Antenna Bias Rigging for Mission-Dependent Performance Objective

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A procedure is described for computation of the optimum paraboloidal antenna rigging angle to minimize the expected average mean square half pathlength surface deviations for gravity loading. Statistics of deep space planetary missions are employed to develop weighting factors for antenna elevation angles during these missions. Comparisons for the existing Mars Deep Space Station antenna show that average gain losses from gravity loading can be reduced by 15 to 30 percent for this antenna by using the optimal rigging angle of approximately 35 degrees rather than the current rigging angle of 45 degrees.

I. Introduction

Bias rigging is a method of optimizing the performance characteristics of the surface-supporting structure of a paraboloidal radio-frequency (RF) transmitting and receiving antenna reflector. The approach is to set the reflecting surface panels to an ideal paraboloid at some particular elevation rigging angle, intermediate between horizon and zenith pointing attitudes. This can provide a built-in bias to improve accuracy of the reflecting surface with respect to the adverse effects of structural deformations. In particular, loss of accuracy from deformations caused by variable gravity loading over the elevation attitude range can be reduced.

An antenna-reflector system, such as shown in Fig. 1, is used extensively in communications, space exploration, and radio astronomy. The function of the reflecting surface during a receiving cycle is to collect RF energy emanating from a distant source and redirect this to a focal collection point or subreflector. A converse function is performed during a transmitting cycle.

During operation, deviations of the reflecting surface from an ideal paraboloid cause losses in performance efficiency from pathlength changes and undesirable phase shifts within the energy beam. These deviations are the result of tolerance accumulation from manufacture and installation, or are the result of response of the structure to environmental loading. Tolerance accumulation deviations can often be effectively controlled by current manufacturing, inspection, and quality assurance techniques. Consequently, the loading deformations from the effects of gravity, wind, temperature, and shock are more significant. Of these loadings, the gravity loading, which is both omnipresent and deterministic, tends to have the most adverse effect upon performance. Consequently, control of the gravity loading deformations is a logical and feasible approach to improve performance reliability. In addition to the intensity of loading, the deformations also depend upon properties of the supporting structure. Compared to comprehensive procedures that can develop the design of the supporting structure (Refs. 1, 2), bias-rigging is a simple procedure that can be readily implemented to help achieve this control.

II. Antenna Surface Performance Efficiency

A convenient measure of antenna surface accuracy is given by Ruze's conventional (Ref. 3) efficiency equation that relates surface accuracy to RF wavelength as

$$e = \exp[-(4\pi \text{rms}/\lambda)^2] \tag{1}$$

in which e is the efficiency of the surface, λ is the wavelength, and rms is the square root of the mean square half pathlength deviation of the reflecting surface from a best-fitting ideal paraboloid. The gain-loss can be computed in decibels from Eq. 1 as

$$G_l = 10 \log_{10} e \tag{2}$$

When the antenna is subjected to gravity loading, the rms term to be used in Eq. 1 is a function of the particular elevation angle attitude at which the antenna is pointing. In Ref. 4, it was shown that the mean square half pathlength deviation, SS_α at elevation angle α can be determined from

$$SS_\alpha = \eta^2 SSY + \zeta^2 SSZ + 2\eta\zeta\, SYZ \tag{3}$$

and the rms half pathlength deviation is

$$(\text{rms}_\alpha) = (SS_\alpha)^{1/2} \tag{4}$$

In Eq. 3, SSY and SSZ are the mean square half pathlength deviations for gravity loading applied parallel to the respective Y and Z axes shown in Fig. 1, and SYZ is the mean inner product of the corresponding half pathlength deviation vectors. The loading coefficients η, ζ, depend upon the elevation angle α and the rigging angle γ and are given by

$$\begin{aligned} \eta &= \cos\gamma - \cos\alpha \\ \zeta &= \sin\gamma - \sin\alpha \end{aligned} \tag{5}$$

The polar plot in Fig. 2 illustrates an example pattern of the change in the rms half pathlength deviation over the elevation attitude range. The deviation can be seen to be zero at the rigging attitude (rms γ) and to have extreme values at the horizon (rms_0) and zenith (rms_{90}) attitudes. It can be observed from this figure that choice of rigging angle can have a significant effect on the performance of the antenna.

Relationships were discussed in Refs. 4 and 5 that considered either the minimization of the extreme rms pathlength deviations over the elevation range, or minimization of the expected average pathlength deviations for antennas that track targets uniformly distributed within the hemisphere above the horizon. Here we will consider bias rigging to minimize the expected average pathlength deviations for an antenna that is required to track missions of a collection of planets with known orbits.

III. Selection of Rigging Angle for Mission-Weighted Elevation Angle Usage Factors

Assume that the projected tracking mission of the antenna is known so that it is possible to determine the elevation angle time history for the antenna for a particular mission or the average time history for a collection of missions. The elevation angle weighting factor W_α can be developed to supply the probability of antenna targets occurring at elevation angle α. A set of weighting factors is developed by dividing the elevation angle range into a set of elevation class marks equally spaced at some constant class interval. Consequently the weighting factors represent the probability of antenna targets occurring at elevations within the class interval centered on particular class marks. When the class marks are closely spaced, and with appropriate normalization of the weighting factors, the elevation angle probability density function is approximately equal to the weighting factor divided by the class interval. In this case it follows that

$$\Sigma W_\alpha = 1.00 \tag{6}$$

Optimum performance for a particular antenna with respect to the mission elevation angle weighting is obtained by choosing the rigging angle to minimize the elevation-weighted mean square half pathlength deviations. In particular, from Eq. (3), we choose γ to minimize the objective expression

$$OBJ = \Sigma_a W_a (\eta^2 SSY + \zeta^2 SSZ + 2\eta\zeta SYZ) \tag{7}$$

This objective represents the expected average mean square half pathlength deviation. Substituting Eqs. (5) in (7), expanding, using Eq. (6), and simplifying leads to the objective in the following form:

$$OBJ = A^2 SSY + B^2 SSZ + 2C SYZ \tag{8}$$

where

$$A^2 = \cos^2 \gamma + \Sigma W_a \cos^2 \alpha - 2 \cos \gamma \Sigma W_a \cos \alpha$$

$$B^2 = \sin^2 \gamma + \Sigma W_a \sin^2 \alpha - 2 \sin \gamma \Sigma W_a \sin \alpha$$

$$C = \sin \gamma \cos \gamma + \Sigma W_a \sin \alpha \cos \alpha - \cos \gamma \Sigma W_a \sin \alpha - \sin \gamma \Sigma W_a \cos \alpha$$

Although there are methods of numerical analysis for choosing to minimize the objective in Eq. 8, a simple and stable method is to consider this as a problem of operations research and use a search method (Ref. 6). Standard subroutines[1] can execute the search rapidly and furnish the optimum rigging angle with any desired precision.

IV. Computation of Elevation Angle Mission Weights

The weighting factor is computed as proportional to the amount of time in a given period that the antenna elevation angle is within the particular class interval. For convenience, we consider an annual period of 365 days. The time in hours is 1/15 of the difference of hour angle (in degrees) of the target in passing through the lower and upper elevation class mark boundaries. The hour angle H is a function of latitude of the site ϕ, the declination angle of the target δ, and the elevation angle α. From spherical trigonometry

$$H = \cos^{-1} \frac{(\sin \alpha - \sin \phi \sin \delta)}{\cos \phi \cos \delta} \tag{9}$$

[1]See, for example, Subroutine FIBMIN, coded by C. L. Lawson, JPL, Sect. 914

Eq. (9) can be solved for the maximum elevation α_{max} at a particular declination, which occurs at 0 degree hour angle, thus

$$\alpha_{max} = \sin^{-1} (\cos(\phi\text{-}\delta)) \tag{10}$$

On a given day, assume that the target's declination δ_j is approximately constant. The tracking time t_j on this day is twice the time interval in following the target from the minimum operational elevation angle α_{min} to the maximum elevation angle given by Eq. (10). Therefore

$$t_j = 2/15\, H\, (\alpha_{min}, \phi, \delta_j) \tag{11}$$

If analysis has been made of the missions to be considered and declination angle weighting factors D_j have been established to give the probability of target declinations being within declination class marks δ_j, the annual hours T_j at this declination class mark are

$$T_j = 365\, D_j t_j \tag{12}$$

At declination class mark δ_j targets will be encountered at elevation class marks between α_{min} and α_{max}. The daily time t_{ij} spent in tracking at elevation class mark α_i with upper and lower class mark boundaries a_i and b_i, respectively, is

$$t_{ij} = 2/15(H(\alpha_i, \phi, \delta_j) - H(b_i, \phi, \delta_j)) \tag{13}$$

On a 365-day basis, declination class mark δ_j contributes the following tracking time hours to elevation class mark α_i

$$T_{ij} = 365\, t_{ij}\, D_j \tag{14}$$

and the total tracking time T_i at this elevation class mark is the sum of the contributions from all declination class marks. That is

$$T_i = \Sigma_j, T_{ij} \tag{15}$$

Finally, the normalized elevation angle mission-weighting factor is

$$W_i = T_i / \Sigma T_i \tag{16}$$

As a check, compare Eq. (14) with Eq. (12)

$$\Sigma_i\, T_{ij} = T_i \tag{17}$$

V. Mission-Weighted Declination Angles

In Ref. 7 an analysis has been made of the most significant NASA-JPL deep space planetary mission tracking orbits to determine composite declination angle weighting factors for missions in the time period from 1973 to 1981. Three mission categories of declination angle weighting factors have been developed for declinations spaced at one degree class intervals from −50 to +50 deg. The mission categories are

(1) Approved Missions
    Pioneer 6–9
    Pioneer 10
    Pioneer 11
    Mariner Venus/Mercury
    Helios A and B
    Mars Viking 1 and 2
    Mariner Jupiter/Saturn

(2) Projected Missions
    Pioneer Venus Probe
    Pioneer Venus Orbiter
    Pioneer Saturn Probe
    Mariner Jupiter/Uranus 1 and 2
    Pioneer Saturn/Uranus FB
    Pioneer Saturn/Uranus Probe
    Mars Viking
    Mariner Jupiter Orbiter
    Pioneer H

(3) Combined Approved and Projected Missions

For comparison, a fourth category has been established, which is equivalent to a solar mission. That is, the mission target is the Sun. In this case, the declination is given by

$$\delta = 23.5 \sin (2\pi d/365) \qquad (18)$$

in which d is the number of days from the vernal equinox. In this case the declination probability density function, which is approximately equivalent to the declination angle weighting factor for a one degree class interval declination angle is given by

$$f(\delta) = \frac{1}{23.5\pi\,[1 - (\delta/23.5)^2]^{1/2}} \quad \text{for } |\delta| < 23.5. \qquad (19)$$

VI. Results and Discussion

Figure 3a shows the declination angle probability density functions for the approved and the projected missions. Figure 3b shows these functions for the combined and the solar missions. The two curves in Fig. 3b exhibit considerable differences, but the combined mission curve in Figure 3b tends to follow the solar mission curve within a reasonable amount of oscillation.

The corresponding elevation angle probability density curves are shown in Figs. 4a and 4b. Again there are differences in the curves of Fig. 4a, while the combined and solar mission curves of Fig. 4b are reasonably similar. These elevation angle probability density curves were computed for the following parameters:

ϕ = 35.4 deg (latitude of Goldstone, Calif.)

Elevation class interval = 2.5 deg

Minimum tracking elevation = 6.0 deg

Maximum tracking elevation cut-off (overrides α_{max}) = 88.0 deg

Table 1 shows the statistics of the elevation weighting factors that are used for computation of antenna rigging angles. Figure 5 is a plot of gain/loss versus antenna elevation angle for the DSN 64-m Mars antenna. The elevation weighting factors used were for combined missions. One curve is for the optimum rigging angle of 35.4 deg computed according to the procedure given here and the other curve is for the same antenna with the current 45-deg rigging angle. Table 2 is a summary of results obtained for this antenna for all mission categories comparing optimum rigging with the existing 45-deg rigging angle.

It can be noted from the last line in Table 2 that, although the expected average gain loss for this antenna is relatively small for all mission categories, the optimal rigging angle results in gain losses of from 15 to 30 percent less than the gain loss with 45 deg rigging. It can also be observed from Fig. 5 that the relatively higher gain losses with optimal rigging, which occur at the elevation angles close to the zenith attitude, occur with low probability (weighting factor).

References

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Table 1. Mission-weighted elevation angle statistics

	Weighting factor			
	Approved missions	Projected missions	Combined missions	Solar missions
ΣW_a	1.0000	1.0000	1.0000	1.0000
$\Sigma W_a \sin^2 \alpha$	0.3886	0.3334	0.3755	0.3641
$\Sigma W_a \cos^2 \alpha$	0.6114	0.6666	0.6245	0.6358
$\Sigma W_a \sin \alpha$	0.5712	0.5282	0.5610	0.5526
$\Sigma W_a \cos \alpha$	0.7523	0.7937	0.7621	0.7717
$\Sigma W_a \sin \alpha \cos {}_a$	0.3796	0.3778	0.3791	0.3805
	Average annual tracking hours			
	4112	3857	4056	3996

Table 2. Comparison for existing Mars antenna

	Approved missions		Projected missions		Combined missions		Solar missions	
	Optimum	Existing	Optimum	Existing	Optimum	Existing	Optimum	Existing
Rigging angle, deg	36.2	45.0	32.8	45.0	35.4	45.0	34.7	45.0
Expected average gain loss for mission, dB at X-Band (= 8.45 GHz)	0.0347	0.0418	0.0296	0.0434	0.0337	0.0421	0.0322	0.0420
Relative gain loss, percent	83	100	68	100	80	100	77	100

SSZ $mm^2 (in.^2) = 0.760 (0.001176)$
SSY $mm^2 (in.^2) = 0.393 (0.000610)$
SYZ $mm^2 (in.^2) = 0.030 (0.000046)$

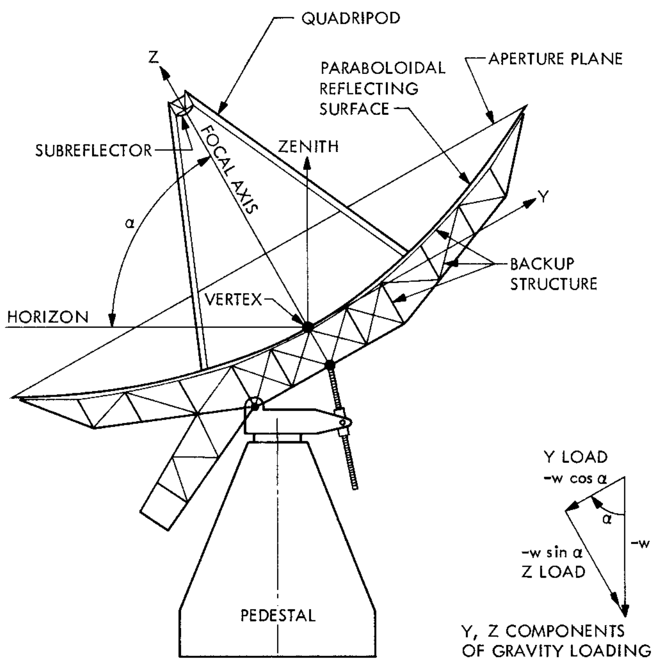

Fig. 1. Antenna-reflector system

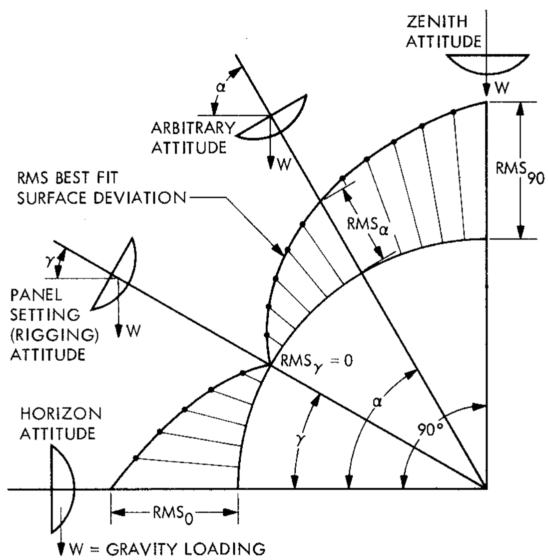

Fig. 2. RMS deviation change with elevation angle

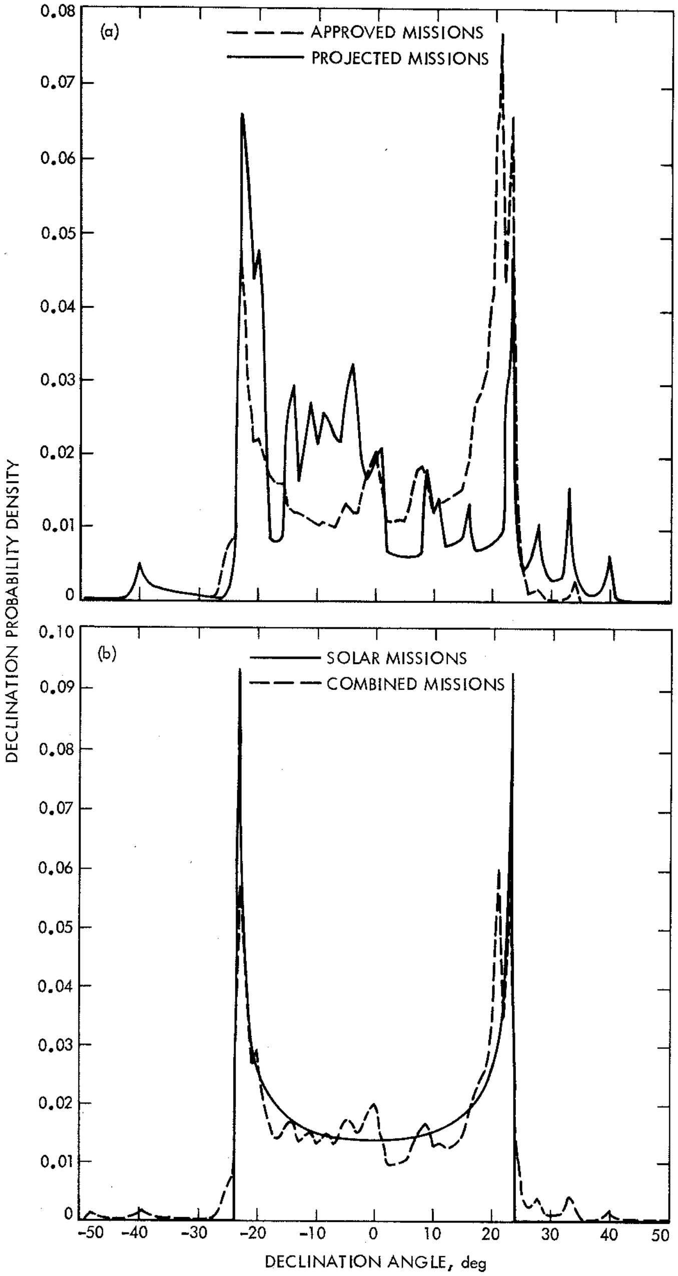

Fig. 3. Declination angle probability density: (a) approved and projected missions, (b) solar and combined missions

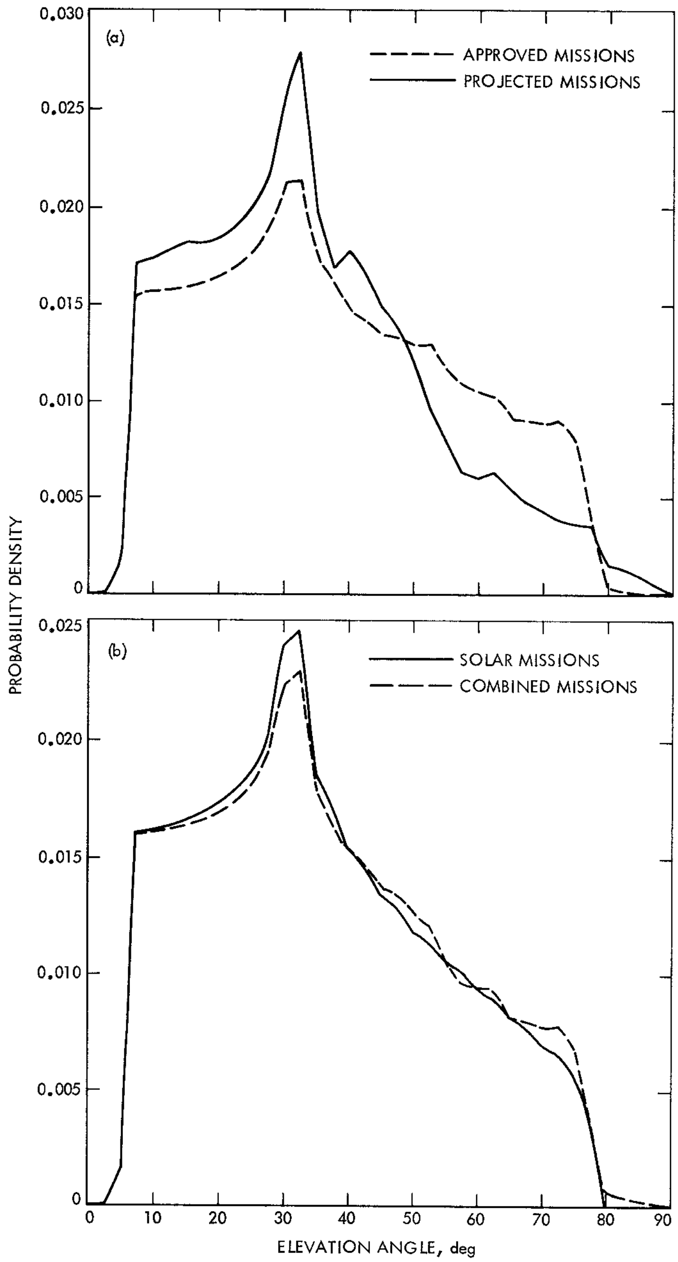

Fig. 4. Elevation angle probability density: (a) approved and projected missions, (b) solar and combined missions

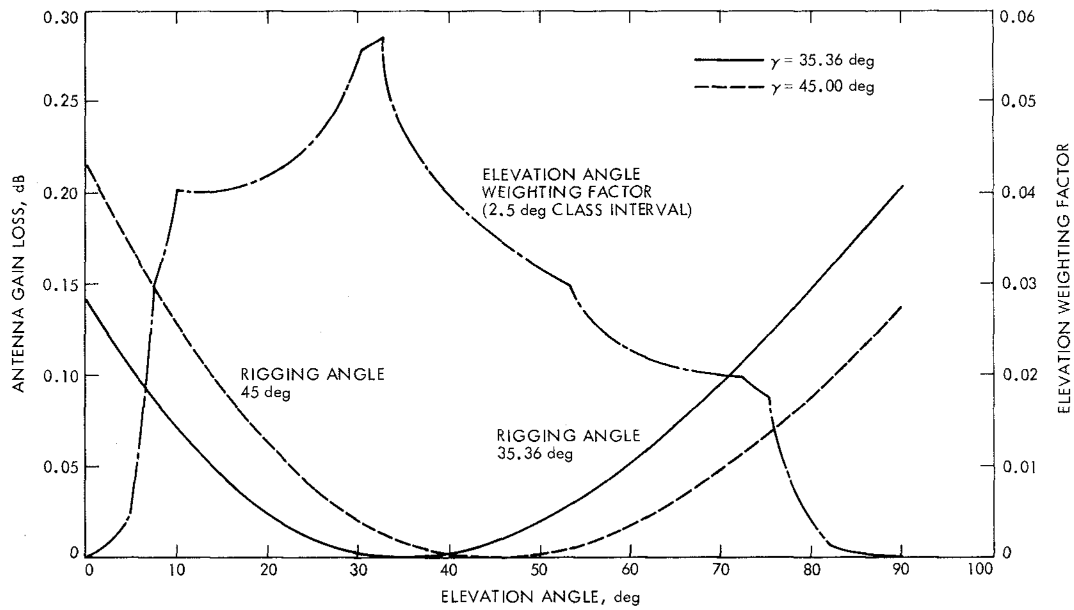

Fig. 5. Mars antenna gravity deformation gain loss at 8.45 GHz for combined missions