, the difference between measured and calculated phases for an assumed position error of half a fringe should be larger than the combined errors of the phase measurements and fringe fitting.

The accuracy for relative position of a spacecraft with respect to a calibrator will depend on the phase calibration errors (due to propagation, instrumental variations, and lack of knowledge about the exact geometry of the observations) and the SNR of the phase estimates for the calibrator and target sources over the observing period.

Calibration errors depend on the angular and temporal separation between calibrator and target source (spacecraft) measurements. For calibration of the VLBI phases with DSN 34-m antennas at 8.4 GHz (X-band), it should be possible to use a calibrator with flux density >50 mJy (Jy $= 10^{-26}$ Wm^{-2}Hz^{-1}) using an 8-MHz signal bandwidth and about a 1-minute integration. A calibrator with X-band flux density of >50 mJy is typically available within about 1 deg of any target position. Temporal separation can be kept small by interleaving observations on the target and calibrator with a short scan time on each source and repeating many cycles of observations on the two sources to provide enough integration on each source.

Based on an estimated error budget, we expect about 1/3-nanoradian (nrad) relative position accuracy using about an hour of (actual) VLBI observations with DSN 34-m antennas at X-band employing a scan time of 40 seconds on each source and 20 seconds to change sources (a cycle time for target–calibrator–target observations of about 2 minutes). The accuracy of the estimated position is limited by calibration errors, so it may be possible to improve the accuracy in a number of ways; for example by (1) reducing errors from Earth orientation parameters and station location, (2) reducing calibrator–spacecraft separation—this may require employing weaker calibrators, which may require increasing observation sensitivity using more G/T (where G is antenna gain and T is system temperature), bandwidth, or longer observations, and (3) using multiple calibration sources on either side of the spacecraft to reduce the effects of errors due to observing geometry and propagation.

III. Theory

The relationship between spatial frequencies (U, V, W), source position (H, δ), and baseline (X, Y, Z) can be expressed as (e.g., see [2, p. 86])

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} \sin H & \cos H & 0 \\ -\sin\delta\cos H & \sin\delta\sin H & \cos\delta \\ \cos\delta\cos H & -\cos\delta\sin H & \sin\delta \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

Here (U, V, W) are Fourier spatial frequency components expressed in wavelengths, H and δ are source hour angle and declination, and (X, Y, Z) are antenna baseline coordinates in wavelengths. X, Y, and Z are measured in $(H = 0 \text{ hr}, \delta = 0 \text{ deg}), (H = -6 \text{ hr}, \delta = 0 \text{ deg})$, and $(\delta = 90 \text{ deg})$ directions, respectively.

The interferometer phase for the source position (H, δ) can be written as

$$\varphi = X\cos\delta\cos H - Y\cos\delta\sin H + Z\sin\delta$$

Then the differential phase for a source at a small offset $(\Delta H, \Delta\delta)$ from this source at (H, δ) can be written as

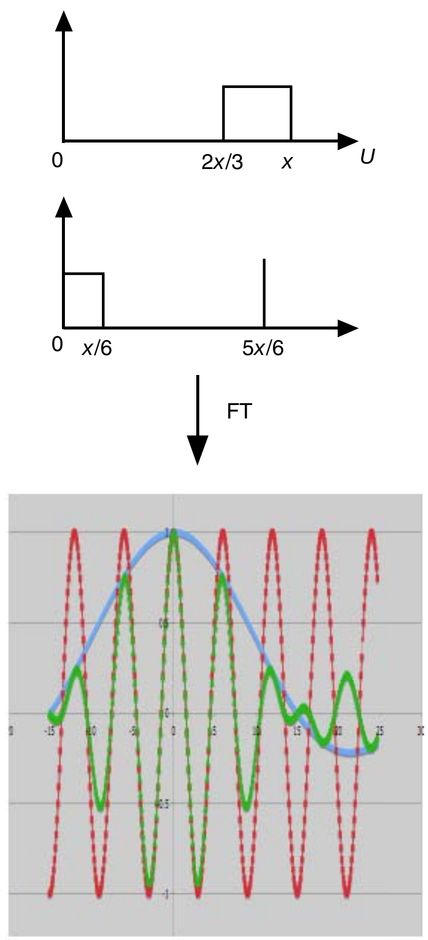
$$\Delta\varphi = (-X \sin\delta \cos H + Y \sin\delta \sin H + Z \cos\delta)\Delta\delta - (X \sin H + Y \cos H)\Delta H \cos\delta$$

$$= \; - U \bullet \Delta H \cos\delta + V \bullet \Delta\delta$$

Knowing the values for (U, V), δ, and the measured value of $\Delta\varphi(U, V)$, one can do a least-squares fit to determine the values of the offset $(\Delta H, \Delta\delta)$ from the assumed spacecraft position, and a phase bias. Caution is required to ensure there are no large gaps in the values $\Delta\varphi(U, V)$ that would allow a cycle (2π) of phase change to be missed; however, this is easily satisfied by other constraints (discussed below) on the assumption of an initial position for the spacecraft. A similar approach, based on fringe phase (delay) and phase change (fringe rate), has been used in radio astronomy to determine accurate relative positions of weak radio sources (e.g. see [3]). The fringe rate can resolve any fringe cycle ambiguity, and then the bias phase can be used to determine an accurate position of the spacecraft.

1. Minimum Spatial Frequency Coverage or Observations Required. The minimum amount of (U, V) coverage required for this to work must provide a phase change that is larger than the error of estimating the value of the phase $\sigma_{\Delta\varphi}$. (It's desirable to have a minimum phase change of at least 2.5 $\sigma_{\Delta\varphi}$.)

Alternatively, one can look at this using a Fourier transform (FT), as shown schematically in Fig. 1. Consider the one-dimensional case of a source with maximum spatial frequency $U_{\max} = x$ and an



Spatial frequency coverage. Here 1/3-rad sky coverage is assumed, so *U* goes from maximum value (=*x*) to 2*x*/3.

Spatial frequency coverage can be expressed as a convolution of two functions (a delta function at 5*x*/6 and a rectangular function from 0 to *x*/6).

A Fourier transform of measured visibility (in green) = cos $(2\pi \times 5xa/6)$ (in red) $\times$ sinc $(xa/3)$ (in blue). Here *a* = angle on sky and *x* = the maximum value of *U*.

That is, there are 2.5 cosine wave cycles in angle 0 to *a* = 6/5*x*.

Consider $x = 2 \times 10^8\ \lambda$; then the first null of the sinc function is at 15 nrad, and it is multiplied by a cosine wave of 6-nrad period.

It means the first peak is at 0 and the negative peak is at 3 nrad, so it will be reduced to 0.935 when it is multiplied by sinc(6/15).

To prevent false detection of a signal a half fringe away, we should have (2.5 sigma noise + 0.935) < 1 $\Longrightarrow$ 2.5 sigma < 0.065 or SNR > 40 for a whole observation.

Fig. 1. Spatial frequency coverage and equivalent beam on the sky using a Fourier transform. See the text for a calculation of the SNR required to resolve fringe cycle ambiguities.

observation of the source for 1/3 rad across the sky (about 1.25 hours of observation for a source on the equator). The spatial frequency coverage (U) is $2x/3$ to x. The interferometer coverage can be Fourier transformed to give the beam response as $\left[\cos(2\pi \times 5xa/6) \times \mathrm{sinc}(xa/3)\right]$. Here a = the angle on sky and x = the maximum value of U. There are 2.5 cosine wave cycles in angle 0 to $a = 6/5x$. If we consider an 8000-km baseline between DSN VLBI antennas, then $x = 2 \times 10^8$ wavelengths (λ), the first null of the sinc function is at 15 nrad, and it is multiplied by a cosine function with a 6-nrad period. The first peak is at 0 and the first negative peak is at 3 nrad, so it will be reduced to 0.935 when it is multiplied by sinc(6/15). To prevent false detection of a signal one-half fringe away, we should have 2.5 × the root-mean-square noise plus the value at the first null, $(2.5\sigma + 0.935) < 1 \longrightarrow 2.5\sigma < 0.065$, or SNR > 40 for the whole observation. The confidence level can be increased from 2.5σ to higher values by increasing the time of observations, which will make the sinc function narrower by increasing spatial frequency coverage and will also improve SNR. Also, it's possible to increase the SNR by using a larger bandwidth on the calibrator and employing bandpass calibration to keep the dispersive errors small.

2. Initial Value of the Spacecraft Position. The initial value of the spacecraft position should not be grossly wrong in order to avoid appreciable loss of coherence over the signal bandwidth or within one integration time. However, there still can be an offset of many fringe cycles between the actual and initial positions without appreciable loss of coherence. For example, consider a 100-MHz signal bandwidth. For a DSN 8000-km baseline, the delay beamwidth will be about 0.3 μrad, and therefore from delay considerations one can easily tolerate an offset of 0.1 μrad. Also, in 1 minute of time, a phase change due to an offset of 10 fringe cycles will be less than roughly (number of fringe offsets × 360 deg × scan time/time from maximum fringe rate to zero fringe rate) = 10 fringe cycles × 360 deg × 1 min/360 min $\sim$ 10 deg for a source at the equator. This phase change has a negligible effect on integrating the signal for 1 minute and can be ignored. Otherwise, one can use short integrations and high spectral resolution to do a least-squares fit to account for these effects.

3. Initial Position of the Spacecraft Determined from Group Delay Estimates. The initial position of the spacecraft can be determined from group delay estimates. If we assume the accuracy required for the initial position of a spacecraft is about 10 fringes, a group delay accuracy of about 1.2 ns is required (σ = 0.4 ns for 3-sigma confidence). Table 1 shows the minimum signal required to give σ = 0.4 ns for VLBI observations with a pair of DSN 34-m antennas under different observing scenarios.

Estimates of the required calibrator strength in Table 1 show that we can determine group delays with the required accuracy from forty-five 40-second scans spread over 75 minutes using DOR tones, or 4 Mbits/s telemetry signals and a calibration source of 50 mJy.

Table 1. SNR and calibrator flux density required to obtain 1-sigma accuracy of 0.4 ns for group delay determination

Object	Condition (T_{sys} at X-band) = 30 K	Required for 0.4-ns delay accuracy	Comment
Spacecraft	DOR tone separation = 40 MHz	SNR > 2	Easily achieved.
Radio source	BW = 16 MHz (64-Mbits/s recording), integration = 40 s/scan × 45 scans	SNR > 4/scan → 16 mJy calibrator required	A calibrator within 0.6 deg is possible.
Spacecraft	Telemetry 4 Mbits/s (BW = 8 MHz)	SNR > 100	4 Mbits/s telemetry will have SNR > 100 in 1 s.
Radio source	BW = 8 MHz (32-Mbits/s recording), integration = 40 s/scan × 45 scans	SNR > 7.4/scan → 50-mJy calibrator required	A calibrator within 1 deg is possible.

Also, it is worth noting that there should be at least an SNR of 3 per scan, so coherent integration over multiple scans can be done without running into the problem of skipping a fringe cycle in the fringe fitting, and a good fringe fitting is achieved. This is easily satisfied by other constraints. Further, a spacecraft signal having a bandwidth corresponding to a data rate of more than about 4 Msamples/s is required to find its position within a few fringe cycles. If that is known from other considerations, then it may be possible to use a much reduced data rate spacecraft signal for this approach to work.

IV. Error of Position Estimates

To get an idea of the expected position accuracy from calibrated phase measurements (e.g., see [2, Eq. 12.13, pp. 374–375]) based on SNR only, consider a DSN baseline of 8000 km. For observations at X-band (wavelength $\lambda = 3.6$ cm), this gives a baseline length of $2.22 \times 10^8 \lambda$. This will give a fringe rate of roughly 17 kHz for a low-declination source. If we observe for an integration time of 1 hour (a clock time of 1.25 hours), we have roughly $1.7 \times 10^4 \times 3600 \times 1.25 = 7.65 \times 10^7$ fringe cycles elapsed during this period. We can measure phase with an SNR of 50 (assuming two DSN 34-m antennas having a system temperature of 30 K, bandwidth of 8 MHz, integration time of forty-five 40-second scans over 60 minutes, and using a calibration source of 50 mJy at X-band). This corresponds to 0.0034 fringe cycles. In the fringe fitting, the spacecraft SNR is much larger than what is achieved on a calibrator, and therefore its effect can be ignored. This translates to 0.0034 fringes out of 7.65×10^7 fringes, or <0.1 nrad due to SNR alone.

V. Calibration Errors and Error Budget Considerations

An error budget for accurate angular position measurements using VLBI has been developed by Jim Border[2] (also see [1]), and is shown in Fig. 2 . The error budget shows the expected (1-sigma) errors from different sources of errors for normal VLBI observing conditions for DSN 34-m antennas at X-band using 10 minutes per scan on the spacecraft, calibration sources separated by 6 deg, and a total observing time of about 1 hour.

The calibration errors depend on the angular and temporal separation between the calibrator and spacecraft and can be reduced by reducing the angular and temporal separation. Generally, tropospheric errors are linearly proportional with temporal separation, and errors due to ionosphere, solar plasma, Earth parameters, and station location are linearly proportional to angular separation between the calibrator and spacecraft [3]. The dispersive effects can be essentially eliminated by bandpass calibrations, but some residual contribution will be left, and here we assume that these are linearly dependent on the temporal separation. Thus, by interleaving short observation scans on the calibrator and spacecraft and repeating the observations as many times as necessary to accumulate enough integration to get the required SNR on each source, and by keeping the angular separation between the two sources small, one can reduce these errors considerably.

An estimate of the expected calibration errors due to various items has been estimated from the error budget developed by Border [1]. In Table 2, we give calibration errors for different items as given by Border (in nanoseconds) and how much potential reduction may be possible by using the dependence discussed above and reducing the angular and temporal separation between calibrator and spacecraft to 1 deg and 1 min.

There is a concern that this requires an observation over a period of 1.25 hours, so at least one of the VLBI stations will be pointing at low elevation and propagation errors will increase by approximately the secant of the change in elevation. Assuming an hour-angle coverage of 1/3 rad, the propagation

[2] D. Shin, "IND Tracking Presentation—Part 2," JPL internal document, Jet Propulsion Laboratory, Pasadena, California, September 21, 2007.

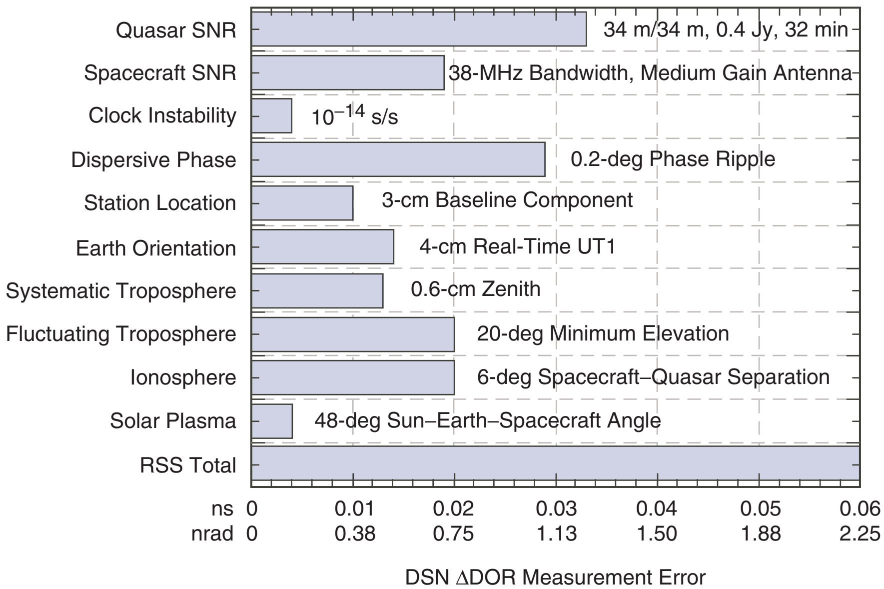


Fig. 2. A conservative estimate of the error distribution for the 2004 DSN ΔDOR measurements at X-band, assuming a separation of 6 deg between the spacecraft and reference sources and a recording data rate of 64 Mb/s for the quasar. The angular accuracy values are obtained by dividing the corresponding delay errors by a baseline length of 8000 km in units of nanoseconds (from [1]).

Table 2. Error budget for angular measurements. The columns labeled "Border error" are from the error budget for angular position measurements for ΔDOR by Jim Border (from the presentation by Dong Shin cited in Footnote 2).

Error budget item	Border error (10-min, 6-deg separation)		Error for 1 min, 1-deg separation, ns	Comment
	value, nrad	value, ns		
Calibrator SNR	1.375	0.033	—	Note 1
Spacecraft SNR	0.7917	0.019	—	Note 1
Clock instability	0.1667	0.004	0.0004	Linear with time
Dispersive phase—instrumental	1.2083	0.029	0.0029	Note 2
Station location	0.4167	0.01	0.0017	Linear with angle
Earth orientation	0.5833	0.014	0.0023	Linear with angle
Troposphere—systematic	0.5417	0.013	0.0013	Linear with time
Troposphere—fluctuating	0.833	0.02	0.002	Linear with time
Ionosphere	0.833	0.02	0.003	Linear with angle
Solar plasma	0.125	0.003	0.0005	Linear with angle
RSS error	—	—	0.0056	—

Note 1: This is not directly applicable here because we use phase error at RF instead of delay. This should be included in the fringe fitting error for both the calibration source and spacecraft.

Note 2: Bandpass calibration and/or use of telemetry data instead of DOR tones would essentially eliminate this, and here we assume any residual effects will reduce linearly with time.

errors in calibration due to the atmosphere will increase by a factor of approximately 1.3. Thus, the total calibration root-sum-squared (RSS) error value will increase from 5.6 ps to about 6.5 ps.

For calibration errors of 6.5 ps ($\sim$2 mm), estimated using 1-min and 1-deg temporal and angular separation, respectively, between calibrator and spacecraft, and an 8000-km baseline, we will have a calibration error of approximately 0.056 fringe cycles. An SNR error contribution of 0.0034 fringe cycles (estimated above) gives a total RSS error of $\sim$0.056 fringe cycles. This is out of a total of 7.65×10^7 cycles, or 1 part in 1.4×10^9, or about 0.7 nrad. This (error) value is much smaller than the mean fringe spacing (6 nrad) and therefore will easily resolve any fringe ambiguity. Now, if we take the phase error in the measured fringe phase offset to also be 0.056 fringe cycles, then the position of the source is uncertain by 0.056 $\times$ fringe spacing of about 6 nrad (for an 8000-km baseline at X-band) or 0.33 nrad. This means we can estimate the relative position of a spacecraft to about 1/3 nrad.

It should be noted that the accuracy of the position error is dominated by calibration errors. The SNR requirement is essentially only to resolve fringe ambiguity. If there is concern about SNR, one can reduce the SNR requirement by increasing the spatial frequency coverage, or alternatively increasing the SNR, either by using a stronger calibration source or by increasing the bandwidth for the calibrator observations. (For spacecraft there is normally plenty of SNR and therefore this is not a concern.) Bandwidth for calibrator observations can be increased easily without affecting the dispersive errors substantially, especially if the bandpass calibration is used to correct for any dispersion effects.

VI. Availability of Suitable Calibrators

The density of radio sources exceeding a given flux density at X-band is known from many studies (e.g., see [4]). Also, the fraction of sources at a given flux density that are compact enough to be used as calibration sources is important. The flux density of interest is 10 to 100 mJy. Samples of radio sources in a few fields with from ten to a few hundred millijanskys at X-band have been studied recently for compactness using the National Radio Astronomy Observatory (NRAO) VLBA. These observations show that about 1/3 of sources with a flux density >10 mJy are compact [5].[3]

From the integrated number density of radio sources as a function of flux density, and knowing what fraction of sources are compact, we can determine availability of sources for calibration. Number density plots show that there is about 1 source per square degree at 50 mJy at X-band, and if 1/3 of these sources are compact, then we should have a source that can be used for calibration within about 1 deg from any direction.

VII. Current Results and Suggested Future Work

Relative angular position measurement errors using VLBI are essentially due to two main reasons—calibration errors and SNR. Errors due to SNR can be calculated easily from actual observing conditions, but estimating the calibration errors is complex. To see whether the above analysis for reducing calibration errors is reasonable, we have made VLBI observations of pairs of known compact radio sources (quasars) of about 0.5 Jy at X-band with separations of about 1 deg. We used an observing cycle of about 2 minutes between the two sources. The observations were made using DSN 34-m antennas at Goldstone and Madrid with a spanned bandwidth of about 80 MHz and 1 hour of total time on each pair. We considered one of the sources from a pair as a calibrator and the second one as a spacecraft, and we estimated the position of the second source using the fringe fitting approach discussed above. Details of the actual observations, data reduction, and results will be presented by the authors in an article in a future issue of this publication. The main results are summarized in Table 3. The results show that we can estimate the relative position of a target source (with respect to the assumed calibrator) to about <0.2 nrad accuracy.

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[3] An article on this subject is being prepared for a future issue of this publication.

Table 3. Estimated position error for target relative to calibrator from observations of known pairs of quasars.

Target and calibrator source pair	Flux density, mJy	Angular separation, deg	Fringe fitting error converted to delay, ps	Equivalent angular position error,[a] nrad	Observation period
0805+410(C) 0814+425(T)	560 550	2.3	3	0.13	December 2005
0805+410(C) 0814+425(T)	560 550	2.3	3	0.13	December 2005
1030+415(C) 1020+400(T)	480 570	2.4	6	0.26	December 2005
1044+719(C) 1053+704(T)	650 350	1.7	3	0.13	December 2005

[a] The angular error values are obtained by dividing the corresponding delay errors by a baseline length of 8000 km in units of nanoseconds.

Future work includes the following:

- Using weaker calibrator and DOR tones from a spacecraft in order to establish that weaker calibrators are adequate.
- Using a telemetry signal instead of DOR tones from a spacecraft in order to establish that telemetry signals, rather than DOR tones, can be used.
- Reducing calibration errors. Reducing the angular separation will reduce systematic effects but will require using weaker calibrators. That can be compensated for by increasing the observing time, which will also increase U, V coverage.

VIII. Conclusion

An approach to measuring the relative position of a spacecraft to about 1/3 nrad with the existing DSN system using DOR tones or a telemetry signal of about 4 Mbits/s was described. The approach employs fringe fitting of VLBI phase measurements to determine the offset from an assumed initial position of the spacecraft. The required initial approximate position of the spacecraft should be within a few fringes and can be estimated using group delay measurements using DOR tones or telemetry signals with a data rate at least about 4 Mbits/s (a bandwidth greater than ~ 8 MHz).

Results from preliminary tests using VLBI measurements, with DSN 34-m antennas, on pairs of known compact radio sources (quasars), using DOR tones at X-band, are consistent with what is expected. More tests need to be made using telemetry signals from a spacecraft, and using weaker calibration sources, to demonstrate that the approach works.

Acknowledgments

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