Pioneer Venus 1978: Telemetry Performance Predicts

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The Pioneer Venus 1978 probe-to-Earth telemetry links will be degraded by fading in the atmosphere of Venus. The severity of this fading is characterized by the variance σ_x^2, where the amplitude fading is represented by the lognormal random variable e^x. For the convolutionally encoded/sequentially decoded telemetry modes, the link performance depends on σ_x^2, the total received signal-to-noise ratio P_T/N_0, the modulation index θ, and the decoder computational capacity N. Using nominal values for anticipated system losses, and assuming a maximum deletion rate requirement of 10^{-2}, the minimum required P_T/N_0 and optimum θ are profiled as functions of N and σ_x^2 for the 256-bps large probe telemetry mode. These predictions are based on a recently developed theoretical model for the combined effects of lognormal fading and noisy carrier reference on sequentially decoded phase-shift-keyed telemetry.

I. Introduction

Current plans call for the Pioneer Venus 1978 (PV'78) large probe to have a 256-bps convolutionally encoded/sequentially decoded telemetry mode. Attention has been focused on this particular mode because initial calculations based on a recently developed theoretical model for the combined effects of atmospheric fading and noisy carrier reference (Ref. 1) predict a marginal link performance with available system parameters. The 256-bps link is examined further to determine the effects of system losses, and variations in the fading level and the sequential decoder computational capacity.

II. Objectives

A standard DSN sequential decoder can perform about 25,000 computations/second in real-time operation: at 256 bps, this is equivalent to a computational capacity N of roughly 100 computations/bit. However, since a frame deletion is a block of *detected* bit errors, whenever a real-time deletion does occur, we do have the option of storing the soft-quantized received data for that frame and attempting to decode it later at a scaled-down rate with a correspondingly higher N. In this article, we will investigate the dependence of the sequential decoding performance for the 256-bps link on N.

The PV'78 probe-to-Earth telemetry links will be degraded by fading in the atmosphere of Venus. The severity of this fading is characterized by the variance σ_x^2, where the amplitude fading is represented by the lognormal random variable e^x. Our previously published link performance predictions (Ref. 1) were based on a supposed worst case fading level of $\sigma_x^2 = 0.014$, a value computed by Woo (Ref. 2, Eq. 14) in his study of the Venus atmosphere. A more careful examination of Woo's report revealed that the value $\sigma_x^2 = 0.014$ applied to the special case of a probe on the surface of Venus transmitting *vertically* through the atmosphere; the atmosphere was assumed to be 55 km in depth. However, the probe-to-Earth telemetry links may be as much as 60° off the vertical, with an atmospheric path length L that could greatly exceed 55 km. Furthermore, during a probe's approximately 2-hour descent to the surface of Venus, L increases from zero initially to a maximum value at the surface. Using experimental data from the Russian Venera space probes, Woo has derived the following relationship between σ_x^2 and L (Ref. 2, Eqs. 12 and 13):

$$\sigma_x^2 = 0.0142 \left(\frac{L}{55}\right)^{11/6}; L \text{ in km} \tag{1}$$

The range of σ_x^2 applicable to the probe telemetry links is roughly $0 \leq \sigma_x^2 \leq 0.06$; the variation in the performance of the 256-bps link over this range of fading is determined below.

The previously published link performance analysis assumed an ideal communication system, with no losses other than noisy carrier reference and fading. In fact, however, at 256 bps, the data channel is expected to have a nominal total loss $L_D = 2.08$ dB due to predetection recording (1.00 dB), multipath interference (0.50 dB), waveform distortion (0.20 dB), symbol synchronizer assembly (0.20 dB), and subcarrier demodulator assembly (0.18 dB); the carrier tracking channel loss $L_C = 1.00$ dB is entirely due to predetection recording. The idealized noisy reference/fading model for sequential decoding performance can be adapted to incorporate these system losses by making two equation modifications (Ref. 1, Eqs. 7 and A-17):

$$\rho \equiv \frac{E_B}{N_0} = \left(\frac{1}{L_D}\right)\left(\frac{P_T}{N_0}\right)\left(\frac{\sin^2\theta}{R_B}\right) \tag{2}$$

$$\eta \equiv \frac{P_c}{N_0 2B_{L0}} = \left(\frac{1}{L_C}\right)\left(\frac{P_T}{N_0}\right)\left(\frac{\cos^2\theta}{2B_{L0}}\right) \tag{3}$$

In these equations, P_T/N_0 is the total received signal-to-noise ratio, θ is the modulation index, the data rate $R_B =$ 256 bps, and the two-sided threshold loop noise bandwidth $2B_{L0} = 12$ Hz. For given values of N and σ_x^2, this modified model is used to compute the minimum P_T/N_0 and optimum θ required to achieve a frame deletion rate of 10^{-2}.

III. Results

Using the formulas for the modified noisy reference/fading model discussed above, a complete telemetry performance profile for the 256-bps sequentially decoded mode was developed. The results of this theoretical study are presented below in a series of graphs. These computations use Layland's formula for the characteristic decoder memory time T_m (Ref. 3, Eq. 6), in which $T_m \sim 2T_B$ (bit time) for sufficiently large N. The validity of this formula is suspect, and it has been identified as a weak link in the modeling efforts (see "Commentary," Ref. 1, p. 60). Stolle (Ref. 4) has suggested that better agreement between the theoretical model and experimental results is obtained when T_m/T_B is increased to about 4 or 5 at 256 bps: this corresponds to more averaging of the fading and noisy reference random processes (Ref. 1, Eq. 11), which should lower the P_T/N_0 requirement for a given system. To investigate the sensitivity of the model to the value of T_m, the sequential decoding performance of several telemetry systems at 256 bps was computed with $T_m/T_B = 2$ and $T_m/T_B = 4$: the difference in required P_T/N_0 in each comparison was small, amounting to about 0.2 dB on the average. The conclusion is that the model is fairly insensitive to refinements in the value of T_m; and the P_T/N_0 requirements presented in this report are reasonably accurate, although they may be interpreted as worst case (slightly high) predictions.

In Fig. 1, the received bit energy-to-noise ratio ρ, required to achieve a frame deletion rate of 10^{-2}, is plotted against the sequential decoder computational capacity N, for the idealized case of perfect carrier reference, no system losses, and no fading ($\sigma_x^2 = 0$). We see that increasing N from 100 computations/bit (real-time decoding) to 1000 or 2000 computations/bit only lowers the required ρ by about 0.5 dB. This special case is intended as a reference point for telemetry design control table (DCT) applications in order to gauge losses (increases in required P_T/N_0) attributed to noisy carrier reference, fading, etc.

Figure 2 still neglects atmospheric fading, but incorporates the noisy reference and non-ideal system losses L_D and L_C. For a given N, the required P_T/N_0 curves have broad minima over the modulation index θ. Increasing N from 100 to 1000 and 2000 computations/bit only lowers the minimum required P_T/N_0 (at the optimal modulation indices, θ_{opt}) by 0.32 and 0.40 dB, respectively.

Figure 3 shows how the minimum required P_T/N_0 and the corresponding θ_{opt} increase with the fading parameter σ_χ^2, for $N = 100$ computations/bit. As an example, suppose that the probe has descended to the surface of Venus and is transmitting to Earth in a direction 60° off the vertical; if we assume a spherical atmospheric envelope 55 km in depth, and take the radius of Venus to be 6050 km, we compute an atmospheric path length L of 109 km. Using Eq. (1), this yields a fading parameter $\sigma_\chi^2 = 0.049$. From the figure, we see that this amount of fading degrades the link by 2.9 dB. Of course, σ_χ^2 would only reach 0.49 at the end of the probe's 2-hour descent; initially, $\sigma_\chi^2 = 0$. For simplicity, we would not want to design the probe telemetry system to permit the modulation index θ to vary continually so as to be always optimized over variations in σ_χ^2. Since the required P_T/N_0 is highest at maximum σ_χ^2, suppose we fix $\theta = 64.3°$ corresponding to θ_{opt} at $\sigma_\chi^2 = 0.049$. Because the required P_T/N_0 varies slowly with θ near its minimum for a given σ_χ^2, the resulting required P_T/N_0 (dashed curve in Fig. 3) is not significantly higher than in the optimal case (e.g., less than 0.2-dB loss at $\sigma_\chi^2 = 0$).

Figures 4 and 5 are the same as Fig. 3, except that $N = 1000$ and 2000 computations/bit, respectively. At $\sigma_\chi^2 = 0.049$, the P_T/N_0 requirement drops by 0.35 dB and 0.42 dB, respectively, due to the increase in N.

Finally, we will discuss the interpretation of these results for telecommunications design control table (Table 1) applications. As an example, we will use information obtained from Figs. 1–3 to form the DCT (shown below) for the real-time decoding case ($N = 100$ computations/bit) with a system optimized for $\sigma_\chi^2 = 0.049$.

Parameter 1 is a current estimate by the PV'78 Project of the available P_T/N_0, based on the indicated transmitter power, receiver noise temperature, and receiving antenna elevation; this estimate is also based on an assumed probe-to-Earth range of 5.75×10^7 km, transmitting and receiving antenna gains of 1.5 and 61.7 dB (64-m antenna), respectively, and some estimated system losses (e.g., transmitter circuit and pointing losses, polarization loss, etc.). This quoted value is subject to change as the design develops, and this will cause a corresponding change in the performance margins.

Next consider the DCT telemetry performance entries. From Fig. 1 we find that the ρ required to achieve a deletion rate of 10^{-2} in the absence of fading, noisy reference, and system losses is 2.58 dB: this is item 9. Figure 3 tells us that we should set $\theta = 64.3°$ to minimize the required P_T/N_0 when $\sigma_\chi^2 = 0.49$. Figure 2 shows that if we neglect the fading, a system with $\theta = 64.3°$ (suboptimal) requires $P_T/N_0 = 31.74$ dB, including system losses L_D and L_C. Using Eq. (2), we find that the effect of the noisy reference is to increase ρ to 4.68 dB: the noisy reference loss (item 10) is then given by $4.68-2.58 = 2.10$ dB. Returning to Fig. 3, we see that the fading increases the required P_T/N_0 to 34.49 dB (item 15), so that the fading loss is simply $34.49-31.74 = 2.75$ dB (item 11). The telemetry performance margin is the difference between the available and required values of P_T/N_0, or 2.45 dB.

Now consider the carrier performance entries in the DCT. Item 2 is the required signal-to-noise ratio in the (two-sided) threshold loop bandwidth $2B_{L0}$, in the absence of fading and system (predetection recording) losses. For telemetry purposes, this parameter has already been fixed: using Eq. (3) with $P_T/N_0 = 31.74$ dB, we find that $\eta = 12.69$ dB. If we entered this value in item 2, and a 2.75-dB fading loss in item 3, we would have 34.49 dB as the required P_T/N_0 in item 7 and a 2.45-dB margin in item 8. However, instead of gauging the performance of the tracking loop according to its noisy reference telemetry requirements, we will use this section of the DCT to examine it from another point of view. The very long baseline interferometry (VLBI) experiment requires a very clean received carrier, with low phase jitter σ_ϕ^2. Suppose an η of 10 dB in the absence of fading will yield an acceptable σ_ϕ^2: enter this value of η in item 2. Using Eq. (A-3) from the Appendix, we find that $\eta = 10$ dB results in $\sigma_\phi^2 = 0.1416$ rad², with $\sigma_\chi^2 = 0$. To achieve this same σ_ϕ^2 for arbitrary σ_χ^2, we can deduce from Eq. (A-3) that we need

$$\eta \cong (10 + 14.1\,\sigma_\chi^2)\ \text{dB} \qquad (4)$$

at $\sigma_\chi^2 = 0.049$, this reduces to $\eta = 10.69$ dB. Therefore, the fading loss is 0.69 dB (item 3). To achieve this performance level, the DCT indicates a required P_T/N_0 of 29.74 dB (item 7), yielding a margin of 7.20 dB (item 8).

Telemetry performance margins for other values of N and σ_χ^2 can be computed from the values of required P_T/N_0 in Figs. 3–5, and the equation

$$\text{margin} = 36.94\ \text{dB} - (\text{required } P_T/N_0)\ \text{dB} \qquad (5)$$

References

1. Levitt, B. K., "Performance Degradation of Uncoded and Sequentially Decoded PSK Systems Due to Log-Normal Fading," in *The Deep Space Network Progress Report 42-23*, pp. 58–67, Jet Propulsion Laboratory, Pasadena, Calif., Oct. 15, 1974.

2. Woo, R., et al., *Effects of Turbulence in the Atmosphere of Venus On Pioneer Venus Radio—Phase 1*, Technical Memorandum 33-644, Jet Propulsion Laboratory, Pasadena, Calif., June 30, 1973.

3. Layland, J. W., "A Model for Sequential Decoding Overflow Due to a Noisy Carrier Reference," *Proceedings of the International Telemetering Conference* (ITC), Oct. 15–17, 1974, Los Angeles, Calif.

4. Stolle, E., Deutsche Forschungs Versuchsanstalt fur Luft und Raumfahrt, Private communication given at Helios Working Group Splinter Session, September 27, 1973.

5. Tausworthe, R. C., *Theory and Practical Design of Phase-Locked Receivers*, Vol. 1, Technical Report 32-819, Jet Propulsion Laboratory, Pasadena, Calif., Feb. 15, 1966.

Table 1. Telecommunications design control table

	Item	Parameter	Nominal value, dB	Comments
	1	Available P_T/N_0	36.94	XMTR: 41.9 watts RCVR: 26.1 K, 30° elevation
Carrier performance	2	Threshold SNR, η	10.00	VLBI requirement
	3	Fading loss	0.69	$\sigma_\chi^2 = 0.049$
	4	System loss, L_C	1.00	
	5	$2B_{L0}$	10.79	12 Hz
	6	$\mathrm{Cos}^2\theta$	−7.26	$\theta = 64.3°$
	7	Required P_T/N_0	29.74	Item 2 + item 4 + item 5 − item 6
	8	Margin	7.20	Item 1 − item 7
Telemetry performance	9	Ideal ρ (lossless)	2.58	Deletion rate = 10^{-2}
	10	Noisy reference	2.10	
	11	Fading loss	2.75	$\sigma_\chi^2 = 0.049$
	12	System loss, L_D	2.08	
	13	Rate, R_B	24.08	256 bps
	14	$\mathrm{Sin}^2\theta$	−0.90	$\theta = 64.3°$
	15	Required P_T/N_0	34.49	Item 9 + item 10 + item 11 + item 12 + item 13 − item 14
	16	Margin	2.45	Item 1 − item 15

Telemetry mode: PV'78 large probe real-time telemetry link (N = 100 computations/bit) at 256 bps, using sequential decoding.

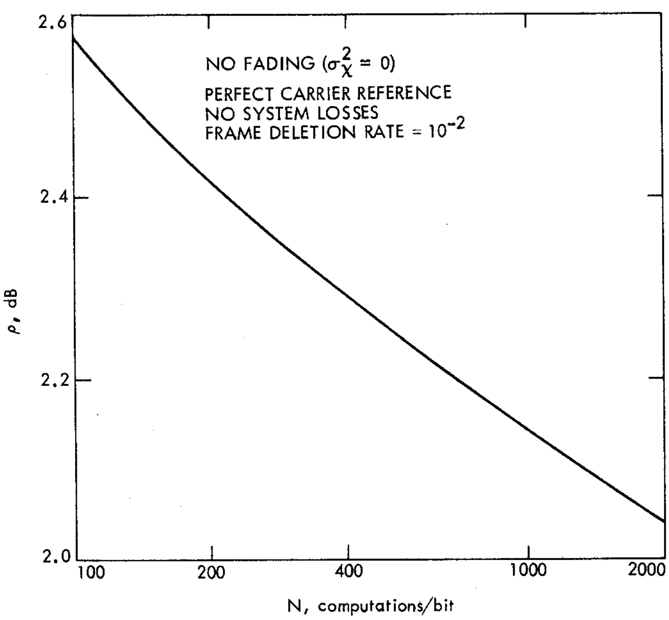

Fig. 1. Received bit energy-to-noise ratio ρ, required to achieve a frame deletion rate of 10^{-2}, as a function of sequential decoder computational capacity N, neglecting fading, noisy carrier reference, and system losses

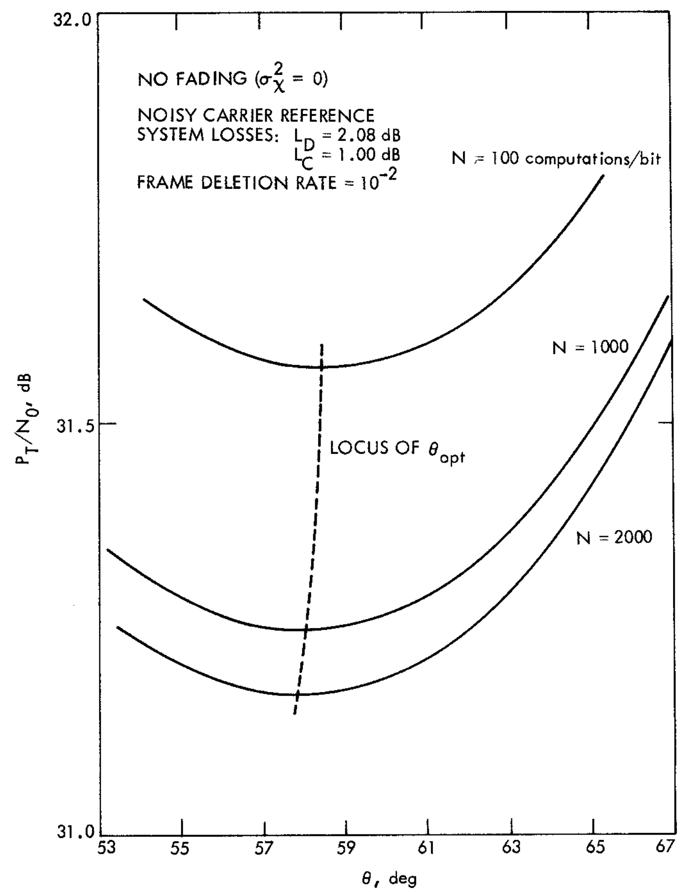

Fig. 2. Total received signal power-to-noise ratio P_T/N_0, required to achieve a deletion rate of 10^{-2}, as a function of modulation index θ, for a given computational capacity N, neglecting atmospheric fading

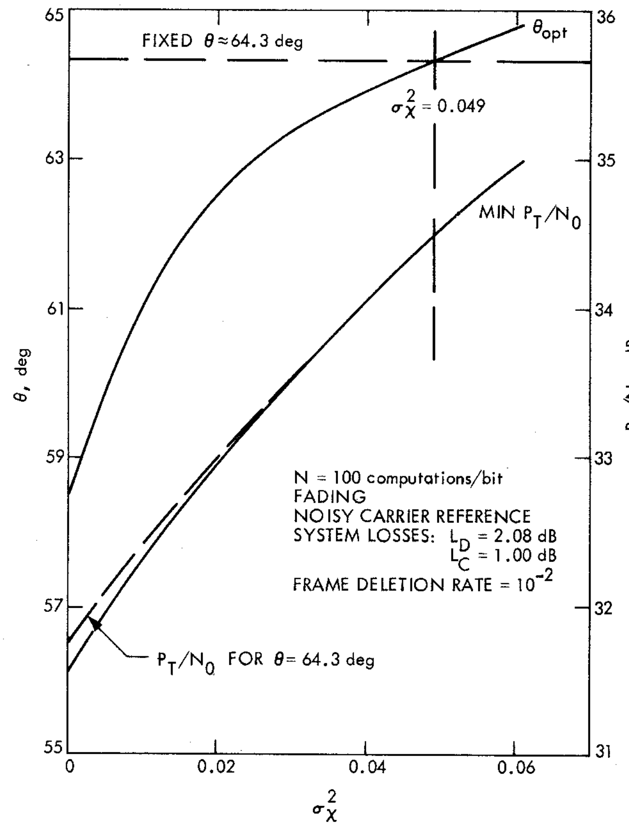

Fig. 3. Minimum received signal-to-noise ratio P_T/N_0 and corresponding optimum modulation index θ, required to achieve a deletion rate of 10^{-2}, as a function of fading variance σ^2_x, when computational capacity $N = 100$ computations/bit

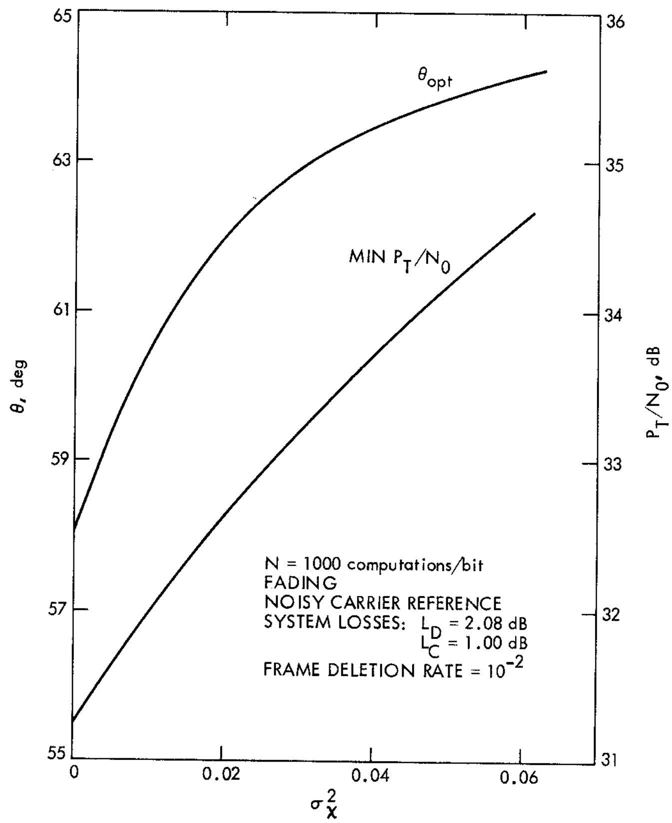

Fig. 4. Same as Fig. 3, except $N = 1000$ computations/bit

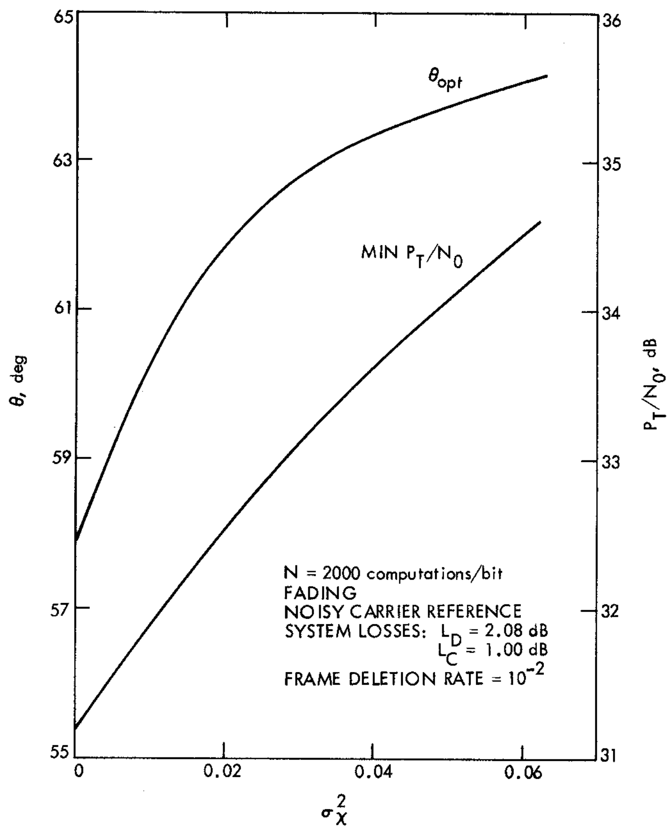

Fig. 5. Same as Fig. 3, except N = 2000 computations/bit

Appendix

Phase Jitter of a Linearized Second-Order Phase-Locked Loop Preceded by a Bandpass Limiter in the Presence of Lognormal Fading

Suppose the input to a second-order phase-locked loop receiver is degraded by lognormal amplitude fading, represented by the random variable e^{χ} at some instant of time. It is assumed that the threshold loop parameter ratio $r_0 = 2$, and the bandpass limiter preceding the phase-locked loop is operating in the limiter suppression region (limiter input signal-to-noise ratio $\eta_{\ell} << 1$). Then, linear phase-locked loop theory has shown (Ref. 1, Eq. A-15) that the effective signal-to-noise ratio in the (one-sided) operating loop bandwidth B_L has the form

$$\rho_L \cong \frac{5.172\,\eta\, e^{2\chi}}{2\sqrt{\eta}\, e^{\chi} + 1} \tag{A-1}$$

where η is the signal-to-noise ratio in the (two-sided) threshold loop bandwidth $2B_{L0}$.

Conditioned on ρ_L, the variance of the loop phase error, called phase "jitter," is approximated by (Ref. 5, Eq. 8–17)

$$\sigma^2_{\phi} \cong \frac{1}{\rho_L} \tag{A-2}$$

using the linear model for the loop. If the mean of the Gaussian random variable χ is the negative of its variance σ^2_{χ} (Ref. 2, Eq. 8), the expected phase jitter is then given by

$$\begin{aligned}\overline{\sigma^2_{\phi}} &\cong \frac{2\sqrt{\eta}\,\overline{e^{-\chi}} + \overline{e^{-2\chi}}}{5.172\eta} \\ &= \frac{2\sqrt{\eta}\exp(3\sigma^2_{\chi}/2) + \exp(4\sigma^2_{\chi})}{5.172\eta}\end{aligned} \tag{A-3}$$