

Statistical Risk Estimation for Communication System Design — A Preliminary Look

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ABSTRACT. — Spacecraft are complex systems that involve different subsystems with multiple relationships among them. For these reasons, the design of a spacecraft is a time-evolving process that starts from requirements and evolves over time across different design phases. During this process, a lot of changes can happen. They can affect mass and power at the component level, at the subsystem level, and even at the system level. Each spacecraft has to respect the overall constraints in terms of mass and power: for this reason, it's important to be sure that the design does not exceed these limitations. Current practice in system models primarily deals with this problem, allocating margins on individual components and on individual subsystems. However, a statistical characterization of the fluctuations in mass and power of the overall system (i.e., the spacecraft) is missing. This lack of adequate statistical characterization would result in a risky spacecraft design that might not fit the mission constraints and requirements, or in a conservative design that might not fully utilize the available resources. Due to the complexity of the problem and to the different expertise and knowledge required to develop a complete risk model for a spacecraft design, this article is focused on risk estimation for a specific spacecraft subsystem: the communication subsystem. The current research aims to be a “proof of concept” of a risk-based design optimization approach, which can then be further expanded to the design of other subsystems as well as to the whole spacecraft. The objective of this research is to develop a mathematical approach to quantify the likelihood that the major design drivers of mass and power of a space communication system would meet the spacecraft and mission requirements and constraints through the mission design lifecycle. Using this approach, the communication system designers will be able to evaluate and to compare different communication architectures in a risk trade-off perspective. The results described in this article include a baseline communication system design tool and a statistical characterization of the design risks through a combination of historical mission data and expert opinion contributions. An application example of the communication system of a university spacecraft is presented.

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Part of the research described in this publication was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration. © 2012. All rights reserved.

I. Introduction

Spacecraft are complex systems that involve different subsystems with multiple relationships among them. The design of a spacecraft is a process that starts from requirements and evolves over time across multiple design phases. The complexity of the systems, the number of people involved, and the time intervals between the reviews inevitably lead to changes in the design. In particular, different studies on major challenges in the design of a mission reveal that there are “Significant deviations from expected mass, power, cost or performance for any element of the spacecraft.” [1]

At each design iteration, the engineers would perform new analysis based on updated information and assumptions, leading to a better understanding of their respective subsystems. This reduces the corresponding uncertainties in the key design metrics like power and mass. However, during the initial design stage, engineers are forced to estimate quantities without a complete knowledge of their subsystems: some components might be in design but not finalized, some can be totally new, some can be fabricated in-house for the first time, and some can be fabricated externally but the knowledge about mass and power is not complete.

For all those reasons, a problem arises: engineers are forced to “speculate” the values for mass and power at the component level and at the subsystem level, and these values inevitably would fluctuate over time. This problem is crucial in spacecraft design as each mission is subjected to constraints in total launch mass and power of the spacecraft. On one hand, the fluctuations can cause the system to exceed its design limitations, which results in a spacecraft that does not fit in the design boundaries and would require a costly redesign. On the other hand, the fluctuations can result in an overly conservative design that greatly reduces the mission’s capabilities and performance. This is the case when excessive overestimate contingencies are used as design margin.

Use of contingencies is a common engineering approach to counteract risks associated with design uncertainties. This approach can be applied at the component level, at the subsystem level, and at the system level. The contingency applied is generally a deterministic number added to the design value. This approach is different from the statistical method that performs probabilistic assessment of the fluctuations in order to develop a representative probability distribution function (pdf). The pdf provides probabilistic characterization of the design metric, and expresses the design value typically as mean and its fluctuation as variance (or sigma, σ). This approach has been used in statistical link analysis [2]. In this article, we extend this approach to the general areas of spacecraft communication system design and spacecraft system design.

The statistical approach uses mathematically tractable techniques to combine uncertainties at the subsystem level, and expresses the design risk at the system level. This allows risk-performance trade-off and enables risk-based system design and optimization.

The aforementioned design challenge of striking the right balance between a conservative approach and a risk-taking approach in the absence of definitive information is evident in all spacecraft subsystems, and in particular for the communication subsystem. Different

publications [2–5] suggest that communication systems typically represent a large fraction of the total spacecraft mass and power. Hence, fluctuations in mass and power of the communication system would have a significant impact on the overall spacecraft design. This phenomenon is valid for spacecraft applications like commercial satellites, space relays, and small satellites.

A considerable amount of work has been done to identify the reasons for which the final values of mass and power fluctuate significantly from the initial values. Specifically, the causes for these fluctuations can be divided into two main categories:

- Fluctuations due to lack of human interaction between designers: a team composed of different engineers working together on a project can, for lack of interaction, develop subsystems that do not fit well together and that require a partial redesign, which results in mass, power, and cost deviations.
- Fluctuations due to lack of knowledge: this is the case in which a lack of definitive information in the mission implementation, or in the fabrication of the components, leads to deviations of mass, power, and cost values.

Different facilities have been established with the goal of improving the interaction among project system and subsystem engineers. Examples are the Project Design Center (Team X) at the Jet Propulsion Laboratory and the Concurrent Design Facility (CDF) at the European Space Agency (ESA).

While there are ongoing efforts to overcome the lack of interaction among engineers, much less work is being done to overcome the lack of knowledge. This is reasonable, since during the initial stage of the design when there is a long lead-time, it is often impossible to create a design with accurate prediction of future capabilities. However, even if we cannot avoid the uncertainties in the design, statistical instruments can be used to quantify and to mitigate the design risk in a mathematically tractable manner.

In this article, we describe a methodology that uses statistical analysis to quantify the risk posture of a system design. This approach enables engineers to evaluate the design risk of different architecture options, to perform risk–performance trade-off, and to make opportune design adjustments through different system design phases up to preliminary design review (PDR) and critical design review (CDR).

The statistical analysis framework uses a combination of two information sources: historical data and expert opinion. The information conveyed by data and by experts is elaborated through a cluster of different statistical techniques, with the objective of calculating the overall design risk.

The following is a summary of prior work that addresses the problem of design risk. Cortellessa [6] categorized the different possible design risks. Meshkat [1] analyzed design risk on the basis of her experience in Team X, and described different cases in which she observed “significant deviations in mass and power” from the initial system design to the final system design. Barrientos [7] explored the causes of design risks, and focused on the lack

of human interaction across engineers. Oberkampf [8] analyzed the causes of risks, while Asnar [9] developed qualitative risk analysis techniques to deal with the problem of selecting across design alternatives.

A more quantitative approach to design risk can be found in the work of Fuchs and Neumaier [10–12]. The authors described a statistical approach based on the creation of an n-dimensional cloud of uncertainties in which the different architectural solutions lie. The dimensions of the cloud are given by the different metrics on which the uncertainty is evaluated, and the cloud becomes smaller as the desired confidence in the solution increases (or risk decreases). The shape of the cloud is defined by expert opinion. This methodology represents the first attempt to statistically model the lack of knowledge in the space system design process. However, this methodology is multidimensional based on expert opinion only, while our approach uses a statistical combination of data and experts.

In terms of previous work in statistical estimation, relevant literature can be found in the fields of probability density estimation and expert elicitation.

In the area of density estimation, the different methods are described in the work of Fix and Hodges [13], Rosenblatt [14], Parzen [15], Loftsgaarden and Quesenberry [16], and Breimen, Meisel, and Purcell [17]. A description and comparison of different methods can be found in the monograph of Silvermann [18].

In terms of expert elicitation, the work of Tversky and Kahneman [19,20] is influential in the field of biases and heuristics in expert elicitation. Another important source is the work of Cooke [21], which focuses on building expert mathematical models to be used in science. Hagan's work [22,23] describes different ways to exploit expert opinion using Bayesian analysis, elicitation processes, subjective probabilities, calibration, and expert cooperation. Other important work in the field of expert elicitation is described in the papers of Garthwaite, Kadane, and Hagan [24], and Cain and Detsky [25].

The article is structured as follows: a methodology overview is presented in Section II. The details on the baseline model are described in Section III. The key statistical analysis techniques are outlined in Section IV, risk analysis in Section V, application examples are discussed in Section VI, and concluding remarks are presented in Section VII.

II. Methodology Overview

The key features of the methodology are summarized in the block diagram in Figure 1.

As inferred from Figure 1, the approach can be decomposed into different modules, each of which represents a specific part of the methodology.

The baseline design is the first conceptual block of the methodology. This block consists of three components:

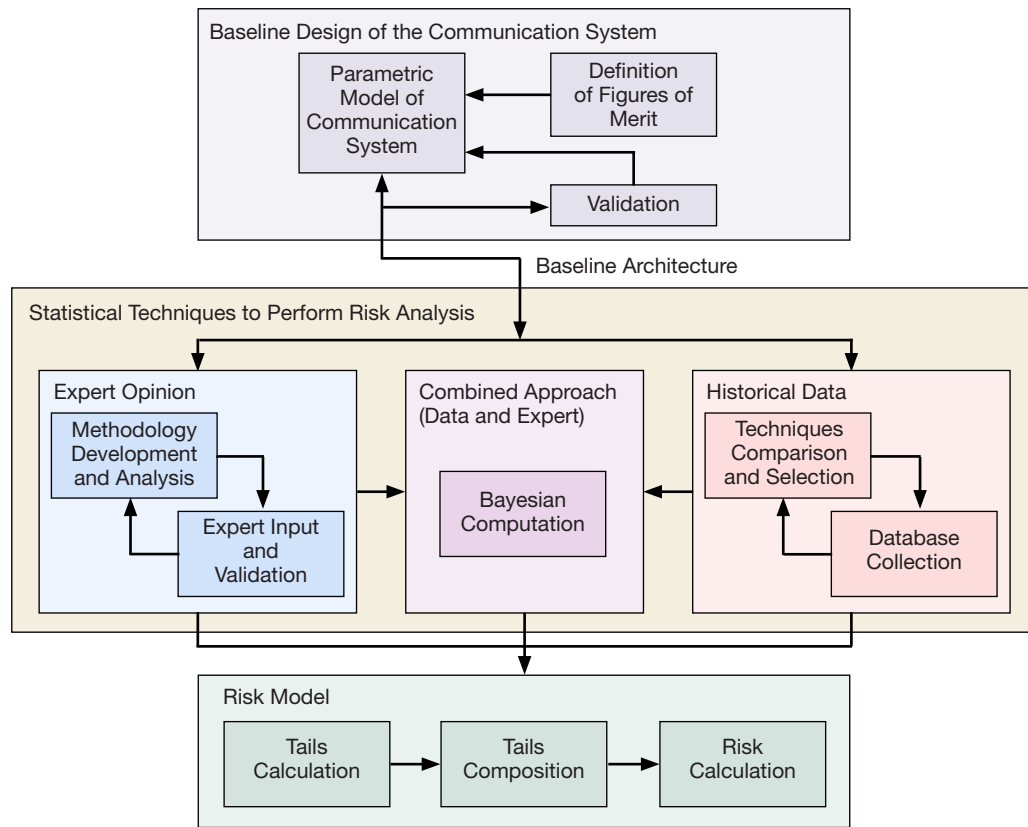


Figure 1. Methodology block diagram.

- (1) A parametric model of a communication system that receives as inputs a set of communication parameters (channels, frequencies, required link quality, weather assumptions, transmitter/receiver characteristics, etc.) and produces as outputs the initial estimates (average values) of communication system performance, mass, and power.
- (2) A list of figures of merit (FOMs) with design requirements that include, but are not limited to, communication performance, mass, and power.
- (3) A validation process that compares the outputs of the parametric communication model against the FOMs, and evaluates if a given set of input communication parameters would produce an architecture that passes the initial test of meeting or exceeding the design constraints.

The objective of this first block is to identify one or more communication architecture candidates that would satisfy the given design constraints. The baseline design tries to emulate what is generally done in the initial concept design phase before applying the contingencies. In the context of the research, this block is important because it represents the starting point at which the risk analysis will be applied to quantify the risk of each solution.

Once the baseline architecture is selected, different statistical techniques are employed to perform risk assessment. The block defined as “Statistical Techniques” collects all the techniques that are used to perform risk estimation. They can be divided into three different categories:

- *Historical data approach.* One way to quantify how the mass and power of a specific component vary over time is to perform statistical analysis of previous samples of similar components. The use of historical data requires different steps: the identification of the appropriate statistical techniques and models, the identification of the number of samples required (which is generally a function of the statistical technique selected), the construction of the database, and the use of the database to shape distributions.
- *Expert opinion approach (expert elicitation).* In the cases when there are not enough samples for classical probability techniques to construct a reliable statistical model, one might have to resort to the subjective opinions of one or more knowledgeable experts. The introduction of expert opinion in the model requires different stages: clear definitions of the design scenarios and the design constraints, a technique to calibrate the expert opinion, a methodology to elicit expert opinions, and an approach to compose multiple expert opinions when more than one expert is available.
- *Combined approach (data and expert).* In some cases, it is useful to combine both sources of information into a unique estimate. This can be done by applying the Bayesian techniques in which the database information is used to model the prior distribution, and expert opinion is used as a likelihood function to construct the a posteriori distribution.

Once the statistical technique to evaluate the risk is selected, one can compute the different tail probabilities (probability of exceeding a certain value) at the component level and at the subsystem level. The overall risk of the system can be estimated by “combining” all the individual tail probabilities.

III. Baseline Model

The baseline model is developed in a similar way as was the case with other analog communication system design methodologies (see Maral and Bousquet [26], Richharia [27], Evans [28], Brown [29], Wertz [5], and Gilchrist [30]). One distinction is that our model is paired with a coverage tool that calculates the minimum transmission rate required. In this way, the system is optimized because the data rate selected is the minimum required to accomplish a certain mission. The model is organized in three submodules: coverage module, link analysis module, and average mass and power calculation module. A description is provided in the block diagram in Figure 2.

The inputs for the baseline design model are number of communication channels required, central frequency for each channel, level of redundancy, transmitter and receiver characteristics, orbital parameters, simulation time, link quality requirements, mission data return requirements, and link margin.

The model performs link budget calculations for each of the communication channels to identify the system feasibility (similar to [31] and [32]), which is defined as the ability of the communication system to transmit the total data required in the specified interval of time. If the system is feasible, the model identifies for each channel a set of architectures solutions that provide an identical equivalent isotropic radiated power (EIRP) that meets the given performance requirements. Each of the solutions corresponds to a different optimized

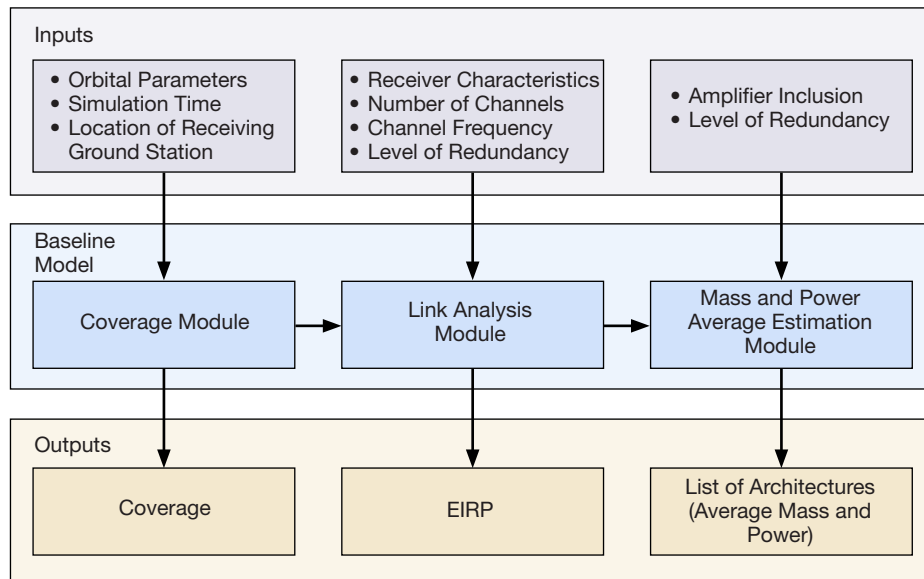


Figure 2. Block diagram for baseline design model.

combination of amplifiers, transceivers, and antennas that achieves the same EIRP for a specific channel. Finally, the results for the different channels are summed together to obtain a list of final possible architectures.

Any architecture represents a collection of components whose power and mass fluctuate over time. The risk model attempts to characterize the fluctuations so as to compute the design risks for the different architecture solutions.

The baseline parametric communication model generates the EIRP required to meet the performance requirements for each channel and a corresponding list of architecture options. The architecture option is associated with a list of constituent components; each is described with the baseline attributes of mass, power consumption (for passive components, it is set to zero), and its gain or loss contribution to the link budget performance.

A validation of the baseline model has been performed using data from current missions and commercial satellites. Specifically, the link analysis part of the model has been validated by comparing the EIRP calculated by the model with the one obtained from the technical documentation of the different missions. The results are shown in Figure 3.

Note that the values of the EIRP computed by the model are very close to those in the project documentation. The variations are due to the slight differences in the noise temperature calculations, or in the modeling of the spacecraft pointing errors.

A similar validation has been performed by comparing the values of mass and power computed by the model with the values obtained from technical documentation.

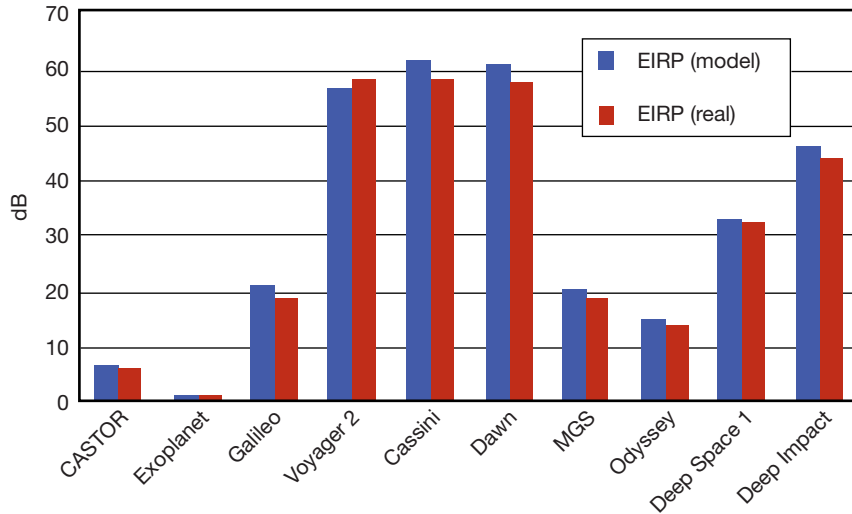


Figure 3. Baseline model validation.

IV. Statistical Techniques

The statistical techniques used in the model can be divided into three categories: database approach, expert elicitation approach, and combined approach.

A. Database Approach

The database approach uses historical and empirical mission data to estimate the mass and power distributions of a component. The reasons are:

- *Heritage.* Many missions inherit their components from previous missions. Hence, the final values of mass and power of components in prior missions can be used to develop a prediction for the mass and power values of components that are being considered in the communication system design.
- *Independence from human opinion.* Ideally, it is desirable to be able to develop analyses that depend as much as possible on data and not on human opinions/experience. However, in many cases, the use of expert opinion is inevitable (more details are provided in the following sections).

A prerequisite to constructing a useful database model is that there are enough samples to generate representative statistics for the parameters of interest. This problem can be challenging as components for space communication systems have hitherto low market demands, and are thus produced in low quantities. Traditional probability density estimation techniques are divided into two main categories: parametric and nonparametric estimations. In the case of this research, it is difficult to estimate the shape of the probability distribution because many factors are affecting the process. For example, if we consider the probability distribution of the mass of a component, it is influenced by functionality of the component, materials used, fabrication processes, etc. It is difficult to model all the effects with governing equations; thus, we cannot always assume a parametric distribution. It is therefore necessary to model the data through nonparametric density estimation.

Nonparametric density estimation techniques process observed data to construct an estimate of the underlying probability density function [18]. Many of these techniques can be shown to converge to the true density function when the sample size approaches infinity [43]. However, there are few works in the literature that assess the usefulness of these techniques when the sample size is small, which is the case for components of spacecraft communication systems. In this article, we develop an experimental approach to investigate the convergence rates of a given set of nonparametric density estimation functions for small sample sizes in the range between 10 and 100. The key steps are summarized as follows:

- We choose the following density estimation techniques for comparison: histogram, naïve estimator, kernel estimator, nearest neighbor estimator, variable kernel estimator, and saddle point estimator.
- To compare the “goodness” of the chosen density estimation techniques, we use the following known distributions as benchmarks: normal, exponential, lognormal, gamma, and beta.
- For a given probability distribution function $f_{PDF}(x)$, we define the tail function $T(x) = \int_x^\infty f_{PDF}(u) du$, and the estimated tail function $\overline{T_j(x)}$ that is constructed by processing the samples at j -th trial using a density estimation technique. We vary the sample size between 10 and 100. We further define two quantities to assess the goodness of each density estimation technique:

— The divergence across the k trials between real tail ($T(x)$) and estimated tail ($\overline{T_j(x)}$) for each point of the distribution (x):

$$\Delta(x) = \frac{1}{k} \sum_{j=1}^k |T(x) - \overline{T_j(x)}| \quad (1)$$

— The average divergence across the distribution for a certain amount of samples n (L1 metric):

$$\overline{\Delta} = \int x \cdot \Delta(x) dx \quad (2)$$

- For a given benchmark distribution $f_{PDF}(x)$, we compare the average divergence $\overline{\Delta}$ generated using different density estimation techniques for different sample sizes.

An example of the average divergence using the Gaussian benchmark is shown in Figure 4.

The experimental analysis reveals the following interesting facts:

- *Histogram*. Generally achieves the worst performance compared to any other technique.
- *Variable kernel estimators and nearest neighbor*. The performances vary strongly according to the benchmark distribution used (e.g., good with normal, bad with beta). These techniques can be helpful in some contexts, but without any previous knowledge of the “real” distributions, they can behave in an unpredictable way.
- *Saddle point estimation*. It achieves good performances for very small sample sizes (10 to 20) compared to the kernel density estimation; however, it does not converge as fast as the kernel techniques when the number of samples increases.

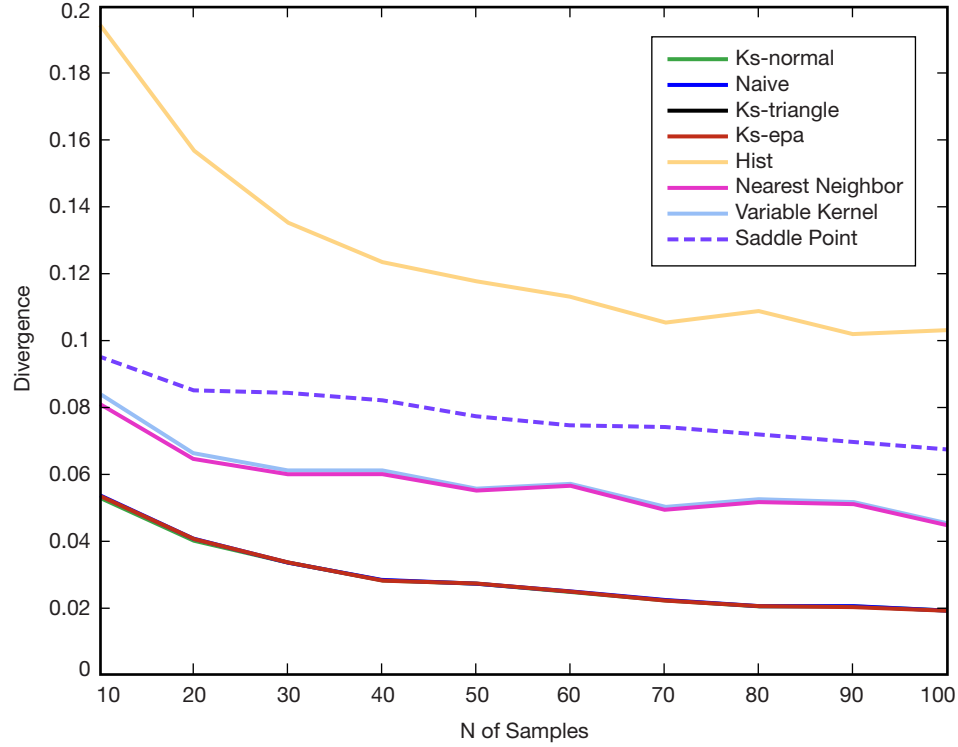


Figure 4. Average divergence in function of the number of samples using Gaussian benchmarks distributions.

- *Kernel density estimators and naive estimator.* They achieve the minimum divergence in most cases.

In conclusion, the numerical analysis discussed above shows a consistent trend on the relative goodness (in terms of average divergence) of different density estimators in the case of small sample sizes. Specifically, the kernel density estimation technique achieves the lower bound in average divergence for all the cases analyzed. For this reason, the kernel density estimation is the technique selected to model database statistics. In the Appendix, we derive the following properties of the kernel density estimator (KDE) that are relevant to risk estimation:

- The mean computed from the KDE is the sample mean, which is an unbiased estimator for the underlying density function. [Equation (A-3)]
- The variance computed from the KDE is the sum of the sample variance and h^2 , where h is the smoothing parameter of the kernel function $K(x)$. This and the above properties imply that the probability distribution function generated by the KDE always overestimates the risk probability. [Equation (A-4)]

Note that Equations (A-3) and (A-4) are general expressions that apply to all KDEs. As all KDEs and the naive estimator¹ use the same h for a given sample size n , their divergence should be identical. This is indeed the case, as shown in Figure 4.

¹ The naive estimator is a form of KDE, with the kernel being a uniform distribution function.

- In general, h decreases in value as n increases. Thus, the overestimation of risk is more severe for small n . In the case of the Gaussian kernel estimator, the overestimation factor of variance compared to the sample variance can be shown to be $\left(\frac{4}{3n}\right)^{\frac{2}{5}}$, and is illustrated in Equation (A-5) and in Figure A-1. Note that $h \rightarrow 0$ as $n \rightarrow \infty$, but the convergence rate is slow for large n .

To establish a database for spacecraft communication system components (and spacecraft systems and subsystems in general), the key challenge has always been overcoming the difficulties in locating the details of engineering specifications of the components. The manufacturers of spacecraft components do not typically reveal these data in the open literature. Up to now, data have been collected from different literature sources. Data are extracted from sporadic articles on the design of specific components for communication systems [29,30,33–35]. Another source of data comes from JPL’s Deep Space Communications and Navigation Systems Center of Excellence (DESCANSO) publications: a collection of design documents for the JPL communication systems of the different missions, e.g., Voyager, Galileo, Cassini, etc. Each document [2–4,36–42] contains a detailed description of the spacecraft communication system, as well as the mass and power consumption data for each component.

The database approach uses samples data to construct the probability distributions of the mass and power of a component. However, one challenge is that the mass and power of a component depends on many other factors. For example, the mass of an antenna depends on the gain and on the frequency. Hence, if we have in the database the mass of a particular antenna, these data can be applied only to estimate masses of antennas with the same gain and frequency. This would require collecting an enormous amount of data, which is not feasible in the area of spacecraft system and subsystem design. In order to avoid this effect, we propose the following parametric approach.

Each sample collected is converted into a coefficient that represents a relation between two parameters. For example, instead of collecting the mass values of the antennas for all the possible gains, the mass data collected are converted to mass per unit of gain. In this way, for the same frequency or band, samples corresponding to antennas with different gains can be used in the same data set that creates the mass distribution of an antenna with the same frequency. The density construction process goes as follows: First, we specify the design metric of the component that needs the probabilistic characterization. For example, if we estimate the probability distribution of the mass of an antenna, the design metric would be the antenna gain. Other inputs include the type of component (e.g., an antenna, a transceiver, an amplifier, etc.), its frequency band, and its category (low-gain antenna, medium-gain antenna, high-gain antenna). Type, category, and frequency band are used in the database to identify the data set that is relevant to a specific type of component. When the vector is extracted from the database, the value of the design metric is used to scale the distribution. At that point, a probability distribution is generated using the kernel density technique. The mathematical formalization of the approach is presented in the following equations, where \bar{a} is a vector of scalar coefficients of mass per unit of gain (G), f is the probability density for the mass of a component, x is the support of the distribution, and k is the kernel density estimator:

$$f(x) = \frac{1}{G} \cdot f(\bar{x} \cdot G) \quad (3)$$

The function k (kernel density estimator) can be expressed as:

$$k(x, \bar{a} \cdot G) = \frac{1}{n \cdot h} \sum_{i=1}^n K\left(\frac{x - a_i \cdot G}{h}\right) \quad (4)$$

where n is the total amount of elements of vector \bar{a} , and a_i represents any single element of the same vector; h is the bandwidth of the kernel estimator, and it is selected to optimize the estimation by reducing the minimum square error [43]. The function K is the kernel: a symmetric but not necessarily positive function that integrates to one. In our testing, multiple kernel functions have been considered, and a normal kernel was selected because it achieves the best performance according to Figure 4.

The result is a probability distribution that takes into account the historical data (using a set of coefficients) that corresponds to the design metric for which the specific component will be designed. Following this procedure, data can be used for different statistical characterizations without the necessity of accumulating too many samples for different specific designs. The only caveat in this approach is that the mathematical operation performed is a transformation of a probability density, which holds true only if the two variables considered (here, mass and gain) have a linear relation with each other. For some components, this is true: Brown [29] shows that for the traveling-wave tube (TWT) amplifiers, there is a linear relation between mass and output power, and between input power and output power. However, in some cases, the relation between the two quantities is not necessarily linear, such as in the case of the antennas. To solve this problem, the nonlinear function is decomposed in a set of linear approximations through a convex piecewise linear approximation. This corresponds to breaking the nonlinear curve into multiple pieces, which, in the case of the antenna, corresponds to different regimes of antenna gain: low gain, medium gain, high gain. This is the reason why one of the inputs of the database model is the component category. When we search for the set of coefficients in the database required to construct the distribution, it needs to know which category the antenna belongs to, in order to select the coefficients that correspond to the correct linear piecewise approximation.

B. Expert Elicitation Approach

The aforementioned database techniques adhere to the “classical” or “frequentist” interpretation of probability, for which the probability density function is estimated from historical data only. Classical statistical methods are ill suited for the situations when the sample size is small.

In the area of spacecraft communication system design, there are experts in the field who can provide subjective assessments of the mass and power of future components based on their experience and their understanding of the market trends, product availability, and technology readiness. In this section, we explore expert elicitation techniques that extract and incorporate expert opinions in the risk-based system design process.

Before describing the approach, the reasons to use expert elicitation in spacecraft system design are given as follows:

- *Required sample size.* Classical probability theory shows that representative statistics can only be estimated from a sample when the sample size is large enough.² Most spacecraft components, which include communication system components, are usually produced in small quantities and do not constitute a large enough sample size.
- *New components.* Completely new components are sometimes tested or used in space missions. If a component is first of a kind, obviously no statistic is available. In this case, a different approach is required.
- *Rapid technical advances.* Some spacecraft components evolve so fast that historical data are not useful to generate representative statistics. Examples are onboard processing units and flash memory. Generating the mass and power statistics for these kinds of components using a historical database will tend to produce overestimations.

Expert opinion can be viewed as a form of external knowledge that is introduced in the model, with the objective of substituting or improving the knowledge given by the historical data. However, as O'Hagan [22] points out, expertise also involves how the person organizes and uses the knowledge. In fact, to have an effective elicitation of the expert's knowledge, the expert has to be able to express his or her uncertainty of the knowledge accurately.

In order to properly model expert opinion, it is necessary to:

- *Identify possible biases and heuristics that the expert may have in expressing his or her knowledge and the corresponding uncertainty.* Biases can be defined as the tendency to make systematic errors in certain circumstances based on cognitive factors rather than on evidence. Heuristics are simple, efficient rules that are applied to make decisions, to express judgments, and to solve problems, typically when facing incomplete information. These rules can lead to systematic errors.
- *Identify the level of adjustment required to calibrate the expert opinion.* In the field of expert elicitation, calibration is a measurement of the agreement between expert opinion (subjective probability) and observed relative frequency. For example, if we asked a weather expert to guess rain probability for a year on the basis of available data and we record the real rain frequency for that year, we can assess how well our expert is calibrated, that is, whether he or she would tend to underestimate or overestimate probabilities.
- *Elicit the probability models from experts.* This last part involves the selection of appropriate statistical quantities that can accurately express the belief and the uncertainty of the experts. They can be bounds, mean, variance, quantiles, etc. In the case of this research, two possible elicitation techniques will be used.

To accomplish the three above steps, we developed a three-part interview technique to calibrate and to extract probabilistic modeling information from the experts.

² Examples include the law of large numbers, Chebyshev's inequalities, and Markov's inequalities.

Part 1 of the interview is called Probabilistic Thinking. This part contains general questions (a total of 16 in this case) on probability that are used to model the expert's ability to think probabilistically. The result of this part is the generation of a quality index that can be used to compare and to weigh opinions from different experts. This part of the interview is focused on mapping the possible biases and heuristics that experts can have. Specifically, the following biases are identified and mapped:

- *Hindsight bias*. It can happen when a person is asked to assess the a priori probability of an event that has actually occurred.
- *Small sample bias*. A person sometime forgets that the size of a sample affects the probability of obtaining certain results.
- *Judgment by availability*. It is used when a person judges the probability of an event according to the ease with which similar events or instances are called to mind.
- *Judgment by representativeness*. Instead of evaluating the conditional probability, a person might tend to judge on the base of the similarity between events.
- *Awareness of underlying conditional probabilities*. It is a bias that occurs when a person does not consider how underlying probabilities can affect conditional probabilities.
- *Judgment by anchoring and adjustments*. It happens when a person is asked to estimate a quantity; he or she might start with an initial estimate ("anchor") and then just makes minor adjustment with respect to the initial estimate as the person's thinking progresses.
- *Coherence*. It is a check on whether a person respects the basic coherence laws of probability (the sum of all probabilities adds to 1, and the intersection of independent events is the product of the respective probabilities).

The result of part 1 is the generation of a total score (S) between 0 and 100 that is the sum of the scores the expert would achieve after answering all the questions. The quality index (Q) is the normalized score that can be used to weigh multiple expert opinions.

$$S = \sum_{i=1}^{16} S_i \quad 0 \leq S \leq 100 \quad (5)$$

$$Q = \frac{S}{100} \quad 0 \leq Q \leq 1 \quad (6)$$

Part 2 of the interview is called Calibration. This part contains technical but general questions on mass and power for communication system components. Experts are required to estimate the mass and power for typical components, and their answers are used to identify if the expert has the tendency to underestimate or overestimate certain quantities. The interview in this part results in a calibration coefficient (C) that can be used to shift the final probability assessments performed by the experts. For each of the questions in the calibration part, the real values of mass and power (r_i) are known. Hence, given the expert estimate (e_i), each calibration coefficient can be computed as

$$c_i = \frac{e_i - r_i}{r_i} \quad (7)$$

The final calibration coefficient (C) is computed as the average of the individual coefficients:

$$C = \frac{\sum_{i=1}^n c_i}{n} \quad (8)$$

The last part of the interview is the Expert Elicitation. In this part, the interview aims at helping the expert to express his or her belief on the mass and power values for certain components and their corresponding uncertainties, and translate the expert's estimate into a density function. Two different approaches are chosen to construct the density function based on the expert's belief and uncertainty of a component: an approach derived from statistical link analysis, and the quantile method.

The approach derived from statistical link analysis [2] first provides the initial design value of the component as the starting point. The interview will then elicit three quantities from the expert: lower bound, upper bound, and the form of the distribution (uniform, triangular, or normal).

In the quantile method, the expert elicits the 50 percent quantile, the 16 percent quantile, and the form of the distribution (uniform, triangular, or normal).

A recursive tool allows the expert to visually check the shape of the distribution that results from the elicitation process, and to modify it until he or she identifies the one that properly expresses his or her belief.

The key aspects of the methodology are summarized in the triangular scheme described in Figure 5.

The interview includes 16 questions for Probabilistic Thinking, 17 questions for Calibration, and multiple test cases for Expert Elicitation. The duration is approximately 1 hour and 30 minutes.

The interview was performed on three different populations: MIT undergraduate students, MIT graduate students, and JPL engineers. The students are not experts in the design of spacecraft communication systems; hence, they were tested only for part 1 (Probabilistic Thinking). The JPL engineers participated in all three parts of the interview. The data collected from the interview are still being processed, and we show some preliminary results in this article.

Figures 6, 7, and 8 show the total score for Probabilistic Thinking across the three populations: undergraduate students, graduate students, and JPL engineering, respectively.

The results are relatively close to each other, proving that the test works well across different populations. When categorized by number of experiences in probability training (each experience can indicate a class or a period of research in which the subject intensively used

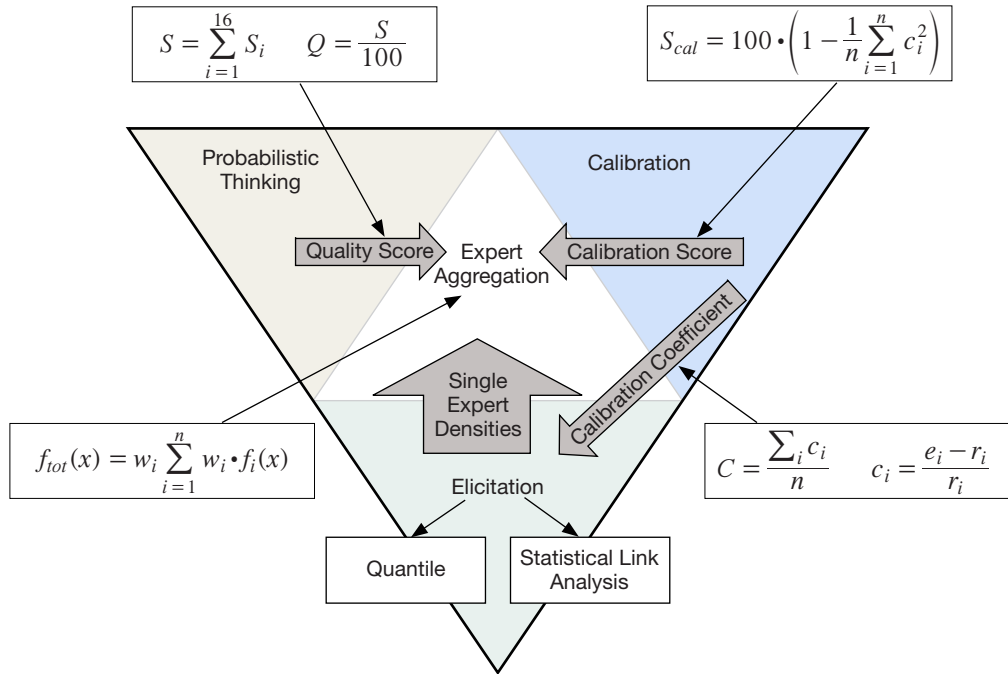


Figure 5. Summary of expert elicitation methodology (expert triangle).

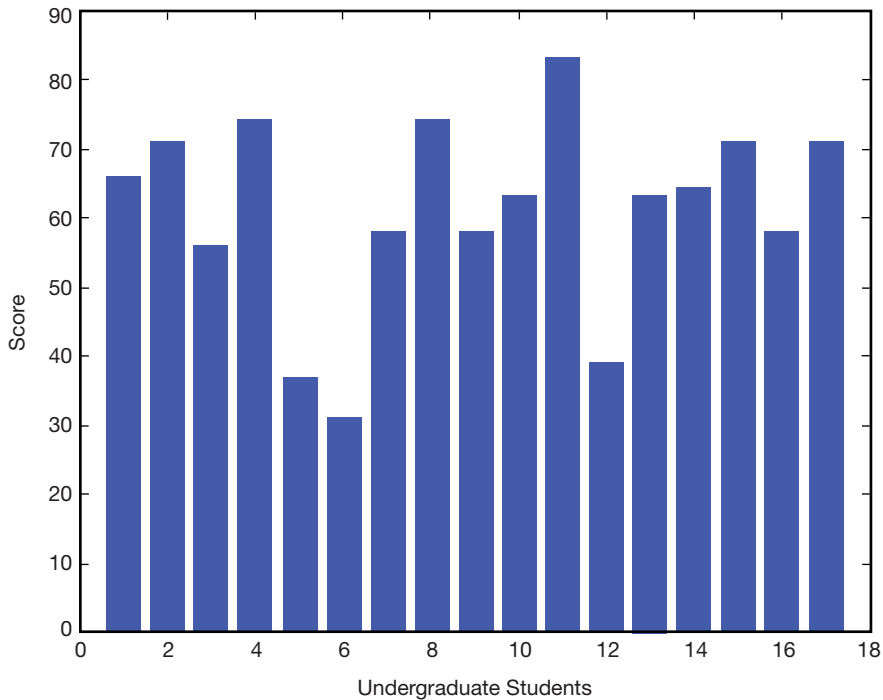


Figure 6. Quality score for part 1 (undergraduate students). Total score: mean is 61 and variance is 198.5.

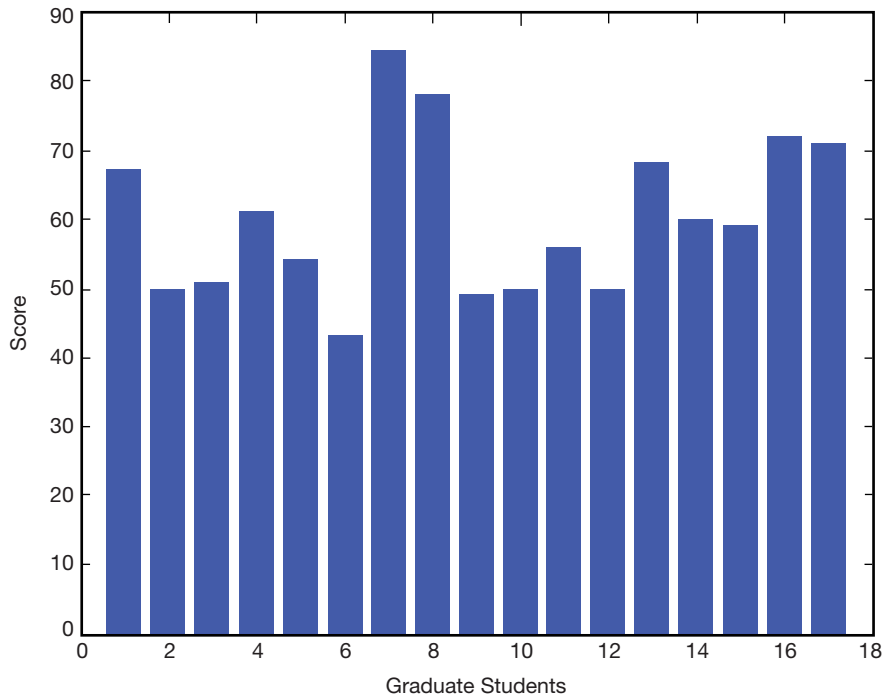


Figure 7. Quality score for part 1 (graduate students). Total score: mean is 60.1765 and variance is 132.6544.

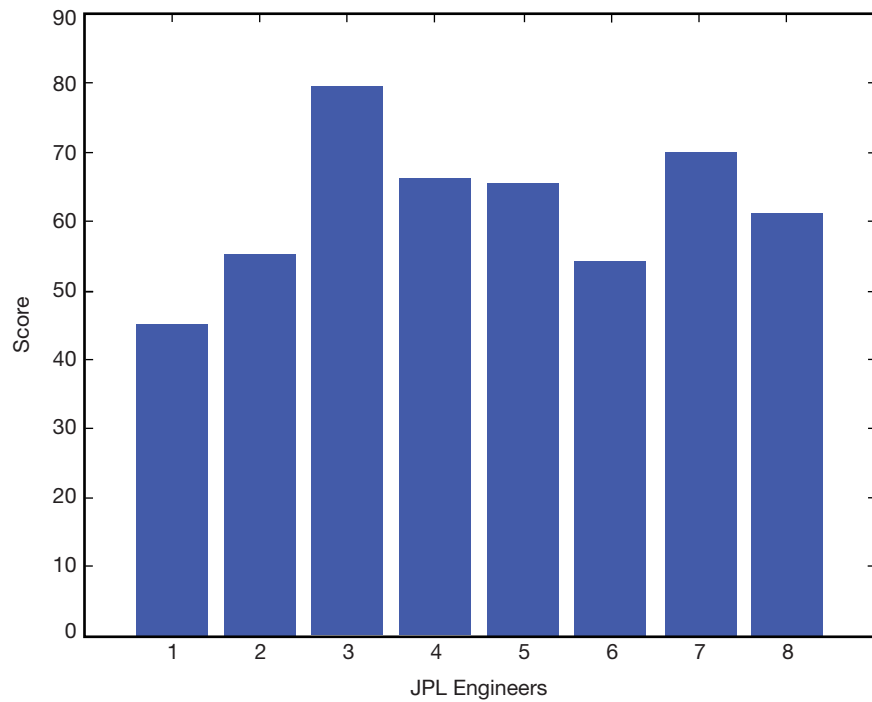


Figure 8. Quality score for part 1 (JPL engineers). Total score: mean is 61.875 and variance is 111.5536.

probability), all categories show similar dispersions of quality scores. This is shown in Figure 9 and suggests that ability in probabilistic thinking is independent from the experience in probability.

The calibration coefficients are shown in Figure 10.

This coefficient is an average across all the coefficients computed for the different calibration questions in part 2. The final coefficients (Figure 10) are all positives, which indicates that the experts generally tend to overestimate quantities. This result is not surprising, as engineers tend to be conservative in their risk estimates.

To evaluate the elicitation part of the interview, test cases have been used. They correspond to missions already developed for which the authors know all the data (from initial concept design up to CDR). The results on the elicitation part are discussed in Section VI on application examples.

C. Database and Expert Integrated: Bayesian Approach

The third possible way to assess risks for mass and power fluctuations is to combine the sources of information given by data and by expert elicitation. We propose to use the Bayesian framework that treats the data statistics as the prior, and the distribution that results from the expert elicitation process as the likelihood function. The resulting posteriori distribution represents the combined estimate:

$$f_{combined}(x) = \frac{f_{data}(\vartheta) \cdot f_{expert}(x|\vartheta)}{\int f_{data}(\vartheta) \cdot f_{expert}(x|\vartheta) d\vartheta} \quad (9)$$

where x is the support of the distribution and ϑ is the set of data (from the database) on which the kernel density is computed. This part of the approach is still under study, so it will not be applied to the examples discussed in Section VI.

V. Risk Analysis

In the context of designing a spacecraft communication system that meets the given mass and power allocations, the risk analysis method receives as input a given design and estimates the probability that the overall mass or power of that design would exceed the given allocations. This probability is computed using one of the three techniques previously discussed: database approach, expert elicitation, or a combination of the two.

In the case of the database approach, density estimation is used to compute the probability distributions. Currently, the size of the database is limited to no more than 40 samples for each category of components, and the samples tend to take on a wide range of values. The distribution constructed from these samples is inclined to have a larger dispersion. This problem will be discussed more in the application examples in Section VI, but it is one reason for which the use of expert opinion is strongly encouraged.

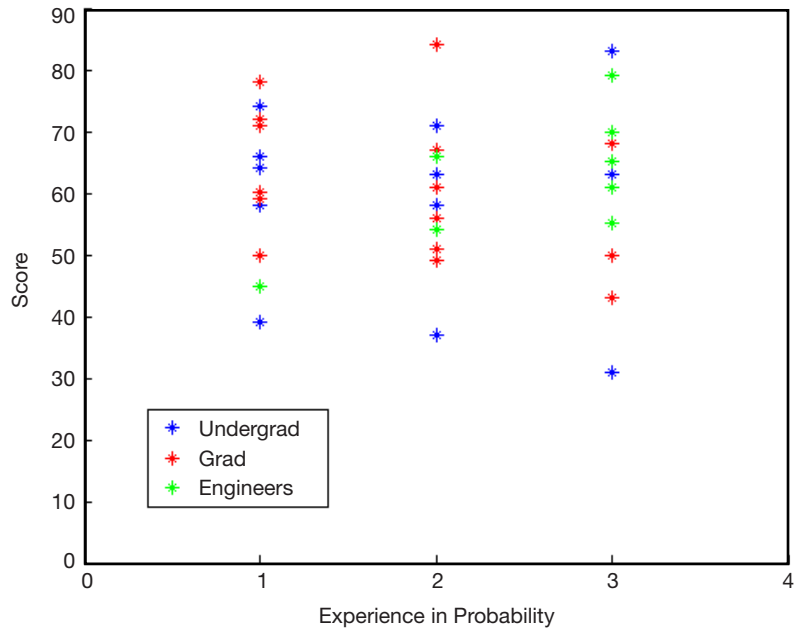


Figure 9. Quality score vs. experience in probability for the three populations.

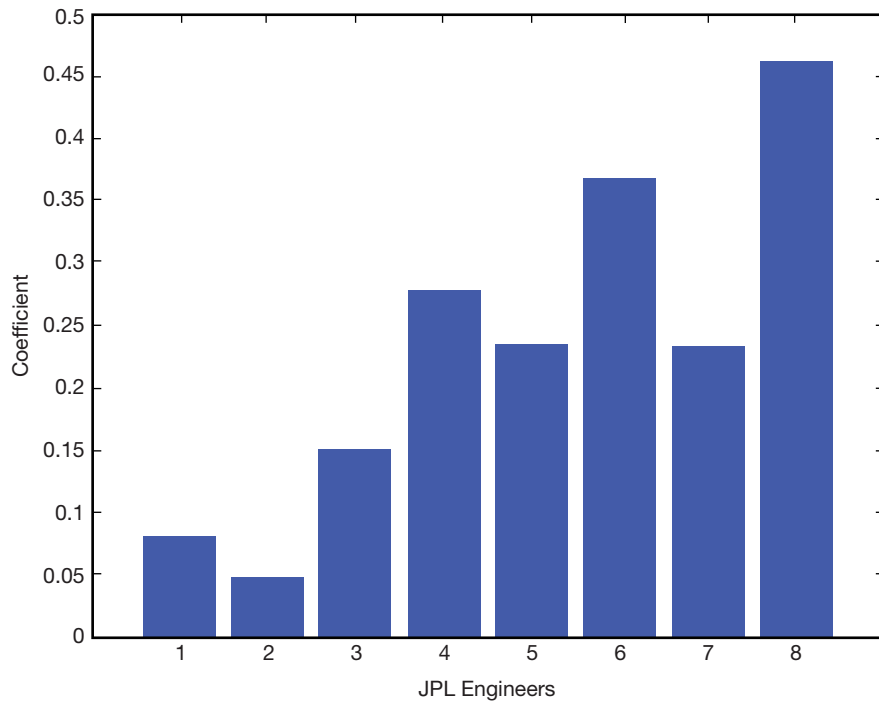


Figure 10. Average calibration coefficient for engineers (part 2). Total calibration coefficient: mean is 0.23079 and variance is 0.019651.

In the case of the expert elicitation, the accuracy of their judgments is difficult to measure, but the three-part interview that includes quality estimation (part 1) and calibration analysis (part 2) can be used to quantify the expert's precision. An additional advantage of using expert elicitation is that when an expert is confident, he or she would impose a smaller uncertainty range with respect to his or her assessment, and this helps to generate a "narrower" distribution function compared to the one constructed by the database approach (more details on this aspect are in Section VI).

The combined approach is currently under development; hence, the application examples will include mostly results derived from the database approach and expert elicitation only.

The next section contains a mission example of the methodology discussed.

VI. Application Example – CASTOR

The example discussed in this section is based on a mission developed at Massachusetts Institute of Technology called Cathode/Anode Satellite Thruster for Orbital Repositioning (CASTOR).

CASTOR³ is a small Evolved Expandable Launch Vehicle (EELV) Secondary Payload Adapter (ESPA) ring class satellite developed at MIT Space Systems Laboratory. The satellite dimensions are $50 \times 50 \times 60$ cm for a total mass of 50 kg. The main goal of this spacecraft is to test the performance of a new type of electric propulsion engine, the Diverging Cusp Field Thruster (DCFT) in the space environment. The DCFT is able to guarantee up to 1 km per second of ΔV . This type of engine is very efficient in terms of mass/impulse ratio, and the whole system is capable of performing rapid orbital transfer maneuvers. The deployed configuration of the CASTOR satellite is shown in Figure 11.

The CASTOR bus has been developed entirely at MIT over a four-year period. This makes CASTOR a suitable example for this analysis, as all the data of the spacecraft communication system that evolved over time are available⁴ [32]. Specifically, the data in which we are interested are:

- *EIRP required.* This input is important to validate the baseline model.
- *Configuration of the communication system.* The types and number of antennas, the types and number of transceivers, and any other additional components.
- *Initial values of mass and power at the PDR level for the components of the communication system.* These values are used to perform the risk estimation using the database approach and the expert opinion approach.
- *Final values of mass and power at the CDR level for any components of the communication system.* These values are used to evaluate the statistical estimation methodology that was developed.

³ *CASTOR Design Document* (internal document), Massachusetts Institute of Technology, Cambridge, Massachusetts, 2011.

⁴ *Ibid.*

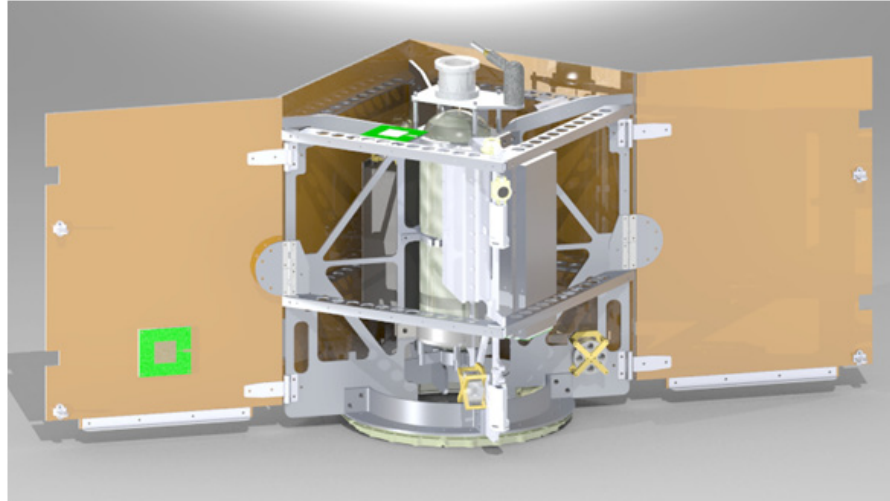


Figure 11. CASTOR satellite.

The CASTOR communication system is a fully redundant system composed of three antennas and two modems. The three antennas are custom-built patch antennas, each with approximately 6 dB of peak gain. The modems are Microhard MHX2420, a commercial-off-the-shelf (COTS) product. The configuration of the system is the following: one antenna is connected directly to one of the modems, while the other two antennas are connected to the second modem through a passive splitter.

The operating frequency of the system is in the S-band, and the ground stations are the same ones used by the High-Energy Transient Explorer (HETE) missions. These consist of a network of ground stations owned by MIT's Kavli Institute. The ground stations are located in Singapore, Kwajalein, and Cayenne, French Guiana.

The CASTOR mission is used in conjunction with other missions to validate the baseline model, as shown in Section III. In this section, the CASTOR mission is used to perform the risk assessment. The risk analysis investigates the fluctuations of the mass and power values of the spacecraft antennas and the transceivers over time. The results are shown in the following subsections.

A. CASTOR Antenna

The three antennas are all identical; hence, we discuss the mass fluctuations of one of them.

The antenna has passed through different design phases. Initially, it was supposed to be a COTS patch, but then, due to a vibration concern, the team resorted to a custom-made antenna. The material selected for the mission changed across the different versions until the final configuration was reached after the CDR.

The initial mass of the antenna was estimated to be around 0.5 kg, while the final value became 0.1 kg. This shows that the initial estimate was clearly overestimated.

The risk analysis was performed using the database approach and the expert opinion approach. For the database statistic, the following steps were followed:

- *Definition of the component type and category.* In this case, it is an antenna that belongs to the low-gain category. This information is used to select the correct vector of data in the database.
- *Definition of the frequency.* In this case, S-band.
- *Definition of the performance metric.* In this case, the performance metric is the peak gain of the antenna pattern, which is used to scale the vector of data in the database.
- *Estimation of the final probability density function of the antenna mass and the evaluation of its corresponding tail.*

For the expert opinion approach, the experts were asked to elicit the probability distribution that the mass of the antenna would exceed the value of 0.5 kg. Different experts have been tested, but since most of the data are still under analysis, we refer only to one of them as an example. The work on composing different expert opinions is ongoing. The expert being interrogated understood immediately that the initial value of the antenna mass was an overestimation. The expert assessment for the probability distribution of the antenna mass was performed using the quantile method, and it resulted in a triangular distribution centered at 0.25 kg, and bounded between 0.05 kg and 0.5 kg, respectively.

The information obtained from the database approach and the expert elicitation approach was used to estimate the probability distribution ($f_{PDF}(x)$) functions and the tail function, which is defined as

$$T(x) = \int_x^{\infty} f_{PDF}(u) du \quad (10)$$

Tail functions are popular in risk assessment applications, since they can express risk in a very visual way. The y-axis represents the risk of exceeding a certain mass value indicated on the x-axis.

The results of the risk estimation performed using the database approach and the expert approach are shown in Figure 12 (probability densities) and in Figure 13 (tail function).

Specifically in Figure 12, both densities (database and expert) have peaks that are closer to the final mass value at CDR than in the PDR. This is important as it shows that both methods are effective in speculating the eventual value, with the database curve providing a better prediction than the expert curve. However, in the database approach, the density function exhibits a stretch-out shape with a fat tail (or heavy tail) as shown in Figure 13, indicating an over-estimation of the mass margin required to retire a given design risk. For example, if the risk requirement of not exceeding the mass allocation is 0.1, the mass margin computed from the database method is 0.7 kg, whereas the mass margin computed resulting from the expert opinion approach is 0.3 kg. We believe that this is due in part to the positive bias on estimating variance using the KDE method, as described in Section IV.

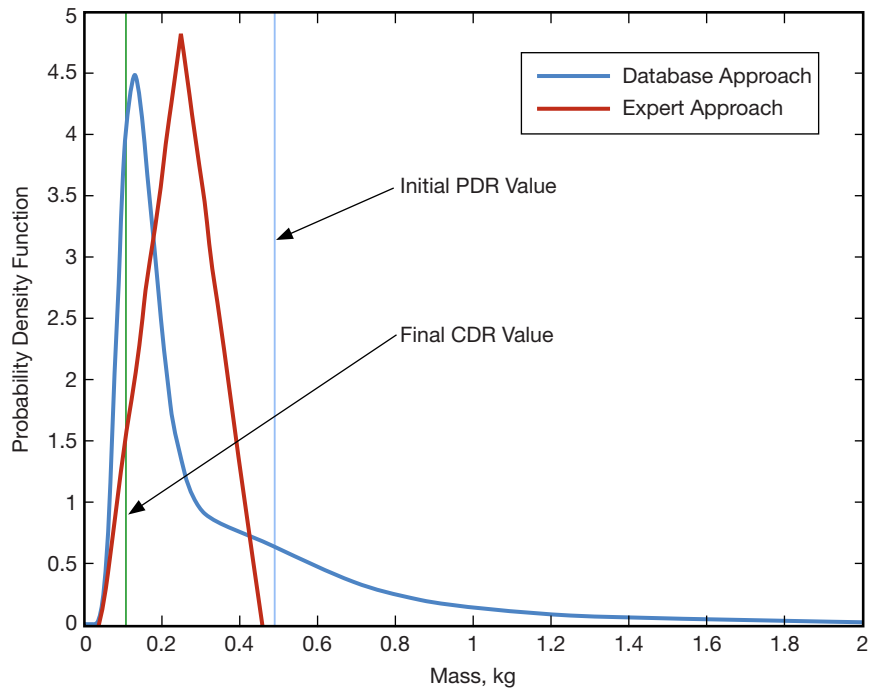


Figure 12. Probability density function for CASTOR antenna mass (database approach vs. expert approach).

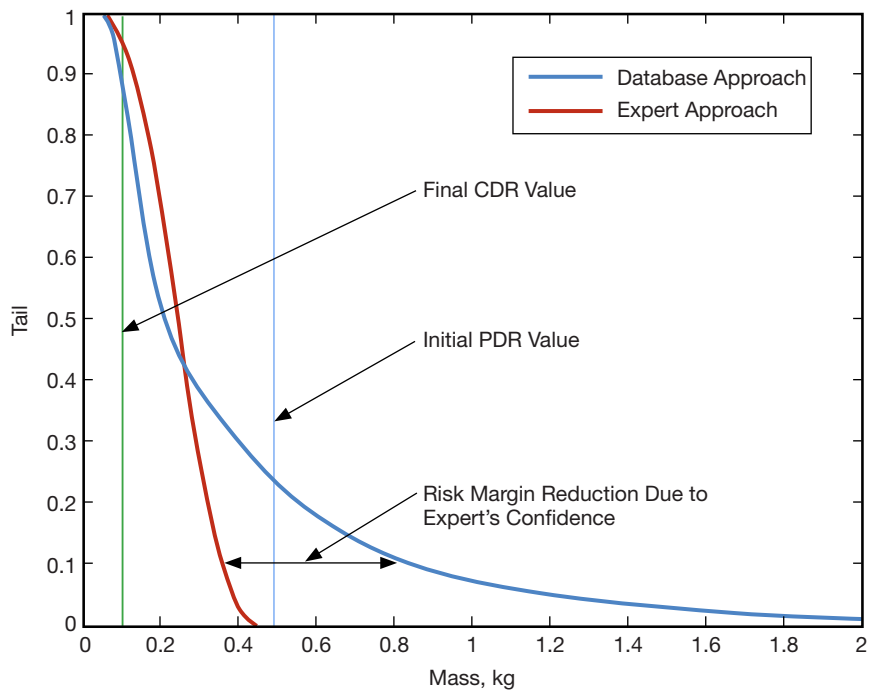


Figure 13. Tail function for CASTOR antenna mass (database approach vs. expert approach).

In summary, both the database and the expert opinion methods generate density functions whose peaks allow us to recognize that the initial design value of 0.5 kg is overestimated, and that the expected value for the antenna mass should be less. The database approach identifies the mass value that is closer to the CDR value than the expert opinion approach. However, when it comes to risk quantification (tail graph), the expert opinion density function offers a much smaller mass margin than the database approach for the same level of design risk.

A similar analysis has been performed for the transceiver that is discussed in the next section.

B. CASTOR Transceiver

An analogous analysis has been performed with the CASTOR transceiver, and the results lead to very similar conclusions. The component is a COTS product from Microhard with an output RF transmitting power of 1 W. For this component, the initial values for mass and power consumption at the PDR stage were 0.11 kg and 1.5 W, respectively. The final values after the CDR were 0.05 kg and 4.5 W. As observed, the initial estimate was an overestimation for the mass and an underestimation for the power. The underlying reason for the fluctuations in the values is mainly due to misunderstandings with the company. The component was already developed and built, but the data sent from the company were incorrect and the final values were assessed only when the component arrived at the laboratory.

As in the antenna case, the risk analysis for the transceiver was performed using the database approach and the expert opinion approach. In the database approach, the steps performed are:

- *Definition of the component type and category.* In this case, it is a low-power transceiver. This information is used to select the correct vector of data in the database.
- *Definition of the performance metric.* In this case, the performance metric is the output transmitting power (1 W), which is used to scale the vector in the database.
- *Estimation of the final probability density function and tail function for mass and power consumption of the transceiver.*

For the expert opinion approach, the experts were asked to elicit the probability distribution that the mass of the transceiver would exceed the value of 0.1 kg, and that the power consumption would exceed 1.5 W. The expert assessed the mass of the transceiver being distributed as a normal distribution centered in 0.15 kg with a 16 percent quantile at 0.11 kg. For the power consumption, the expert estimated a uniform distribution between 3.5 W and 5 W.

The probability density functions and the tail functions for the transceiver mass are shown in Figure 14 and Figure 15, respectively. The probability distribution functions and the tail functions for the transceiver power consumptions are given in Figure 16 and Figure 17, respectively.

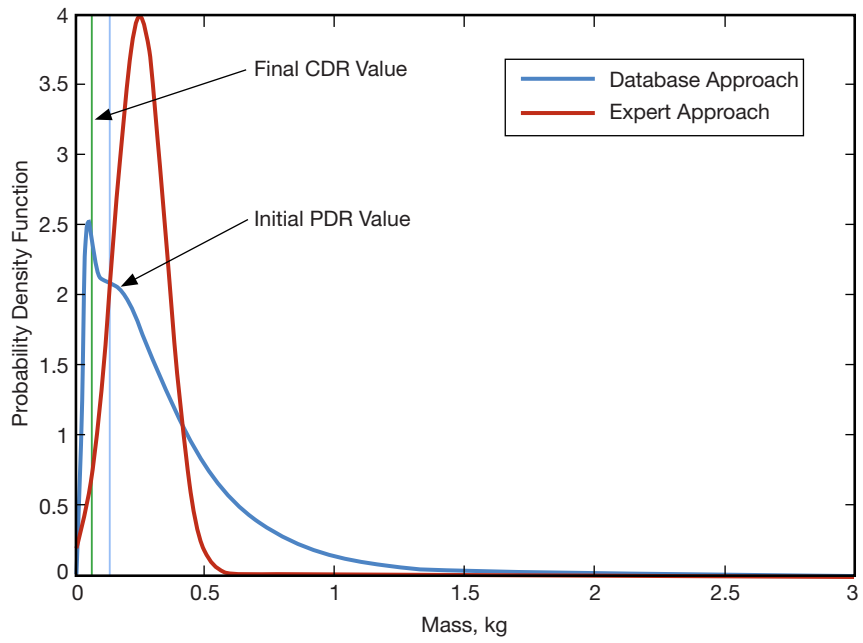


Figure 14. Probability density function for CASTOR transceiver mass (database approach vs. expert approach).

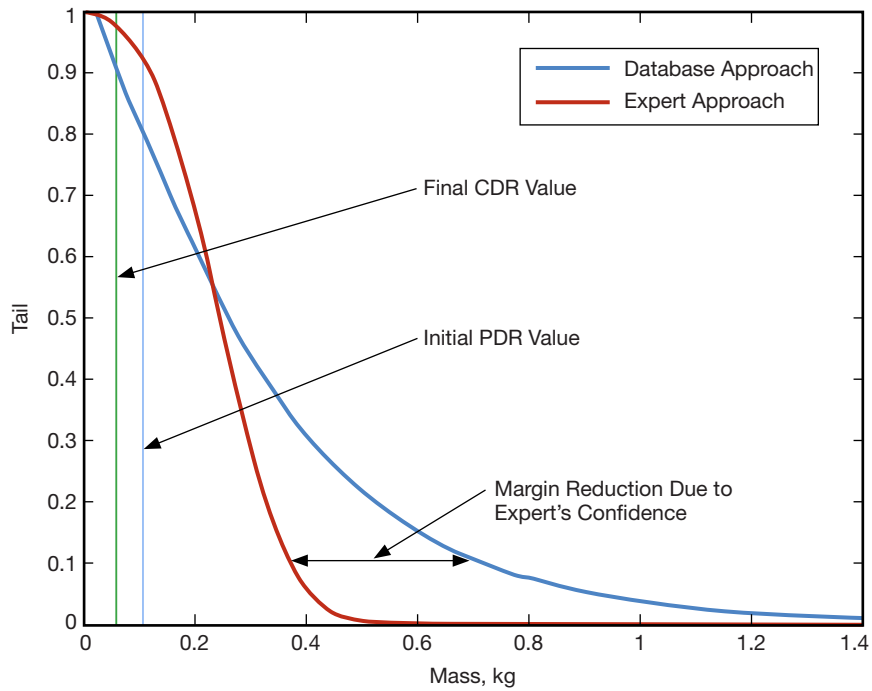


Figure 15. Tail function for CASTOR transceiver mass (database approach vs. expert approach).

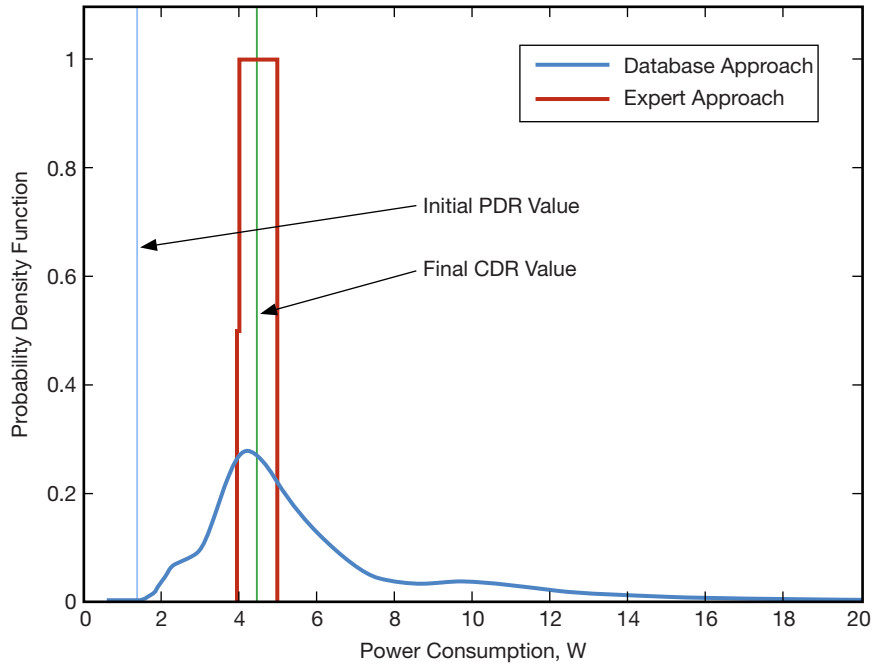


Figure 16. Probability density function for CASTOR transceiver power consumption (database approach vs. expert approach).

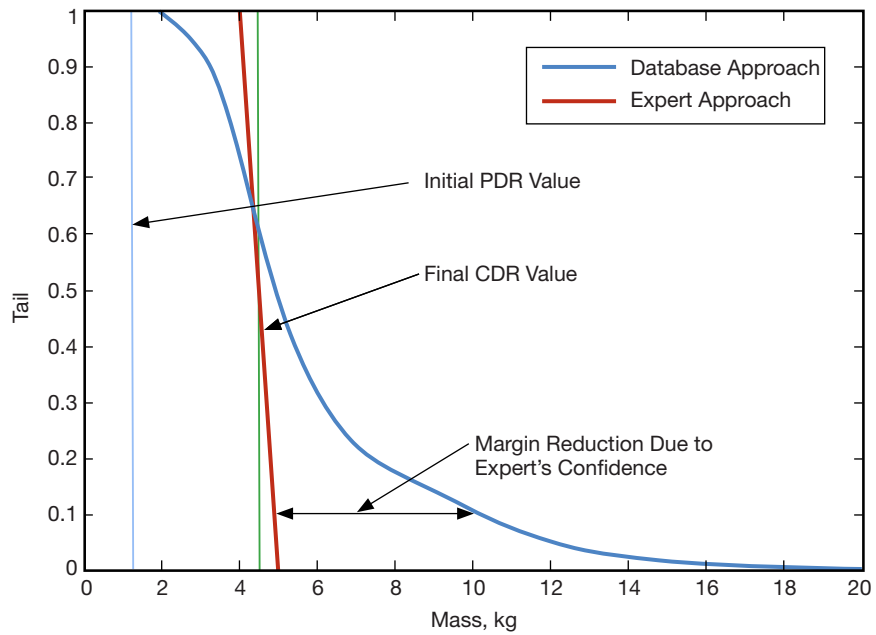


Figure 17. Tail function for CASTOR transceiver power consumption (database approach vs. expert approach).

We notice that the peak of the database probability density (Figure 14) is closer to the final CDR value, while that of the expert opinion distribution overestimates the transceiver mass. However, as in the antenna mass case, the opinion approach generates a narrower distribution than the database approach, and the overall effect is that it requires a smaller transceiver mass margin, 0.2 kg versus 0.6 kg, to mitigate the risk of exceeding the mass allocation at 0.1 risk level (Figure 15).

In the case of the power consumption (Figure 16), the peaks of both probability densities are close to the final CDR value, showing that the initial guess on the power consumption was clearly an overestimation. The tail functions (Figure 17) show once again that the expert approach is helpful to reduce the power margin compared to that using the database approach.

The conclusion from this analysis is that the database approach seems to provide good insights as to what the final value of component mass should be. This is evident by the fact that in both cases (antenna and transceiver), the peaks of the probability density functions are close to the final values of the system. However, the weakness of the database approach is that, due to the limited number of scattered data, the distribution constructed typically reveals a wide shape, and requires more margin to mitigate a given level of design risk.

On the other hand, probably due to the conservative mindset of most engineers, expert opinion tends to overestimate the mass and power consumption, as we observed in the above CASTOR antenna and transceiver experiments. However, an expert's confidence helps to limit the variance (or spread) of the distribution, thus reducing the required margin needed to retire the design risk.

Since both techniques have advantages and weaknesses, future work will focus on a Bayesian approach that combines both sources of information in a mathematically tractable manner.

VII. Summary

This article describes an approach to quantify the design risks for communication systems. The approach includes a baseline model that performs a preliminary design of the system. The baseline tool is useful in cases when a preliminary design is not yet available, or as an instrument to check if the preliminary design is reasonable. The risk assessment can be performed using one of three approaches: database approach, expert elicitation approach, and a combined approach.⁵ For the database approach, we describe a method used to evaluate the best density estimation technique for small sample size scenarios. We also created a database of components for spacecraft communication systems. In the case of the expert elicitation, we developed a three-part interviewing process to assess expert bias and heuristic thinking, and to calibrate the expert's tendency to underestimate or to overestimate. Finally, the risk model is tested on a university mission, CASTOR, for which the risk assessment is

⁵ Work on the technique to combine the database approach and expert elicitation approach is ongoing.

focused on the antenna and on the transceiver. The results reveal advantages and disadvantages in both the database and the expert opinion approaches. Hence, a combination of the two approaches is suggested.

Acknowledgments

This research was carried out at Massachusetts Institute of Technology and at the Jet Propulsion Laboratory, California Institute of Technology, and was sponsored by MIT Space Systems Laboratory, MIT Department of Aeronautics and Astronautics, the NASA/Jet Propulsion Laboratory Graduate Fellowship Program, and the Amelia Earhart Fellowship Program. The authors would like to thank Prof. David W. Miller, Prof. Moe Z. Win, and Dr. Alvar Saenz Otero for their support and suggestions.

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Appendix

Properties of KDE

Kernel density estimation (KDE) is a popular nonparametric technique used to construct a probability density function from a finite set of samples of unknown distribution. Let x_1, x_2, \dots, x_n be an independent and identically distributed (iid) sample drawn from some unknown distribution $f_{PDF}(x)$ of x with mean m_x and variance σ_x^2 . The kernel estimator $\hat{f}_{PDF}(x)$ is defined as

$$\hat{f}_{PDF}(x) = \frac{1}{n} \sum_{i=1}^n K_h(x - x_i) \quad (\text{A-1})$$

where $K_h(\cdot)$ is the kernel function such that $\int_{-\infty}^{\infty} K_h(x) dx = 1$, and h is a smoothing parameter.⁶ Note that h is a free parameter that has to be chosen such that it is large enough to smooth out the irregular data artifacts, and at the same time small enough to preserve the underlying structure. Also, $K_h(\cdot)$ has to be continuous and symmetric with respect to 0. Common kernel functions include the uniform and Gaussian. Equation (A-1) can be rewritten as

$$\hat{f}_{PDF}(x) = \left(\frac{1}{n} \sum_{i=1}^n \delta(x - x_i) \right) * K_h(x) \quad (\text{A-2})$$

where $\delta(\cdot)$ is the Dirac delta function, and $*$ corresponds to the convolution operator between the two functions. From the form of Equation (A-2), one can interpret the kernel estimation process as a probability experiment of adding two independent random variables x_d ⁷ and k , where x_d is a discrete random variable that assumes values of x_1, x_2, \dots, x_n with equal probability $1/n$, and k is a continuous random variable with distribution $K_h(x)$.

Estimating the mean \hat{m}_x of x using $\hat{f}_{PDF}(x)$ corresponds to adding the means of x_d and k . As k has zero mean, \hat{m}_x is given by the mean of x_d :

$$\hat{m}_x = \frac{1}{n} \sum_{i=1}^n x_i \quad (\text{A-3})$$

which is the sample mean and is an unbiased estimator of x . On the other hand, estimating the variance $\hat{\sigma}_x^2$ of x using $\hat{f}_{PDF}(x)$ is equivalent to adding the variance of x_d and k , which is given by the following expression:

$$\hat{\sigma}_x^2 = \frac{1}{n} \sum_{i=1}^n (x_i - \hat{m}_x)^2 + h^2 \quad (\text{A-4})$$

Note that the first component in Equation (A-4) represents the sample variance of x_1, x_2, \dots, x_n , and let us denote it by σ_x^2 . Thus, the variance $\hat{\sigma}_x^2$ estimated using kernel density estimator $\hat{f}_{PDF}(x)$ is consistently greater than the sample variance.

⁶ If $K_h(x)$ is interpreted as a probability density function, h^2 corresponds to its variance.

⁷ This is in contrast to the continuous random variable x .

In the case of a Gaussian kernel

$$K_h(x) = \frac{1}{\sqrt{2\pi}h} e^{-\frac{x^2}{2h^2}}$$

it is suggested that the optimal choice of h is⁸

$$\left(\frac{4\sigma_x^2}{3n}\right)^{\frac{1}{5}}$$

The ratio

$$\frac{\hat{\sigma}_x^2}{\sigma_x^2}$$

is then given by

$$\frac{\hat{\sigma}_x^2}{\sigma_x^2} = 1 + \left(\frac{4}{3n}\right)^{\frac{2}{5}} \tag{A-5}$$

and is displayed in Figure A-1 as a function of n .

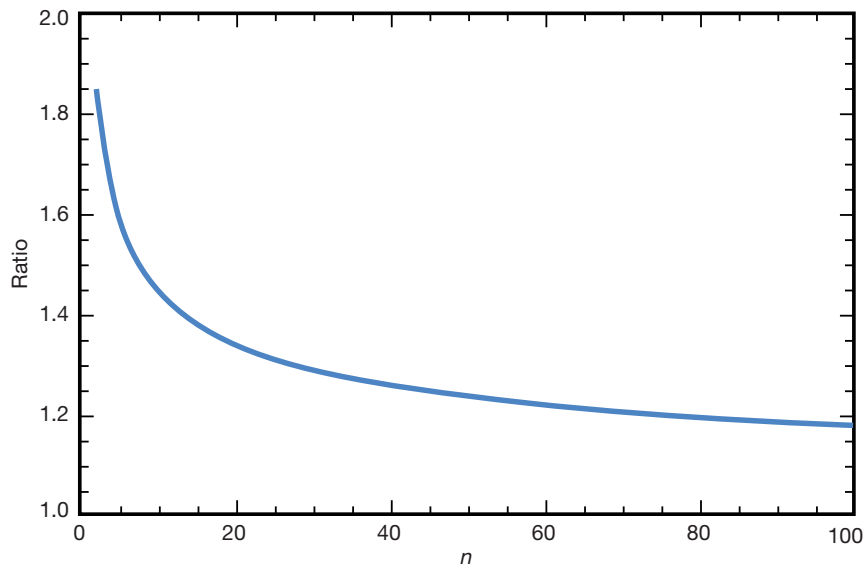


Figure A-1. Ratio of variances.

Note that the ratio decays rather slowly as n increases. For example, when $n = 20$, the variance estimated by the Gaussian kernel estimator $\hat{\sigma}_x^2$ is still 34 percent larger than the sample variance σ_x^2 .

⁸ See B. A. Turlach, "Bandwidth Selection in Kernel Density Estimation: A Review," Institut de Statistique, Biostatistique, et Sciences Actuarielles, 1993 — download from the site at <http://www.stat.ucl.ac.be/ISpub/dp/1993/dp9317.ps>