S-Band/X-Band Doppler Two-Way Nonlinear Jitter Analysis Using Simplified Series-Expansion Techniques

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Using a simplified nonlinear approach by Yuen together with simplified series expressions to obtain the nonlinear values for Alpha, Gamma, and Sigma, a full program was developed that outputs estimated two-way jitter in degrees as a function of ground and spacecraft signal-level margin above design point. Required modified ("hyperbolic") bessel functions were calculated individually, rather than by recursion, with the "defining" series to reduce the error. Results were replaced by linear approximation when the series was no longer feasible, well into the "linear range." As an experiment, and to establish feasibility of the entire model, programming was done entirely on the table calculator (four card-sides; about 1700 steps).

I. Introduction

The S-band/X-band (S/X) doppler jitter experienced by DSN stations to date has been small, once hardware problems were identified and resolved. However, as signal levels decrease in the future, both spacecraft and ground margins may drop into the range where thermal noise contributions are a significant parameter. The jitter amplitude in this region is not well documented nor understood, because the large S/X multiplier in the spacecraft drastically alters the model from that of the S-band/S-band (S/S) case. Modified linear results were first described in Ref. 1, where an unusual "bottoming-out" of jitter at medium margins was predicted. Reference 1 did *not* use the recently established "½ factor" on linear σ^2, so predicted jitter was much larger.[1] In order to deter-

[1]The term "margin," as used in this analysis, is simply the ratio of signal power to the noise power in the loop at its design-point bandwidth, or $2BL_0$. By "conventional" theory (now proven obsolete), this would also be the point where linear analysis, by former criteria, would yield a variance of unity. The original program (and Ref. 1) was based on this assumption, which leads to expected jitter values larger than the data normally exhibit. New theory has demonstrated that variance at "design point," calculated by linear methods, is only one-half that formerly used, yielding a factor of $\sqrt{2}$ improvement in sigma. This latter number was subsequently incorporated into the subject program, and all results are based on it. Values are therefore somewhat less than those of Ref. 1, although direct comparison is not possible because of bandwidth differences. Results agreed, at several "check points," with those of J. H. Yuen (Ref. 2 and associated program), within 2 deg. Differences in the calculation of "GAMMA" could account for this.

mine the effects shown in Ref. 1 more accurately, a nonlinear model was subsequently written and programmed, using simplifications suggested by J. H. Yuen (Ref. 2). This paper describes the model, its derivation, and presents and discusses results. It is planned that the model will be used as the basis for S/X doppler system testing in the DSN.

II. Doppler Phase Jitter: A Definition

Doppler is a ground frequency measure yielding an estimate of the difference between the transmitted and received frequencies, the former appropriately translated and biased so that the difference can be reduced to give an estimate of spacecraft velocity (among other things).

The frequency is a cycle count (resolved to 10^{-5} cycle) divided by a counting period, with appropriate time-tag. The data are "curve-fit," and a frequency standard deviation from the curve, or "doppler frequency noise" is calculated for operational use.

Simultaneously, the entire cycle count over a sample period is cumulatively recorded, and differenced each sample period with the previous count. The difference, each sample point, is the true phase change during the period plus the sum of noise displacements at the instants of sampling. This latter figure is instantaneous doppler phase jitter (ϕ_T), and its standard deviation is the measure that the subject program estimates as a function of ground and spacecraft margin. The square of σ_T, or $(\sigma_T)^2$, is the doppler phase jitter variance, or "phase noise," a relative power with respect to the carrier.

As with frequency jitter, ϕ_T data cannot be read directly, but must be differenced from a "curve fit," since true cumulative phase is a variable with spacecraft distance. The data reduction program deals with discrete samples, which I describe here as total phase differences:

$$\hat{\phi}_n(t_n) = \{[\phi_M(t_n) - \phi_C(t_n)] - [\phi_M(t_{n-\tau}) - \phi_C(t_{m-\tau})]\} \tag{1}$$

where

$\phi_M(t_n)$ = cumulative total phase measure to t_n

$\phi_C(t_n)$ = cumulative "curve fit" total phase measure to t_n

$t_n, t_{n-\tau}$ = time to nth and $(n+\tau)$th sample and τ = sample period

$\hat{\phi}_n(t_n)$ = the nth estimate of phase displacement from "true phase," of which ϕ_C is the cumulative estimate.

$\hat{\phi}_n$ can have other forms than Eq. (1); I show it as a sample expression only. The ground processing, however, no matter how mechanized, yields a measure of $\hat{\phi}_n(t)$ and, after appropriate processing of a large number of such samples, calculates the sigma and variance of this quantity. This is well covered in the literature, and will not be repeated here. The result as stated is $\hat{\sigma}_T$ or $(\hat{\sigma}_T)^2$, and the model developed here is to predict this parameter for S/X band (Mariner Venus/Mercury Mission) application.

III. Phase Jitter Model: General Form

Refer to Fig. 1. The final model output $\hat{\sigma}_T$ consists of two components, a fixed "system" contribution, and the X-band receiver total contribution. The latter has two components also, one of which is modified from a "linear sum" to a nonlinear form; this is a more accurate representation, particularly at small ground margins. The notations on Fig. 1 have the preliminary definitions:

$$\left.\begin{aligned}
\hat{\sigma}_T^2 &= \sigma_{RT}^2 + \sigma_S^2 \\
\sigma_{RT}^2 &= (G\sigma_U^2) + (\sigma_{NL})^2 \\
\sigma_{NL}^2 &= F(\rho) \\
\rho &= 1/[\sigma_{R_2}^2 + K(G\sigma_{R_1})^2] \\
\sigma_P^2 &= (G\sigma_U)^2 + (G\sigma_{R_1})^2
\end{aligned}\right\} \tag{2}$$

where (all σ in radians except as expressed otherwise)

$\hat{\sigma}_T^2$ = total predicted jitter variance (converted to degrees for actual program output)

σ_S^2 = composite variance of all fixed (and considered fixed) system sources, such as doppler extractor references (from standards and exciter), 1 pulse/s, counter resolver, receiver VCO residual, etc. For this model $\sigma_s = 6$ deg, which was computed earlier for the linear model.

σ_{RT} = total jitter contributed by X-band receiver

G = 880/221, the spacecraft X/S multiplier

σ_U = uplink jitter arising from the exciter frequency sources and multipliers. The spectrum is not well defined, but the jitter is specified as "less than 8 deg in a 3-Hz loop." This will be insignificant by comparison to thermal noise at weak signal levels. A "fixed" value of 2 deg was selected, and treated as a residual, in this model. Since σ_U is multiplied by G, the rms total of σ_U and σ_S is about 10 deg.

σ_{NL} = composite nonlinear contribution of both receivers

$F(\rho) = \sigma_{NL}$ = function that converts the linear receiver calculations to nonlinear results

ρ = "system" signal-to-noise (S/N), as defined by J. H. Yuen (Ref. 2)

σ_{R_1} = spacecraft total receiver loop output jitter (S-band) due to "white" (gaussian) input noise

σ_{R_2} = same as σ_{R_1}, but for ground X-band receiver

K = a variable multiplier on the spacecraft noise output spectrum to account for filtering of this spectrum by the ground receiver

σ_P = "path" jitter, as present on the down-link prior to reception and filtering. It would be the ground receiver output at very strong signals, if the bandwidth was quite large and the system very low in noise.

If only strong signals were present, without significant thermal noise, this would end the model:

$$\sigma_{R_1}, \sigma_{R_2} \rightarrow 0, \qquad \rho \rightarrow \infty, \qquad F(\rho) \rightarrow 0$$

For the strong signal model:

$$\sigma_T = \left[\sigma_S^2 + \left(\frac{880}{221}\sigma_U\right)^2\right]^{1/2} \approx 10\,\text{deg} \qquad (3)$$

This is the lower bound for σ_T. As receiver noise contributions begin to be significant, the functions become more sophisticated, as described in the next section.

IV. Phase Jitter Model: Detailed Expressions

The functions σ_{R_1}, σ_{R_2}, K, and (finally) ρ and σ_{NL} depend on the two receiver bandwidths and noise spectral density. Assuming conventional design point definition and damping ($r_0 = 2$), as well as second-order loops containing band-pass filters and limiters (the Block IV/Mariner spacecraft mechanization), the signal levels may be expressed as a margin above design point, initially in dB, but convertible to a power ratio. For this program, the following fixed input parameters were used:

(1) Spacecraft design point loop bandwidth = 2 BL_0 S/C = 18 Hz.

(2) Spacecraft predetection bandwidth = BC S/C = 5000 Hz.

(3) Ground design point loop bandwidth = 2 BL_0 GND = 10 Hz.

(4) Ground predetection bandwidth = BC GND = 2000 Hz.

Although the program was designed for arbitrary bandwidth and spacecraft margin, with ground margin running from 0 to 50 dB in 2-dB steps, spacecraft margin inputs of 15, 17, 20, 25, and 40 dB were processed. Indicating any margin by the notation "dB," and using the modifier "S/C" or "GND" for spacecraft/ground receiver when necessary (as for bandwidths above), the definitions and expressions in Table 1 complete the program preliminary description.

In Table 1, the functions (4) through (10) are straightforward and together with (2) define the entire model in detail. The series (9) and (10) were not, of course, carried to an infinite number of terms; the simplified form of (9), whose integral form was determined as valid by Yuen (Ref. 2), also requires comment. It was found that (10) was accurate to better than 2×10^{-6}, for small angles at the limit of machine capability, and that (9) followed to better than 5×10^{-6} for jitter angles down to about 8 deg, where linear theory was adequate. This approach is not only simplified, but, if necessary, could be programmed easily in DSN field software. A detailed error analysis of the series in (9) and (10) is covered in Ref. 5.

V. Results and Conclusions

Program results are plotted on Fig. 2. The "dip down" in jitter at moderate signal levels, resulting from the large S/X multiplier, showed even more strongly than the analysis with linear theory (Ref. 1). This was due in part to the different input bandwidths, chosen to agree with hardware of the current mission.

The decreasing jitter with decreasing ground margin is attributed to the parameter K, which reflects the narrowing ground bandwidth. This bandwidth reduction "filters" the spacecraft noise, but little ground noise is generated "by comparison." The jitter eventually "bottoms out," then climbs rapidly near design point as the ground noise becomes significant.

It is evident that tracking will be impossible when the spacecraft margin drops below 20 dB, for jitter will never reach tracking levels. Meanwhile, expect jitter levels to improve before they get worse; a rather new phenomenon.

Parameters from Ref. 6 were also programmed (favored values) to confirm results of that memo. Spacecraft margin of 36.3 dB and ground 2 BL_0 of 3 Hz ($BC =$ 200 Hz) were entered, and the X-band receiver jitter at a ground margin of 36.6 dB was interpolated. The input numbers agree with expected conditions when the 100-kW transmitter is used, together with aided tracking (3-Hz loop) on the ground, as covered in Ref. 6. The result, also shown on Fig. 2 (designated point), was 2.88 deg, even below that of the predicted value of 4 deg.

All results shown in Fig. 2 are without residual contribution. If σ_U is set at 2 deg, and the 6 deg of σ_S is also contributed, a minimum residual of about 10 deg occurs, of which 6 deg is "outside" the loop. An expression for adding these contributions to σ_{NL} is given in Fig. 2, resulting in the total expected jitter.

References

1. Bunce, R. C., *S-Band/X-Band Two-Way Doppler Jitter*, IOM 421E-74-073, Apr. 4, 1974 (JPL internal document).
2. Yuen, J. H., *On Coherent Two-Way Doppler Phase Probability Distribution*, IOM 3395-72-81, June 27, 1972 (JPL internal document).
3. Lindsey, W. C., and Simon, M. K., *Telecommunication Systems Engineering*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1973.
4. *Standard Math Tables*, Edition 17, Chemical Rubber Co., Cleveland, Ohio, 1969.
5. Bunce, R. C., *S-Band/X-Band Two-Way Doppler Jitter, Non-Linear Model*, IOM 421E-74-085, Apr. 26, 1974 (JPL internal document).
6. Brunn, L., *Doppler Phase and Ranging Performance at M_e Using the 100 KW Transmitter*, IOM 3396-73-300, Oct. 22, 1973 (JPL internal document).

Table 1. Detailed expressions for S/X doppler jitter

$$\left.\begin{aligned} |\text{margin}| &= M = 10^{(\text{dB}/10)} \qquad G = 880/221 \\ \text{predetection S/N ratio} &= A = (2\,BL_0/BC)\cdot M \\ A_0 &= (2\,BL_0/BC) \\ \text{design point loop bandwidth ratio} &= \zeta = (2\,BL_o\,\text{S/C})/(2\,BL_o\,\text{GND}) \end{aligned}\right\} \tag{4}$$

$$\left.\begin{matrix}\text{jitter variance} \\ \text{(either receiver)}\end{matrix}\right\} = \sigma_R^2 = \left.\frac{1}{2M}\left[\frac{1}{3} + \frac{2}{3}\frac{\alpha}{\alpha_0}\right]\Gamma_p\right\} \tag{5}$$

$$\left.\begin{aligned} \alpha(A) &= \sqrt{\frac{\pi}{4}A}\; e^{-A/2}\left[I_0\left(\frac{A}{2}\right) + I_1\left(\frac{A}{2}\right)\right] \\ \alpha_0 &= \alpha(A_0) \qquad \text{(Ref. 3, p. 50)} \end{aligned}\right\} \tag{6}$$

$$\left.\begin{aligned} \Gamma_p(A) &= \frac{1 - e^{-A}}{\frac{\pi}{4}A\,e^{-A}\left[I_0\left(\frac{A}{2}\right) + I_1\left(\frac{A}{2}\right)\right]^2\left[1 + \left(\frac{3.448}{\pi} - 1\right)\exp\left[-A\left(1 - \frac{\pi}{4}\right)\right]\right]} \\ &\text{(Ref. 3, p. 52; inexact)} \end{aligned}\right\} \tag{7}$$

$$\left.\begin{aligned} &K\sigma_{R_1}^2 = \frac{1}{4\pi M}\int_{-\infty}^{\infty} |H_1(j\omega)\,H_2(j\omega)|^2\,d\omega \\ &H_1(S), H_2(S) = \text{S/C, GND loop transfer functions} \\ &\text{Solving yields:} \\ &K = \left[\frac{K_1}{2 + K_1}\right]\left[\frac{K_1(2 + K_1) + 2(K_1 + K_2 + 2)(\zeta + \zeta^2) + K_2(2 + K_2)\zeta^3}{K_1^2 + 2K_1\zeta + 2(K_1 + K_2 - 2K_1K_2)\zeta^2 + 2K_2\zeta^3 + K_2^2\zeta^4}\right] \\ &K_1 = [\alpha_0/\alpha]\,\text{S/C} \qquad K_2 = [\alpha_0/\alpha]\,\text{GND} \qquad \text{(Ref. 3, p. 100)} \end{aligned}\right\} \tag{8}$$

$$\left.\begin{aligned} \sigma_{NL} &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \phi^2\,\frac{\exp[\rho\cos\phi]}{I_0(\rho)}\,d\phi \qquad \text{(modulo } 2\pi) \\ &\qquad \{\rho = 1/[\sigma_{R_1}^2 + K(G\sigma_{R_2})^2]\} \\ &= \frac{\pi^2}{3} + 4\sum_{N=1}^{\infty}(-1)^N\cdot\frac{1}{N^2}\cdot\frac{I_N(\rho)}{I_0(\rho)} \qquad \text{(Ref. 2)} \end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned} I_N(x) &= \left(\frac{x}{2}\right)^N \sum_{M=0}^{\infty}\frac{\left(\frac{x}{2}\right)^{2M}}{(M)!\,(M+N)!} \\ &\text{(Ref. 4; defining series for } I_N(x)) \end{aligned}\right\} \tag{10}$$

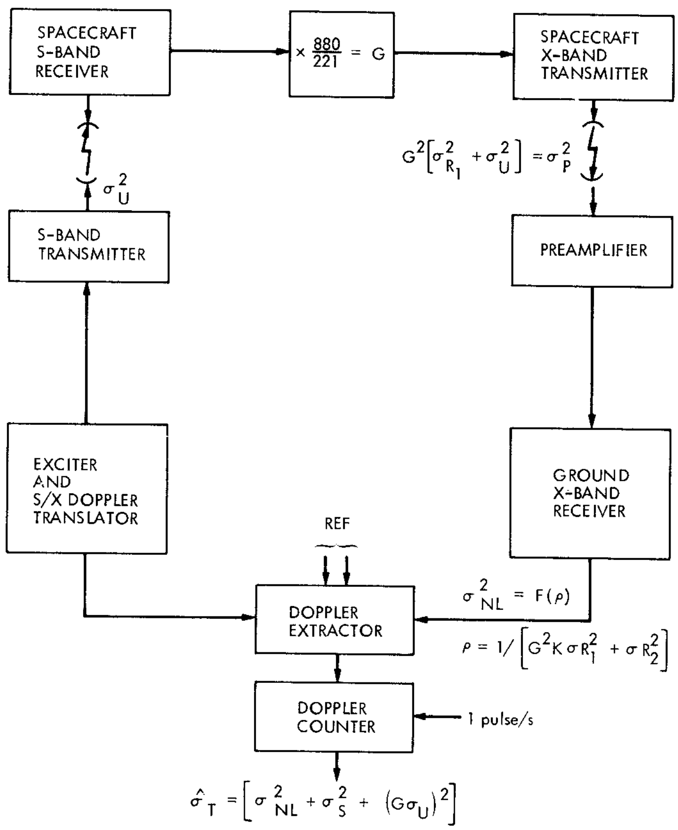

Fig. 1. S/X two-way doppler model diagram

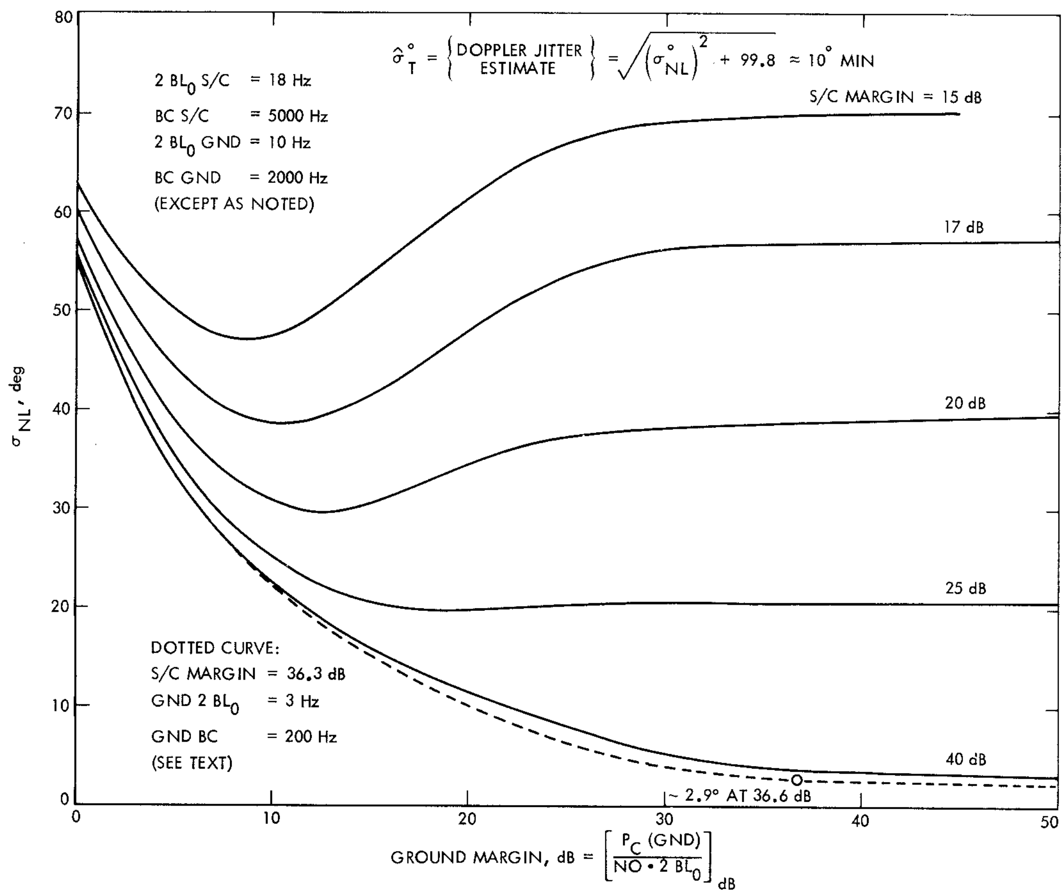

Fig. 2. Ground X-band receiver output jitter, degrees (nonlinear) vs ground and spacecraft margin